A SENSITIVITY METHOD FOR CANDU CORE ANALYSIS

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ABSTRACT

A sensitivity method has been developed for the analysis of Canada deuterium uranium (CANDU) reactor performance. The sensitivity equations were derived based on the physics method of RFSP code. The sensitivities of physics parameters such as the maximum channel power, maximum bundle power, and channel power peaking factor were expressed in terms of the unconstrained sensitivity and the zone controller level sensitivities, which were obtained by the generalized perturbation theory. Sample calculations were performed for the perturbation of lattice parameters to assess the sensitivity method, and the results were satisfactory.

1. INTRODUCTION

Recently, advanced Canada deuterium uranium (CANDU) fuel development programs have been conducted, which included 43-element fuel bundle, recovered uranium, slightly enriched uranium, mixed oxide, and direct use of spent pressurized water reactor fuel in CANDU reactors (DUPIC).[1] For the DUPIC fuel, it is inevitable that the fuel composition varies depending on the spent pressurized water reactor (PWR) fuel condition,[2] which will affect the core performance parameters. The purpose of this study is to develop a sensitivity method that can be used to estimate the uncertainties of key performance parameters of a CANDU reactor due to fuel composition variation. The sensitivity method was developed specifically for the current CANDU core physics design and analysis code RFSP[3] by considering both the spatial and bulk controllability of the reactor regulating system.

2. PHYSICS METHOD OF RFSP CODE

The two-group neutron diffusion equation solved by RFSP is as follows:

$$\begin{aligned}
&-\nabla \cdot D_1(\vec{r})\nabla\phi_1(\vec{r}) + \{\Sigma_{a1}(\vec{r}) + \Sigma_m(\vec{r})\}\phi_1(\vec{r}) - \frac{\nu\Sigma_f(\vec{r})}{k_{eff}}\phi_2(\vec{r}) = 0 \\
&-\nabla \cdot D_2(\vec{r})\nabla\phi_2(\vec{r}) + \Sigma_{a2}(\vec{r})\phi_2(\vec{r}) - \Sigma_m(\vec{r})\phi_1(\vec{r}) = 0
\end{aligned} \tag{1}$$

where the fast fission and up-scattering cross-sections are merged to the thermal fission and down-scattering cross-sections, respectively, by the flux weighting. This convention is used in the RFSP code because the thermal flux is dominant in the CANDU core.

The above static diffusion equation can be rewritten in a matrix form as follows:

$$\boldsymbol{M}\phi = \lambda \boldsymbol{F}\phi\,, \tag{2}$$

where $\boldsymbol{M}$ is the removal operator consisting of leakage, absorption, and scattering terms, $\boldsymbol{F}$ is the fission production operator, and λ is the eigenvalue having the form of $\lambda = 1/k_{eff}$.

During normal operation, the excess reactivity of the system provided by the refueling operation is controlled by varying the zone controller unit (ZCU) light water level (bulk and spatial control function), which acts as a neutron absorber, in each of the zone controller compartments. During the bulk control, the individual zone controller level is adjusted by ΔZ to achieve the target power level. During the spatial control, the individual zone controller level is adjusted to achieve the target zone power.

3. SENSITIVITY METHOD FOR CANDU CORE ANALYSIS

In the reactor physics calculations, the fuel composition heterogeneity is represented by the variation of lattice parameters. In a CANDU reactor, if there are perturbations in lattice parameters (e.g., refueling operation), the bundle and channel powers in the core change. If the power distribution changes, the zone power will change accordingly, which results in the ZCU water level change to maintain the criticality and reference power distribution. Therefore, the perturbed channel and bundle powers are affected by the lattice parameter and ZCU level variation together.[4]

3.1 SENSITIVITY COEFFICIENT

For CANDU physics problems, a general response R (e.g., channel power or bundle power) is determined by the lattice parameter α and ZCU level z , which are two physical variables of the physics calculation:

$$R = R(\alpha, z_i)\,, \quad i = 1, 2, \cdots, 14 \tag{3}$$

The constrained sensitivity $\hat{S}$ of the response R is written in terms of the unconstrained sensitivities of R to the lattice parameter and ZCU level and the constrained sensitivity of ZCU level such as:

$$\hat{S}_{\alpha}^{R} = S_{\alpha}^{R} + \sum_{i=1}^{14} S_{z_i}^{R} \hat{S}_{\alpha}^{z_i} \tag{4}$$

During the ZCU level search by RFSP code, the bulk ZCU level is adjusted at first to maintain the core criticality. By the definition, the sensitivity of k is written as follows:

$$\hat{S}_{\alpha}^{k} = S_{\alpha}^{k} + S_{z_b}^{k} \hat{S}_{\alpha}^{z_b} \tag{5}$$

Because the reactor maintains the criticality during the normal operation, the constrained sensitivity of k is essentially zero, and therefore,

$$\hat{S}_{\alpha}^{z_b} = -\frac{S_{\alpha}^{k}}{S_{z_b}^{k}} . \tag{6}$$

If z_b is the bulk ZCU level which is the average of 14 ZCU levels, the constrained sensitivity of z_b to the lattice parameter can be expressed with the constrained sensitivities of ZCU level as follows:

$$\hat{S}_{\alpha}^{z_b} = \frac{1}{14} \sum_{i=1}^{14} f_i \hat{S}_{\alpha}^{z_i} , \tag{7}$$

where $f_i = z_i / z_b$ and $z_b = \frac{1}{14} \sum_{i=1}^{14} z_i$.

The ZCU level is also searched to maintain the reference power of each zone. For zone i, the zone power is a function of the lattice parameter and ZCU levels such as:

$$P_i = P_i(\alpha, z_j), \quad j = 1, 2, \cdots, 14 \tag{8}$$

which in turn results in a sensitivity equation of zone i power as follows:

$$\hat{S}_{\alpha}^{P_i} = S_{\alpha}^{P_i} + \sum_{j=1}^{14} S_{z_j}^{P_i} \hat{S}_{\alpha}^{z_j} \tag{9}$$

In fact, the ZCU level changes to maintain the reference zone power, and therefore, the constrained sensitivity of the zone power is essentially zero, which results in 14 equations to be solved for the sensitivity of ZCU level to the lattice parameter:

$$
\begin{aligned}
&S_{z_1}^{P_1}\hat{S}_{\alpha}^{z_1} + S_{z_2}^{P_1}\hat{S}_{\alpha}^{z_2} + \cdots + S_{z_{14}}^{P_1}\hat{S}_{\alpha}^{z_{14}} + S_{\alpha}^{P_1} = 0 \\
&S_{z_1}^{P_2}\hat{S}_{\alpha}^{z_1} + S_{z_2}^{P_2}\hat{S}_{\alpha}^{z_2} + \cdots + S_{z_{14}}^{P_2}\hat{S}_{\alpha}^{z_{14}} + S_{\alpha}^{P_2} = 0 \\
&\qquad\vdots \\
&S_{z_1}^{P_{14}}\hat{S}_{\alpha}^{z_1} + S_{z_2}^{P_{14}}\hat{S}_{\alpha}^{z_2} + \cdots + S_{z_{14}}^{P_{14}}\hat{S}_{\alpha}^{z_{14}} + S_{\alpha}^{P_{14}} = 0
\end{aligned} \tag{10}
$$

Now the objective function can be constructed to satisfy the constraints of the criticality (see Eq. (7)) and the reference zone power (see Eq. (10)). That is,

$$
\begin{aligned}
Q = {} & w_1\left(S_{z_1}^{P_1}x_1 + S_{z_2}^{P_1}x_2 + \cdots + S_{z_{14}}^{P_1}x_{14} + S_{\alpha}^{P_1}\right)^2 \\
& + w_2\left(S_{z_1}^{P_2}x_1 + S_{z_2}^{P_2}x_2 + \cdots + S_{z_{14}}^{P_2}x_{14} + S_{\alpha}^{P_2}\right)^2 \\
& + \cdots \\
& + w_{14}\left(S_{z_1}^{P_{14}}x_1 + S_{z_2}^{P_{14}}x_2 + \cdots + S_{z_{14}}^{P_{14}}x_{14} + S_{\alpha}^{P_{14}}\right)^2 \\
& + \zeta\left(\hat{S}_{\alpha}^{z_b} - \frac{1}{14}\sum_{i=1}^{14} f_i x_i\right)^2
\end{aligned} \tag{11}
$$

where w_i is weighting factor, ζ is Lagrangian multiplier, and $x_i \equiv \hat{S}_{\alpha}^{z_i}$.

Using a stationary condition of Eq. (11) by taking the partial derivative of Q with respect to x_i, the following equation can be obtained:

$$
\begin{aligned}
&a_{11}x_1 + a_{12}x_2 + \cdots + a_{1N}x_N = b_1 \\
&a_{21}x_1 + a_{22}x_2 + \cdots + a_{2N}x_N = b_2 \\
&\qquad\vdots \\
&a_{N1}x_1 + a_{N2}x_2 + \cdots + a_{NN}x_N = b_N
\end{aligned} \tag{12}
$$

where

$$
a_{ij} = \sum_{k=1}^{N} w_k S_{z_j}^{P_k} S_{z_i}^{P_k} + \zeta\,\frac{f_j f_i}{N^2} \tag{13}
$$

$$
b_i = -\sum_{j=1}^{N} w_j S_{\alpha}^{P_j} S_{z_i}^{P_j} + \zeta\,\hat{S}_{\alpha}^{z_b}\,\frac{f_i}{N} \tag{14}
$$

In summary, the sensitivity of ZCU level $\hat{S}_{\alpha}^{z_i}$ can be obtained by solving Eqs. (6) and (12) iteratively. In order to get these sensitivities, the unconstrained sensitivities S_{α}^{k}, $S_{z_b}^{k}$, S_{α}^{P}, and $S_{z_i}^{P}$ should be provided. In this study, these sensitivities are generated by the generalized perturbation theory (GPT). When the unconstrained sensitivities of a response to the lattice parameter and ZCU level are also provided by the GPT, the constrained sensitivity of a response to the lattice parameter perturbation, Eq. (4), is obtained.

3.2 PERTURBATION METHOD

The sensitivities of k to the lattice parameter and the bulk ZCU level can be written as follows:

$$S_{\alpha}^{k} = -\frac{\alpha}{\delta\alpha}\frac{\left\langle \phi_o^*, (\delta \boldsymbol{M} - \lambda_o \delta \boldsymbol{F})\phi_o \right\rangle}{\left\langle \phi_o^*, \boldsymbol{F}_o \phi_o \right\rangle} \tag{15}$$

$$S_{z_b}^{k} = -\frac{z_b}{\delta z_b}\frac{\left\langle \phi_o^*, (\delta \boldsymbol{M} - \lambda_o \delta \boldsymbol{F})\phi_o \right\rangle}{\left\langle \phi_o^*, \boldsymbol{F}_o \phi_o \right\rangle} \tag{16}$$

The CANDU reactor is divided into 14 control zones, and ZCU's located at the center of each zone are controlled individually to maintain the reference zone power. If the ZCU water level is within the operating range, it is true that the reference power distribution is achieved within the convergence limit. The zone powers and the total reactor power can be represented as a linear functional such as:

$$P_i = \left\langle H_o, \phi_o \right\rangle_i \tag{17}$$

where H is the energy release factor due to the fission reaction.

When there is a refueling perturbation or any kind of reactivity perturbation, the zone power will change accordingly, which will be compensated by changing the ZCU water level. Therefore the variation in the zone power is used as a response function P_i which is constructed as an augmented function K in the variational approach as below:[5]

$$K\left[\alpha, \phi_{o,} \Gamma^*, P^*, \lambda_o\right] = P_i - P_i^*\left(\left\langle H_o, \phi_o \right\rangle_C - P_T\right) - \left\langle \Gamma_i^*, (\boldsymbol{M}_o - \lambda_o \boldsymbol{F}_o)\phi_o \right\rangle, \tag{18}$$

where α and ϕ_o are the lattice parameter and the reference flux, respectively. Γ_i^* is the generalized adjoint flux and P_i^* is the generalized adjoint power.

If ϕ_o is the exact solution to the system equation, then the two constraint conditions in Eq. (18) vanish. Thus, the functional K equals to the response functional P_i. A perturbation in some data α will result in the variations of the functional K, and if ϕ is the exact solution to the perturbed system equation, the perturbed functional K' becomes the perturbed response functional P_i'.

$$K' = K\left[\alpha', \phi, \Gamma^{*'}, P^{*'}, \lambda\right] = P_i' \quad (19)$$

Expanding K' about the unperturbed state and using the first order approximation gives the following expression for the sensitivity of zone power P_i to the lattice parameter:

$$S_{\alpha}^{P_i} = \frac{1}{P_i}\frac{\alpha}{\delta\alpha}\left\{\langle \delta H, \phi_o \rangle_i - p_i \langle \delta H, \phi_o \rangle_C - \left\langle \Gamma_i^*, (\delta\boldsymbol{M} - \lambda_o \delta\boldsymbol{F})\phi_o \right\rangle\right\}, \quad (20)$$

where p_i is the ratio of zone i power to total reactor power.

When there is a reactivity perturbation, the ZCU water level changes accordingly. The ZCU water level is represented by incremental cross sections in CANDU core simulation. Like the perturbation of a lattice parameter, the ZCU motion is described by modifying the cross sections (δH , $\delta\boldsymbol{M}$, and $\delta\boldsymbol{F}$) along the ZCU region. As a result, the zone power will also change depending on the amount of ZCU water level variation. The variation of zone i power caused by a unit ZCU level displacement perturbation in zone j can be obtained by the same approach used to get zone power change due to the lattice parameter perturbation. Therefore, the sensitivity of zone i power to the zone j ZCU level can be written as:

$$S_{z_j}^{P_i} = \frac{1}{P_i}\frac{z_j}{\delta z_j}\left\{\langle \delta H, \phi_o \rangle_i - p_i \langle \delta H, \phi_o \rangle_C - \left\langle \Gamma_i^*, (\delta\boldsymbol{M} - \lambda_o \delta\boldsymbol{F})\phi_o \right\rangle\right\} \quad (21)$$

The sensitivities of other performance parameters (channel power, bundle power, and channel power peaking factor) can be derived in the same way.

4. SENSITIVITY COEFFICIENTS

The unconstrained sensitivity coefficients (similar to Eq. (20)) were calculated for a DUPIC fuel core as shown in Table I for the perturbation of the thermal absorption cross-section. The perturbation was made by increasing the thermal absorption cross-section by 5% for bundles where the maximum channel/bundle power or channel power peaking factor occurs. It can be seen that the sensitivity coefficients are consistent with the results of direct calculations. In order to calculate the constrained sensitivity of the zone controller level, the unconstrained sensitivities of the zone power to the zone controller level and the thermal absorption cross-section were prepared by GPT calculations. The unconstrained sensitivities of the zone power to zone controller level were also compared with the results of direct calculations in Table II. Finally, the constrained sensitivities (see Eq. (4)) were obtained as shown in Table III. The sensitivity coefficients estimated by the sensitivity method provide agreeable results to the RFSP direct calculation though they show somewhat large discrepancy for the sensitivity of maximum channel power, which may come from the large truncation errors in direct calculations.

5. CONCLUSIONS

A sensitivity method has been developed to estimate the effect of the lattice parameter on the core performance parameters. The results of sample calculations have shown that the sensitivity coefficients obtained by the sensitivity method are consistent with those by direct calculations.

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Table I. Unconstrained Sensitivity to Thermal Absorption Cross Section

Response	Direct calculation	GPT calculation	Difference
Maximum channel power	-0.16	-0.19	-0.02
Maximum bundle power	-0.96	-0.99	-0.04
Channel power peaking factor	-0.23	-0.26	-0.03

Table II. Unconstrained Sensitivity of Zone Power to Zone Controller Level

Zone Controller Unit	Direct calculation	GPT calculation	Difference
Zone 1	-0.07	-0.08	-0.01
Zone 2	-0.11	-0.12	-0.01
Zone 3	-0.04	-0.05	-0.01
Zone 4	-0.03	-0.04	0.00
Zone 5	-0.07	-0.07	0.00
Zone 6	-0.08	-0.09	-0.01
Zone 7	-0.11	-0.12	0.00
Zone 8	-0.07	-0.08	0.00
Zone 9	-0.11	-0.12	-0.01
Zone 10	-0.04	-0.04	-0.01
Zone 11	-0.04	-0.04	0.00
Zone 12	-0.07	-0.08	0.00
Zone 13	-0.09	-0.10	-0.01
Zone 14	-0.11	-0.11	0.00

Table III. Constrained Sensitivity to Thermal Absorption Cross Section

Response	Direct calculation	Sensitivity method	Difference
Maximum channel power	-0.08	-0.05	0.02
Maximum bundle power	-0.81	-0.88	-0.06
Channel power peaking factor	-0.15	-0.15	0.01