STATSTICAL DISTRIBUTIONS OF THE RESONANCE PARAMETERS

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ABSTRACT

The random statistics analysis of resonance width distributions was performed. The unequal contributions of individual channels forming these widths is taken into account. The analytic forms of the corresponding functions for two as well as for three channels are derived. In general case the distribution density function is presented in the integral form. The simple relations for the moments of the function of density distribution in dependence of the contributions of various channels in the total (summary) width are found. The moments of the distribution function form a system of equations for the coefficients characterizing the relative contribution of channels in the average width and can be estimated if the values of the corresponding moments are known for a sufficiently large parameter set. The variant with a big number of channels for one part of the resonance width and one or two separate channels for the remainder is of interest too. This corresponds, for example, to the problem of estimation of (n,γf)-process contribution in the total fission width. The obtained distribution functions are used for a generalized statistical analysis of the fission widths of ^{235}U resolved resonances, determined by the ORNL group. The preliminary results of this analysis are presented.

The neutron resonances are outstanding from the scientific point of view as long living nuclear states, which enable the study of various processes - neutron capture, nuclear fission, neutron scattering, charged particle emitting nuclear reactions realized from the resonant states with known energy, spin and partial resonance widths - the resonance parameters. The significant practical meaning of the knowledge of resonance parameters stimulates further investigations on the resonance analysis of the neutron cross sections and the statistical properties of resonance parameters. The last can open a useful link from the neutron resonance field treated preferably as a practical area to the study of nuclear structure. The creation and practical application in the resonance analysis of a large set of a experimental data of the fitting code SAMMY[1], which uses Rich-Moore formalism and the generalized least squares method, is a remarkable achievement. The results about ^{235}U are very impressive revealing the resolved resonance structure for this nucleus in a wide energy interval[2]. We are presenting here the results of an attempt to perform the statistical analysis of the set of the obtained resonance widths.

The physical concept of compound resonances involves the hypothesis about the random (Gaussian) distribution of the resonance width amplitudes $\gamma_{\lambda c}$ for sufficiently large in statistical sense resonance level set. This hypothesis can be considered as a consequence of the statistical central limit theorem in application to the problem of the creation (decay) in the channel c of the complicate multicomponent compound states λ. The fluctuations of the resonance widths in the separate channels $\Gamma_{\lambda c} = p_c \gamma^2_{\lambda c}$ ($p_c(E)$ - are the penetration factors) in the frame of this hypothesis must obey the χ^2_1 distribution or the distribution of Porter-Thomas[3].

$$P_1(x_c)dx_c = \frac{1}{\sqrt{2\pi x_c}} \exp(-x_c/2)dx_c \,, \tag{1}$$

where $x_c = \gamma^2_{\lambda c} \big/ \overline{\gamma^2_c}$. The data about resonance neutron widths agree rather well with this distribution.

The extension of the hypothesis of the amplitudes random distribution on the problem of the fission widths fluctuation analysis, where the several (ν) independent fission channels are effective, is realized by the usual χ^2_ν-distribution, or the distribution of Porter-Tomas

$$P_\nu(x)dx = \left(\frac{\nu x}{2}\right)^{(\nu-2)/2} \exp\left(-\frac{\nu x}{2}\right) \Gamma^{-1}\left(\frac{\nu}{2}\right) dx \,, \tag{2}$$

where

$$x = \frac{1}{\nu} \sum_{c(f)=1}^{\nu} x_c \tag{3}$$

For the fission over threshold the energy dependence of the penetration factor in the fission channels is unessential and practically we have:

$$x = \Gamma_{\lambda f} / \overline{\Gamma_f} = \sum_{c(f)=1}^{\nu} \Gamma_{\lambda c} / \overline{\Gamma_c}, \qquad \overline{\Gamma_f} = \sum_{c(f)=1}^{\nu} \overline{\Gamma_c} = \nu \overline{\Gamma_c}$$

The relative contribution of different channels here is supposed to be equal and the channel number can be estimated by means of the dispersion of the analyzing set of the fission widths (for fixed total number)

$$\overline{x^2} - 1 = 2/\nu \tag{4}$$

The channel number can be determined also by using the position of the distribution maximum (for $\nu \geq 3$)

$$x_{max} = (\nu - 2)/2$$

The method for statistical interpretation of the observed statistical fluctuations of the resonance fission widths can be modified. This is actual now, when the big parameter sets derived from the experimental data are available - about 400 resonances for ^{239}Pu, more than 3000 for ^{235}U and the total moments of these are identified. The following distribution, more generalized than (2) had been created for the variant with

$$x = \sum_{c(f)=1}^{\nu} \beta_c x_c, \qquad x_c = \Gamma_{\lambda c} / \overline{\Gamma_c}, \qquad \beta_c = \overline{\Gamma_c} / \overline{\Gamma_f}, \tag{5}$$

where the possibility of unequal contribution β_c of different channels in the total width has been taken into account. The characteristic function (Laplace transform) of this distribution, let us note it as $\wp(\beta_1,...\beta_\nu, x)$, has been found in the following form[4]

$$\Phi(p) = \int_0^{\infty} e^{-px} \wp(\beta_1,...\beta_\nu, x) dx = \prod_{c=1}^{\nu} e^{-\beta_c x_c p} P_1(x_c) dx_c = \prod_{c=1}^{\nu} (1 + 2\beta_c p)^{-1/2}, \tag{6}$$

and the original in general case is the convolution of the transforms in (6)

$$\wp(\beta_1,...\beta_\nu, x) = \int_0^{x} P_1\left(\frac{t_1}{\beta_1}\right)\frac{dt_1}{\beta_1} \int_0^{x-t_1} P_1\left(\frac{t_2}{\beta_2}\right)\frac{dt_2}{\beta_2} ... \int_0^{x-t_1-...t_{\nu-2}} P_1\left(\frac{t_{\nu-1}}{\beta_{\nu-1}}\right)\frac{dt_{\nu-1}}{\beta_{\nu-1}\beta_\nu} P_1\left(\frac{x - t_1 - ...t_{\nu-1}}{\beta_\nu}\right) \tag{7}$$

The characteristic function $\Phi(p)$ determines the moments of the distribution as the corresponding derivatives of (6) at p=0:

$$\overline{x} = \sum_{c=1}^{\nu} \beta_c = 1, \qquad \overline{x^2} = 1 + 2\sum_{c=1}^{\nu} \beta_c \ldots \tag{8}$$

It can be easily seen, that in the case of equal $\beta_c = 1/\nu$ the modified distribution coincides with (2).

For two channels, as seen in reference 5, the inverse transform of the corresponding function $\Phi(p)$ is

$$\wp(\beta_1, \beta_2, x) = \frac{1}{2\sqrt{\beta_1\beta_2}} \exp(-x/4\beta_1\beta_2)\, I_0\left(-\frac{\beta_1 - \beta_2}{4\beta_1\beta_2} x\right), \tag{9}$$

and in the limit case $\beta_2 << 1$ we will have $P_1(x)$ distribution (1), as well as at $\beta_1 = \beta_2 = 1/2$ the distribution $P_2(x)$ - Figure1

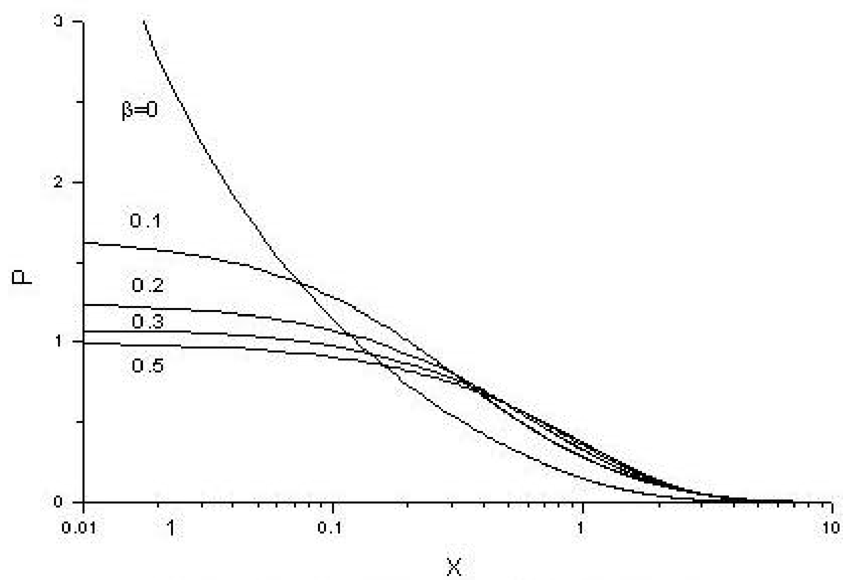

Fig.1 Modified distribution for ν=2 at $\beta_1=\beta$, $\beta_2=1-\beta$

In the three channels case $\beta_1 = \beta_2 = \beta$, $\beta_1 = 1 - 2\beta$ we obtain

$$\wp(\beta_1, \beta, \beta, x) = \left(\frac{x}{2\pi\beta_1\beta^2}\right)^{1/2} \exp\left(-\frac{x}{2\beta}\right) \int_0^1 dt\, \exp\left(-\frac{\beta - \beta_1}{2\beta\beta_1} x t^2\right) \tag{10}$$

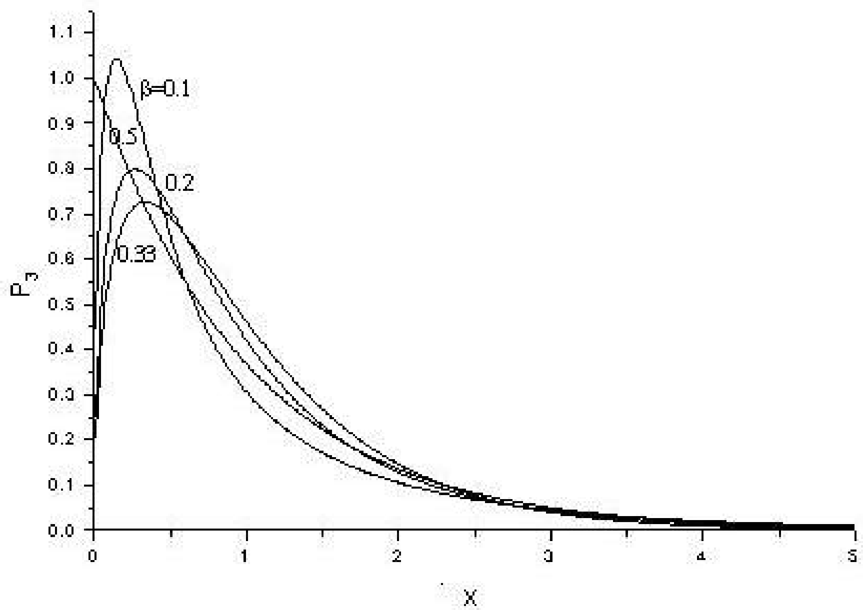

Fig.2 Modified distribution for ν=3 at $\beta_1=\beta_2=\beta$, $\beta_3=1-2\beta$

The variance with the predominance of 1-3 channels and a big number of weak channels, corresponding e.g. to the (n, γf) process is interesting too. Here the characteristic function (6) can be presented approximately as

$$\Phi(p) \approx e^{-\varepsilon p} \prod_{c'=1}^{\nu_c} (1 + 2\beta_{c'} p)^{-1/2} \; ,$$

where ε is the summary relative contribution of the weak channels in the average width, and the index c' is relative to the isolated "strong" ν_0 channels (direct fission), so that

$$\varepsilon + \sum_{c'=1}^{\nu_0} \beta_{c'} = 1$$

The corresponding distribution function is evidently

$$\wp(\beta_1, \ldots \beta_{\nu_0}, x - \varepsilon) \qquad \text{for} \qquad x > \varepsilon$$

$$0 \qquad \text{for} \qquad x < \varepsilon$$

The distributions of the resonance widths, presented in the Reference 2, are shown on Figure 3 for the levels with the J - spin value equal to 3 and on Figure 4 for these with spin value equal to 4. Generally the full set of the resolved levels has been included into the consideration. We rejected only small number of levels - the levels with negative values of energy and the

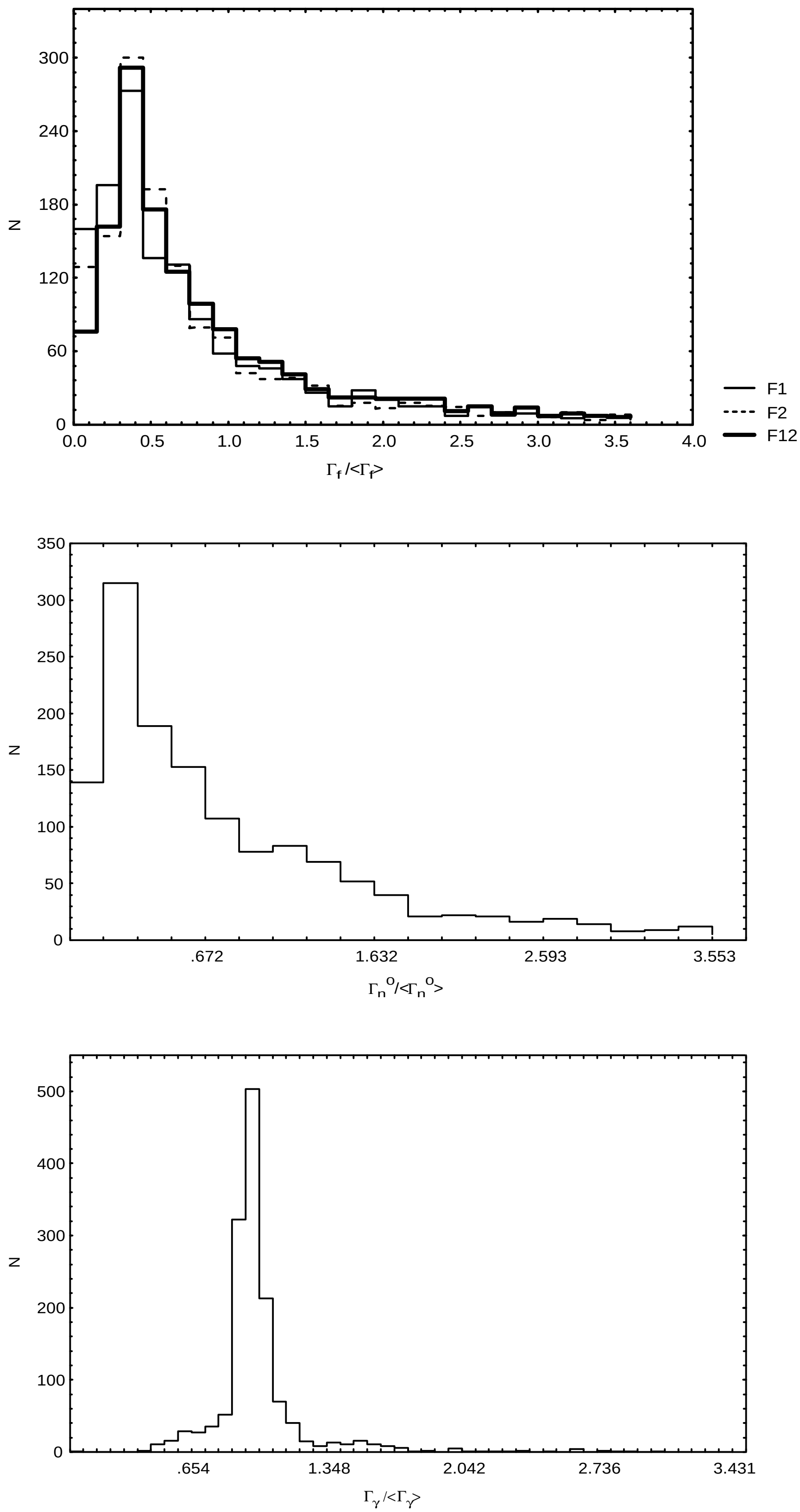

Figure 3. The distribution of the spin 3 widths-fission (upper), neutron (middle) and radiation (lower) graph

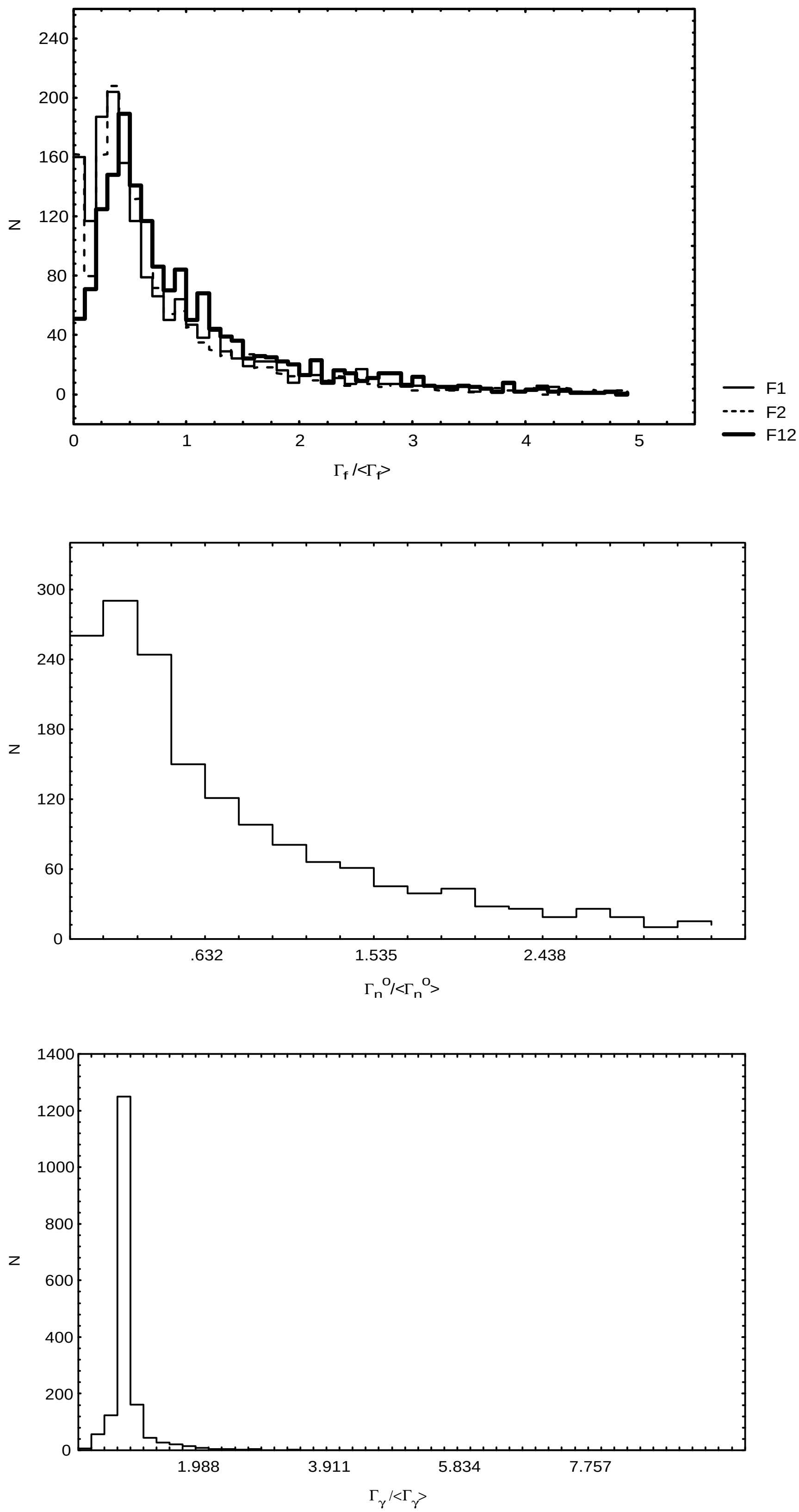

Figure 4. The distribution of the spin 4 widths-fission (upper), neutron (middle) and radiation (lower) graph

levels with very big fission and neutron widths. We considered these as added for the needs of multilevel analysis. The channel number has been determined for each one of the width sets presented on the figures 3 and 4. The value of ν is close to 1 for neutron widths and for each one of the two fission widths corresponding to given J. The ν is close to 2 for the distribution of the total fission width. The ν for the distributions of the radiation capture widths is big, which is expected.

Some preliminary conclusions can be done considering the shape of the distributions shown on the Figures 3 and 4. In the frame of the random statistics these distributions are close to the assumption for one neutron channel and two fission channels. The widths for each of the two assumed channels, derived in the multilevel analysis, probably have not very clear physical interpretation. If we treat these as partial fission widths the distribution suggest the availability of more fission channels. Naturally all these interpretation is connected strongly with the problem of the uniqueness of the multilevel parameters and the dependence of this on the energy [6].

In the region of the unresolved levels the distribution function of the resonance parameters are needed for the determination of the resonance averaged cross sections and the cross section functionals like averaged transmission <exp(-nσ)> as well as the resonance self-shielding factors. Here the resonance cross section structure can be unfolded only by a modelling, which can be done by using statistical approaches[7]. The corresponding fluctuation coefficients, however, are estimated as a rule in the approximation of SLBW (single level Breit-Wigner).

The modified distributions of resonance parameters can be used also for the estimation of the resonance width fluctuations in the resonance averaged cross sections. In the traditional scheme of the averaging this is taken into account by the factor F_{nc} in the Hauser-Feshbach formula

$$\overline{\sigma_{nc}}^{J} = \pi k^{-2} g(J) \frac{T_n T_c}{T} F_{nc} \qquad (T_c = 2\pi\Gamma_c / D_J),$$

where, as is seen in Reference 8, in the SLBW-approximation

$$F_{nc} = \int_0^\infty dp\,(1+2pT_n/T)^{-3/2}(1+2pT_c/T)^{-3/2}\prod_{c'\neq c}(1+2pT_{c'}/T)^{-1/2} e^{-T_\gamma/T}. \qquad (11)$$

Here is considered the case of 1 neutron channel and several fission channels c(c') in competition with the multichannel radiation capture ($T = \sum_c T_c + T_n + T_\gamma$). Then the average fission cross section summarised over all own channels can be presented as

$$\sigma_{nf}^{J} = \sum_{c(f)} \sigma_{nc}^{J} = \pi k^{-2} g(J) \frac{T_n T_f}{T} F_{nf} \qquad (T_f = \sum_{c(f)} T_c) \qquad (12)$$

where

$$F_{nf} = \sum_{c(f)} \frac{T_c}{T_f} F_{nc}$$

This is equivalent to the result of averaging, which uses the corresponding distributions of the summary fission widths.

However, for the estimation of the averaged cross sections in the unresolved region it is more convenient to separate different reaction channels, except non fluctuating widths of the radiation capture (and probably this of (n, γf) process). Then in each channel we will have one-channel distribution density. This permits to standardise the presentation of the fluctuation factors keeping the correctness of the mathematical scheme and without using special distribution functions for different reactions. The expression for the fluctuation factors are known for the case of one-channel scattering and multichannel radiation capture, the two-channel problem - one neutron and one fission channel probably plus multicascade radiation capture (or two neutron channels)[8]. Two identical variants for the determination of fluctuation factors are possible - as a twofold integral with two-channel distribution function for the fission widths or separately for each one of the channels. The advantage of channel by channel accounting of the fluctuation factors in calculating the average cross section is in the uniform algorithmization of these.

The similar factor for the cross section of the radiation capture is determining as an integral

$$F_{n\gamma} = \int_0^\infty dp\,(1+2pT_n/T)^{-3/2} \prod_c (1+2pT_c/T)^{-1/2}\, e^{-pT_\gamma/T}$$

The SLBW-approximation, used here seems to be dubious in application to the fissile nuclei. However the more rigorous accounting of the fluctuations in averaging the multilevel (R-matrix) cross section formulas gives rather close values for the corresponding factors F_{cn}, when using the approximate results.

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