AN ASSESMENT OF CONSISTENT BILINEAR WEIGHTED TWO-GROUP SPATIAL KINETICS FOR MOX FUEL APPLICATIONS

Changho Lee, T. J. Downar, Karl O. Ott
School of Nuclear Engineering
Purdue University
W. Lafayette, IN 47907-1290
changho@ecn.purdue.edu

Han G. Joo
Korea Atomic Energy Research Institute
150 Dukjin-Dong, Yusung-Ku, Taejon, Korea
joohan@nanum.kaeri.re.kr

ABSTRACT

The bilinear weighted 2-group kinetics equations and the consistent and inconsistent formulations of the linear weighted 2-group kinetics equations have been analyzed with a one-dimensional, fine-mesh diffusion code for MOX and UO2 applications. A 97-group consistent formulation of the kinetics equations is used as the reference. The steady-state comparisons show that there is no significant difference between the linear and bilinear equations. In the transient case, however, there is a large discrepancy between the reference and the linear weighted 2-group results when using the beta-physical. The discrepancies were significantly reduced when the beta-physical was replaced by the beta-effective, even though the 2-group beta-effective formulation is inconsistent. The bilinear weighted 2-group formulation with beta-physical also shows good agreement with the reference result, as do the linear weighted 4-, 8-, and 22-group results using the consistent formulation with beta-physical.

1.0 INTRODUCTION

The energy collapse of fine group cross section data into few group constants for light water reactor analysis is traditionally performed with simple linear flux weighting. Several previous authors have investigated the merits of bilinear energy collapsing in which both the real and adjoint fine group fluxes are used to form the few group constants[1-6]. Because of the recent interests in irradiating MOX fuel in light water reactors, the merits of bilinear weighted group constants were reexamined and the results are reported in this paper.

As reported by several previous authors, the linear weighting energy collapse method preserves eigenvalue, flux spectrum, and reaction rate but does not preserve the adjoint spectrum or the reactivity. On the contrary, bilinear weighting preserves all quantities except the reaction rate.

One of the primary motivations for previous bilinear studies has been that the bilinear weighting approach preserves reactivity worth which is particularly important for kinetics analysis. Much of the early bilinear research was related to fast breeder reactors (FBRs) for which superprompt critical transient analysis was a particular concern. There has been bilinear research[6] specifically for Light Water Reactors (LWRs) loaded with UO2 fuel. However, the results were not conclusive since only a restrictive range of core conditions was considered.

Because the delayed neutron fraction of MOX fuel is significantly smaller than cores loaded with uranium dioxide, the accuracy of predicting the magnitude of the delayed neutron fraction becomes particularly important in analyzing the transient behavior of MOX fueled cores. The current procedure for solving the 2 group spatial kinetics equations for LWR transient analysis is to employ group constants which are collapsed using linear flux weighting. However, the delayed neutron fraction used in the 2 group spatial kinetics equations is weighted with both the real and adjoint in order to account for the difference in the importance of delayed and prompt neutrons. This introduces an inconsistency in 2 group spatial kinetics analysis and the research here is designed to investigate the impact of this inconsistency for cores partially loaded with MOX fuel. In addition, the use of more than 2 energy groups is also investigated.

2.0 TWO-GROUP KINETICS EQUATIONS

2.1 LINEAR WEIGHTING METHOD

Integrating the space-, energy-, and time-dependent diffusion equation over energy yields the traditional form of the two-group space and time-dependent diffusion equations[7-8]:

$$\frac{1}{\mathrm{v}_1}\frac{\partial f_1(r,t)}{\partial t} = (1-b)\sum_g \nu\Sigma_{fg}(r,t) f_g(r,t) + \nabla\cdot D_1(r,t)\nabla f_1(r,t) - \Sigma_{a1}(r,t) f_1(r,t) - \hat{\Sigma}_{12}(r,t) f_1(r,t) + \sum_k l_k C_k(r,t), \tag{1a}$$

$$\frac{1}{\mathrm{v}_2}\frac{\partial f_2(r,t)}{\partial t} = \nabla\cdot D_2(r,t)\nabla f_2(r,t) - \Sigma_{a2} f_2(r,t) + \hat{\Sigma}_{12} f_1(r,t), \tag{1b}$$

$$\frac{\partial C_k(r,t)}{\partial t} = -l_k C_k(r,t) + b_{dk}\sum_g \nu\Sigma_{fg}(r,t) f_g(r,t), \tag{1c}$$

where $f_g = \int_g j(E)dE$, $\mathrm{v}_g = \dfrac{\int \frac{1}{\mathrm{v}(E)} j(E)dE}{f_g}$, $D_g = \dfrac{\int_g D(E) j(E)dE}{f_g}$,

$\Sigma_{ig} = \dfrac{\int_g \Sigma_i(E) j(E)dE}{f_g}$, $\Sigma_{g'g} = \dfrac{\int_g \int_{g'} \Sigma_s(E'\rightarrow E) j(E')dE'dE}{f_{g'}}$,

$$b_{dk} = \frac{\int n_{dk}\Sigma_f(E)j\,(E)dE}{\int n\Sigma_f(E)j\,(E)dE}, \quad b = \sum_k b_{dk}\,.$$

For simplicity, the up-scattering term is included in the down-scattering cross section[9,10]:

$$\hat{\Sigma}_{12} = \Sigma_{12} - \Sigma_{21}\frac{f_2}{f_1} \tag{2}$$

It should be noted that in this 2-group formulation the delayed neutron fraction b_{dk} is defined without the fission source spectra since all neutrons are born in the fast energy group. This differs from the formulation of the multigroup effective delayed neutron fraction which contains the group-wise fission source spectra. For two energy groups, the formulations would be identical except for the adjoint weighting of the effective delayed neutron fraction. In this paper, b_{dk} will be called the "beta-physical" so that it can be distinguished from the "beta-effective" which will be discussed in the following section.

2.2 BILINEAR WEIGHTING METHOD

Unlike the linear weighting method, the bilinear weighting employs both the real and adjoint spectra for group collapsing:

$$\begin{aligned}\frac{1}{\mathrm{v}_1^*}\frac{\partial f_1(r,t)}{\partial t} &= (1-b_1^*)\sum_g \nu\Sigma_{fg}(r,t)f_g(r,t) + \nabla\cdot D_1^*(r,t)\nabla f_1(r,t) - \Sigma_{a1}^*(r,t)f_1(r,t) \\ &\quad - \Sigma_{12}^*(r,t)f_1(r,t) + \Delta\Sigma_{11}^*(r,t)f_1(r,t) + \Sigma_{21}^{**}(r,t)f_2(r,t) + \sum_k l_k V_{k1}(r,t)\end{aligned} \tag{3a}$$

$$\begin{aligned}\frac{1}{\mathrm{v}_2^*}\frac{\partial f_2(r,t)}{\partial t} &= \nabla\cdot D_2^*(r,t)\nabla f_2(r,t) - \Sigma_{a2}^*(r,t)f_2(r,t) \\ &\quad - \Sigma_{21}^*(r,t)f_2(r,t) + \Delta\Sigma_{22}^*(r,t)f_2(r,t) + \Sigma_{12}^{**}(r,t)f_1(r,t)\end{aligned} \tag{3b}$$

$$\frac{dV_{kg}(r,t)}{dt} = -l_k V_{kg}(r,t) + b_{dkg}^*\sum_g n\Sigma_{fg}(r,t)f_g(r,t) \tag{3c}$$

where $\dfrac{1}{\mathrm{v}_g^*} = \dfrac{\int_g j^*(E)\dfrac{1}{\mathrm{v}(E)}j\,(E)\,dE}{f_g}$, $D_g^* = \dfrac{\int_g j^*(E)D(E)\,j\,(E)\,dE}{f_g}$,

$$\Sigma^{*}_{ig} = \frac{\int_{g} j^{*}(E)\Sigma_{i}(E)\, j(E)\, dE}{f_g},$$

$$\Sigma^{*}_{g'g} = \frac{\int_{g} j^{*}(E')\int_{g'}\Sigma(E'\rightarrow E)\, j(E')\, dE'\, dE}{f_{g'}}, \quad (g \neq g')$$

$$\Sigma^{**}_{g'g} = \frac{\int_{g} j^{*}(E)\int_{g'}\Sigma(E'\rightarrow E)\, j(E')\, dE'\, dE}{f_{g'}}, \quad (g \neq g')$$

$$\Delta\Sigma^{*}_{gg} = \frac{\int_{g}\int_{g}[j^{*}(E) - j^{*}(E')]\Sigma(E'\rightarrow E)\, j(E')\, dE'\, dE}{f_g}, \quad V_{kg}(r,t) = \int_{g} c(E) j^{*}(E) dE\, C_k(r,t),$$

$$b^{*}_{dkg} = \frac{\int_{g} c_{dk}(E) j^{*}(E) dE \int_{0}^{\infty} n_{dk}\Sigma_f(E')\, j(E')\, dE'}{\int_{g} c(E) j^{*}(E) dE \int_{0}^{\infty} n\Sigma_f(E')\, j(E')\, dE'}, \quad b^{*}_{g} = \sum_{k} b^{*}_{dkg}.$$

Note that b^{*}_{dkg} is the group-wise effective delayed neutron fraction unlike that defined in Eq. (1). If the adjoint spectrum is constant over all the energies, the bilinear collapsed results would be the same as the linear flux weighting scheme. However, since the adjoint spectrum varies with energy, the within-group scattering terms become different on both sides of the equations. In addition, the down-scattering and up-scattering terms appearing in the fast and thermal equations are not the same as in the linear weighting formulation. The differences of the scattering terms can be treated as below in order to preserve the structure of the conventional two-group diffusion equations:

$$\Sigma^{**}_{a1} = \Sigma^{*}_{a1} - \Delta\Sigma^{*}_{11} - \Delta\Sigma^{*}_{12}, \quad \Sigma^{**}_{a2} = \Sigma^{*}_{a2} - \Delta\Sigma^{*}_{22} - \Delta\Sigma^{*}_{21}, \quad \hat{\Sigma}^{**}_{12} = \Sigma^{**}_{12} - \Sigma^{**}_{21}\frac{f_2}{f_1} \tag{4}$$

where $\Delta\Sigma^{*}_{gg'} = \Sigma^{*}_{gg'} - \Sigma^{**}_{gg'}, \ (g \neq g')$.

Eqs. (3a) and (3b) can then be simplified as:

$$\frac{1}{v^{*}_{1}}\frac{\partial f_1(r,t)}{\partial t} = (1 - b^{*}_{1})\sum_{g} v\Sigma_{fg}(r,t) f_g(r,t) + \nabla\cdot D^{*}_{1}(r,t)\nabla f_1(r,t) - \Sigma^{**}_{a1}(r,t) f_1(r,t) - \hat{\Sigma}^{**}_{12}(r,t) f_1(r,t) + \sum_{k} l_k V_{k1}(r,t) \tag{5a}$$

$$\frac{1}{v_2^*}\frac{\partial f_2(r,t)}{\partial t} = \nabla \cdot D_2^*(r,t)\nabla f_2(r,t) - \Sigma_{a2}^{**}(r,t)f_2(r,t) + \hat{\Sigma}_{12}^{**}(r,t)f_1(r,t) \quad (5b)$$

which are the same as Eqs. (1a) and (1b) of the linear weighting method in form. This means that the cross section structure of the conventional method can be kept for the bilinear weighting one without any modification.

In the linear weighted two-group equations, the net current at the interface between adjacent nodes, i and $i+1$, is determined using the current continuity condition:

$$J_{i^+} = J_{i+1^-} = -\tilde{D}_{i,i+1}(f_{i+1} - f_i) \quad (6)$$

where $\tilde{D}_{i,i+1} = \dfrac{2D_{i+1}D_i}{D_{i+1}\Delta x_i + D_i \Delta x_{i+1}}$.

However, if the bilinear weighted cross section is used, the current is no longer continuous at the interface between different compositions since the bilinear diffusion coefficient has been changed arbitrarily with the adjoint weighting:

$$J_{i^+}^* \neq J_{i+1^-}^* \quad (7)$$

Since only the flux is continuous at the interface, $\tilde{D}_{i,i+1}^*$ must be defined differently for each side of the interface:

$$\tilde{D}_{i^+}^* = \frac{2D_{i+1}D_i^*}{D_{i+1}\Delta x_i + D_i \Delta x_{i+1}}, \quad \tilde{D}_{i+1^-}^* = \frac{2D_{i+1}^*D_i}{D_{i+1}\Delta x_i + D_i \Delta x_{i+1}}. \quad (8)$$

This means that the linear weighted diffusion coefficient is still required for the current continuity condition in the bilinear approach.

3.0 RESULTS

The lattice transport calculation for generating group constants for a single fuel assembly is an eigenvalue problem with reflective boundary conditions. A leakage spectrum is obtained in the fundamental mode calculation as a B^2-mode eigenvalue problem. The adjoint spectrum is calculated by simply transposing cross section matrices during the fundamental mode calculation. Cross sections for a few energy groups were computed using the leakage spectra for both the real and adjoint fluxes.

In order to verify the equations shown in the previous sections, a fine-group one-dimensional fine-mesh finite difference code (FGOD) was developed and used to perform the steady-state and transient calculations with Doppler temperature feedback. The steady-state results are first compared for the linear and bilinear weighted methods using homogeneous and checkerboard configurations of UO2 and MOX fuels. Table I shows the error in the eigenvalue prediction for

both approaches when compared to the 97-group reference results. Even though agreement of the bilinear weighting method is a little better than the linear weighting method, the results basically suggest there is no significant difference between the linear and bilinear weighting methods in the steady state.

The 2-group time-dependent scheme of FGOD was validated using the PARCS code which has been well-benchmarked[11]. A comparison of transient results between FGOD and PARCS for a superprompt critical control rod ejection using two energy groups was in very good agreement. Several superprompt critical transients were then performed with FGOD to compare the linear and bilinear weighting methods. In the event modeled here, the control rod is fully inserted and then ejected in 0.1 sec resulting in superprompt criticality. The 97-group cross sections were modified by the insertion fraction of the control rod in order to simulate control rod movement.

The transient power predictions for the consistent and inconsistent formulations of the 2-group linear weighted method, the 2-group bilinear weighted method, and the 97-group "reference" are compared in Figs. 1 and 2. In all cases significant differences were observed between the reference and the 2-group results with beta-physical. In general, replacing the beta-physical by the beta-effective as defined in Eq. (3) resulted in a more accurate 2-group result. As shown in the figures, the bilinear weighted results are in close agreement with the reference in all cases.

An important sensitivity was observed in the inconsistent 2-group results. The accuracy achievable using the beta-effective depended on the choice of the adjoint spectrum used to compute the beta-effective. The transient results shown in Figs. 1 and 2 used a leakage adjoint spectrum, whereas those shown in Figs. 3 and 4 were determined using a beta-effective computed with an infinite medium adjoint spectrum. The beta-effective computed with the leakage spectrum is larger than the beta-effective computed with the infinite medium spectrum since the relative importance of the delayed neutrons is greater in the leakage spectrum. Because the beta-effective is larger with the leakage spectrum, the peak in the power prediction is lower and occurs later in the transient as observed by comparing Figs. 1 and 3 or Figs. 2 and 4. As reported earlier[12], there currently appears to be some ambiguity regarding the proper choice of spectrum for computing the beta-effective. However, there is no ambiguity in the consistent bilinear results since the leakage adjoint is always used in preparing the group constants and kinetics parameters.

A comparison of the energy depositions for the cases shown in Figs. 1-4 is provided in Table II. The impact of using more than 2 energy groups in the consistent linear weighted method was also investigated. The transient results are shown in Fig. 5 when the number of energy groups is increased to 4, 8, and 22 using the beta-physical. The energy cutoffs were chosen such that Pu resonances were isolated. An improvement in the 2-group results is achieved with a 4-group structure because the prompt and delayed neutrons can be distinguished with 4 energy groups.

CONCLUSIONS

The bilinear weighted 2-group kinetics equations and the consistent and inconsistent formulations of the linear weighted 2-group kinetics equations were analyzed with a one-dimensional, fine-mesh diffusion code for MOX and UO2 applications. A 97-group consistent formulation of the kinetics equations was used as the reference. The steady-state comparisons show that there is no significant difference between the linear and bilinear equations. In the transient case, however, there were significant discrepancies between the reference and the linear weighted 2-

group results when using the beta-physical. The discrepancies were significantly reduced when the beta-physical was replaced by the beta-effective, even though the 2-group beta-effective formulation is inconsistent. However, the accuracy of the inconsistent 2-group results with the beta-effective was observed to be very sensitive to the method used for computing the adjoint spectrum. The bilinear weighted 2-group formulation with beta-physical showed good agreement with the reference result, as did the linear weighted 4-, 8-, and 22-group results using the consistent formulation with beta-physical.

REFERENCES

1. T. A. Pitterle and C. W. Maynard, “Bilinear-Averaging for Diffusion Theory Parameters,” Trans. Am. Nucl. Soc., 8, 205, (1965).

2. A. F. Henry, “Few-Group Approximation Based on a Variational Principle,” Nucl. Sci. Eng., 27, 493-510, (1967).

3. A. J. Buslik, “Interface Conditions for Few-Group Neutron Diffusion Equations with Flux-Adjoint Weighted Constant,” Nucl. Sci. Eng., 32, 233-240, (1968).

4. E. Kiefhaber, “Improved Weighting Procedures for Energy Group Condensation in Nuclear Reactor Calculation,” Annual Meeting of Nuclear Technology 83, Bonn, Germany, 51-56, (1983).

5. D. C. Wade and R. G. Bucher, “Conservation of the Adjoint Neutron Spectrum by Use of Bilinear-Weighted Cross Sections and Its Effect on Fast Reactor Calculations,” Nucl. Sci. Eng., 64, 517-538, (1977).

6. Myung H. Kim and A. F. Henry, “Flux-Adjoint Weighted Few-Group Cross Sections Used for Reactor Transient Analysis,” Nucl. Sci. Eng., 103, 276-282 (1989).

7. Allan F. Henry, *Nuclear Reactor Analysis*, The MIT Press, Massachusetts (1975).

8. Karl O. Ott, *Introductory Nuclear Reactor Dynamics*, American Nuclear Society, Illinois (1985).

9. Dean B. Jones, et al., “CPM-3 Theory and Numerics Manual,” ERPI, RP-3418 (1997).

10. M. Edenius, H. Haggblom and B. H. Forssen, “CASMO-3 Methodology Manual,” STUDSVIK /NFA-89/2, Studsvik of American, Inc. (1989).

11. Han G. Joo et al, “PARCS: A Multi-Dimensional Two-Group Reactor Kinetics Code Based on the Nonlinear Analytic Nodal Method,“ PU/NE-98-26, Sept. (1998).

12. C. Lee, T. Downar, and D. Jones, “Implementation of the Effective Delayed Neutron Fraction with Adjoint Spectrum Weighting into CPM-3,” ANS Trans., Vol. 80, p. 278.

TABLE I. Comparison of Eigenvalues in the Steady-State
for Heterogeneous Cores with Zero-Flux Boundary Condition

Fuel Compositions	97-Group	Δk (pcm)	
		Normal 2-Group	Bilinear 2-Group
UO2 (2 w/o, 2500ppm) + UO2 (4 w/o, 2500ppm)	1.00392	57	8
MOX (7 w/o, 3000ppm) + UO2 (3 w/o, 3000ppm)	0.99979	145	91

TABLE II. % Error of Deposited Energy during the Transient

CASE	% Error (Reference: 97-Group)		
	2-Group (b -phy)	2-Group (b -eff)	Bilinear 2-Group
UO2 (2 w/o, 2500ppm)+ UO2 (4 w/o, 2500ppm)	-17.2	-8.2[1)]	-3.7
		1.0[2)]	
MOX (7 w/o, 3000ppm) + UO2 (3 w/o, 3000ppm)	-12.4	-0.6[1)]	-0.3
		8.8[2)]	

1) Leakage adjoint spectrum is used for b -effective

2) Infinite-medium adjoint spectrum is used for b -effective

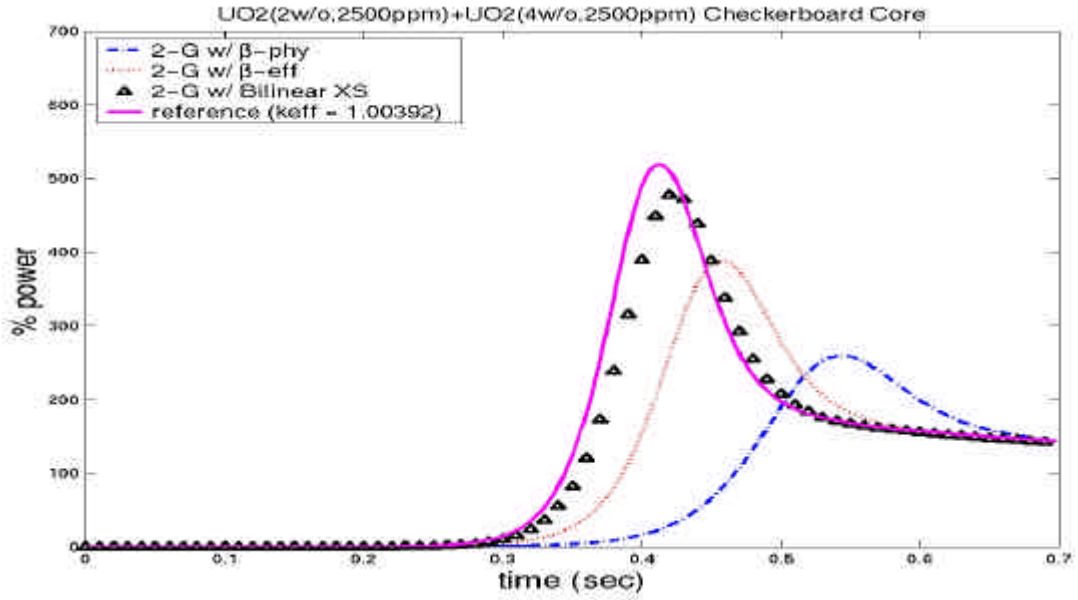

Figure 1. Rod Ejection Transient Results for UO2/UO2 Heterogeneous Core (r =1.07$)
(Leakage Adjoint Spectrum is used for b -effective)

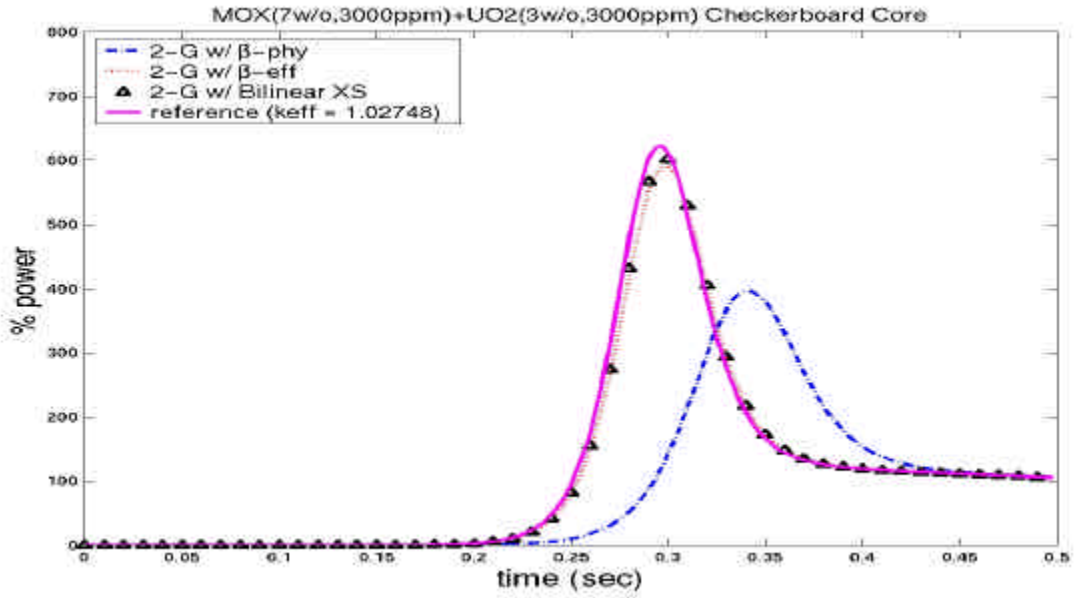

Figure 2. Rod Ejection Transient Results for MOX/UO2 Heterogeneous Core (r =1.09$)
(Leakage Adjoint Spectrum is used for b -effective)

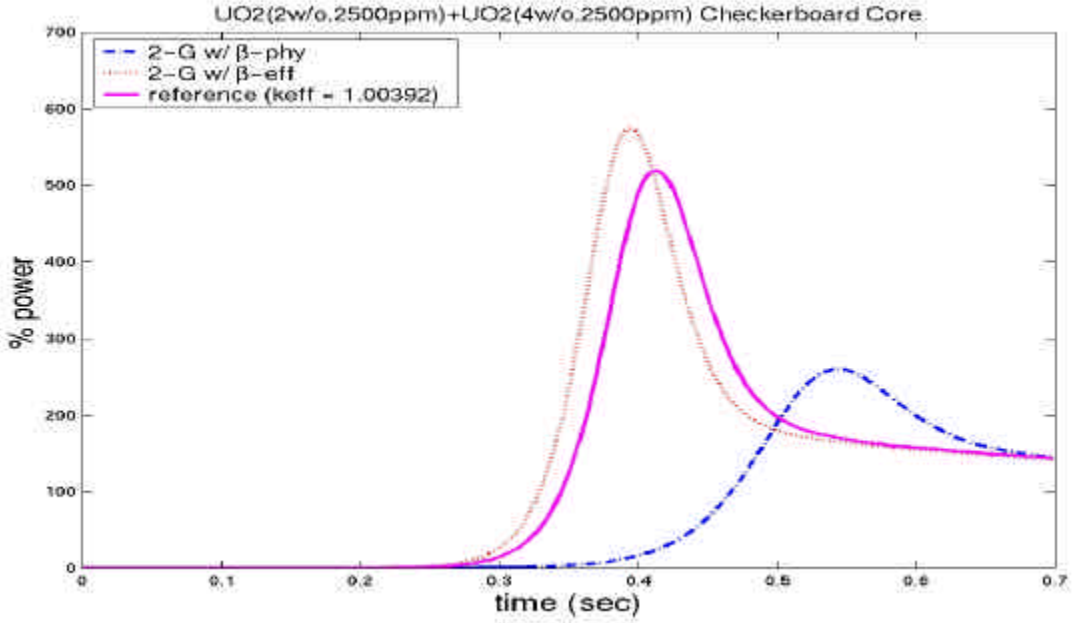

Figure 3. Rod Ejection Transient Results for UO2/UO2 Heterogeneous Core
(Infinite-Medium Adjoint Spectrum is used for b -effective)

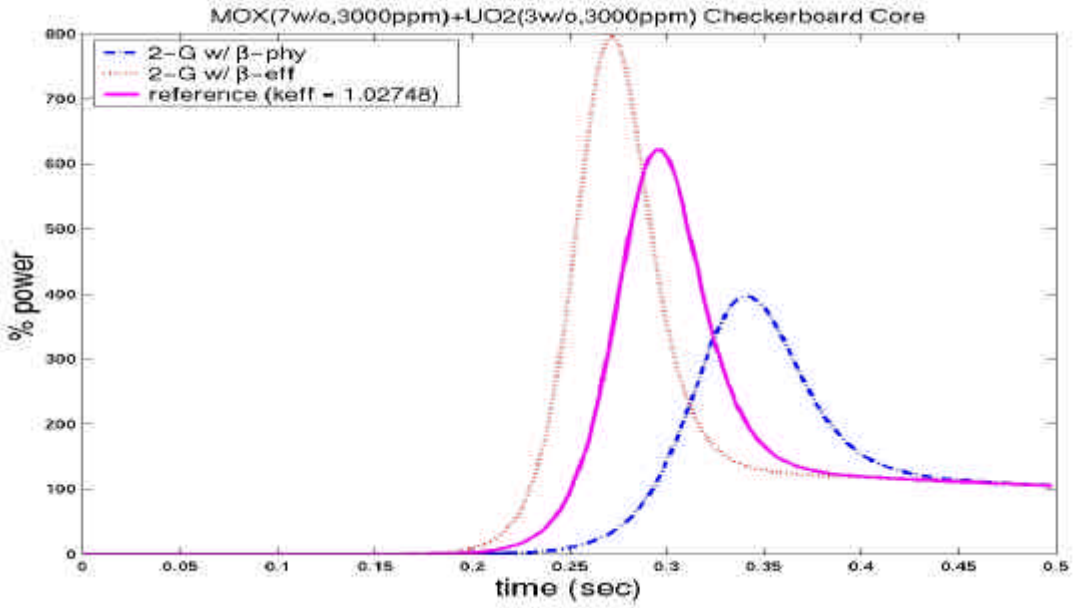

Figure 4. Rod Ejection Transient Results for MOX/UO2 Heterogeneous Core
(Infinite-Medium Adjoint Spectrum is used for b -effective)

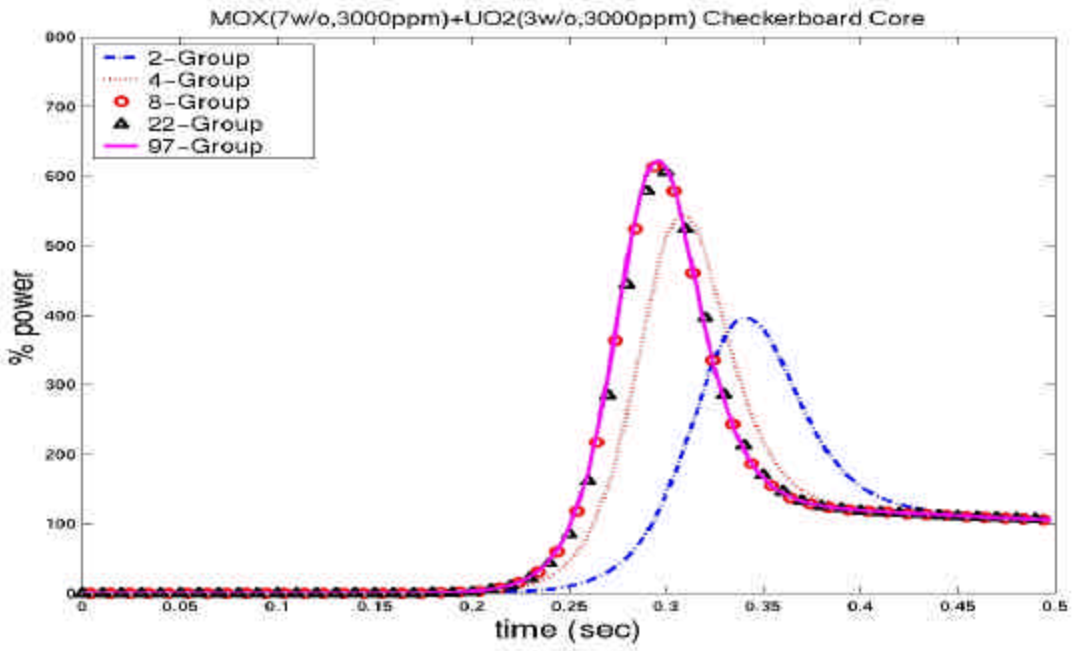

Figure 5. Comparison of RE Transient Results with Various Number of Energy Groups