New Developments in Representativity Approach to study Advanced Assembly Concepts in the EOLE Critical Facility

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ABSTRACT

A new representativity approach based on sensitivity analysis of integral parameters to nuclear data, in the field of Advanced Assemblies Concepts (AAC) design is developped. The adopted scheme proposes an original approach to the problem, going from the initial « microscopic » pin-cells integral parameters to the whole « macroscopic » assembly integral parameters.. The originality of the present work is to develop an enhanced representativity theory based on pin-by-pin power distribution sensitivity to the nuclear data by using a particular application of the Generalized Perturbations Theory. This new development is shown to bring the complementary informations necessary to optimize the initial design and to meet target accuracy on the integral parameters to be measured.

1. INTRODUCTION

Sensitivity studies have always been of major concern in nuclear reactor theory since the forties (ref.1,2,3). The associated perturbations theories (standard, generalized) are a fabulous and very powerful tool for nuclear engineers and physicists. An astonishing quantity of papers and publications devoted to this subject can be found in related journals and conferences, in particular in the last twenty years with the emergence of new calculational schemes and tools (ref .4,5)·The sensitivity analysis is the systematic study of effects of some variations on basic parameters p_i (cross sections, concentrations, etc...) on integral quantities R characterizing the reactor (k_{eff}, spectral indices, etc...). The basic idea is to link the relative perturbation of the response of interest to the modification of these p_i through an expression of the type:

$$\frac{dR}{R} = \sum_i \left(\frac{p_i}{R} \frac{\partial R}{\partial p_i} \right) \frac{dp_i}{p_i} = \sum_i S_R^{p_i} \frac{dp_i}{p_i} \qquad (1)$$

The sensitivity coefficient, noted $S_R^{p_i}$, or simply S_R, of the integral quantity R to the parameter p_i, is defined as the ratio

$$S_R = \frac{p_i}{R}\frac{\partial R}{\partial p_i} \tag{2}$$

Perturbation theories enable to obtain these coefficients. The sensitivity analysis is generally used to understand and interpretate perturbations of neutronic key-parameters, to connect these perturbations to the basic parameters modifications and/or to propagate the nuclear data uncertainties to the integral quantities, through the known « sandwich rule » widely used in the present work.

2. REPRESENTATIVITY FORMALISM

The representativity method is used to extract a quantitative relationship between a particular integral response of an experimental mock-up and the same response in a power reactor that want to be designed. It is based on the similarity of the sensitivity profiles of both integral responses. The representativity is linked to the definition of a correlation coefficient (called by analogy representativity coefficient r_{RE}), defined, as far as we know, for the first time by Orlov (ref.6), as :

$$r_{RE} \equiv \frac{\underline{S}_R^+ \underline{\underline{V}} \underline{S}_E}{\left[\left(\underline{S}_R^+ \underline{\underline{V}} \underline{S}_R\right)\left(\underline{S}_E^+ \underline{\underline{V}} \underline{S}_E\right)\right]^{1/2}} \tag{3}$$

where

$\underline{S}_E$ is the sensitivity vector of the experimental response E to the nuclear data (the superscript « + » denotes the transpose of the vector)

$\underline{S}_R$ is the sensitivity vector of the reactor response R to the nuclear data

$\underline{\underline{V}}$ is the variance-covariance matrix between the nuclear data

and

$\underline{S}_R^+ \underline{\underline{V}} \underline{S}_R = \Delta R_0^2$ and $\underline{S}_E^+ \underline{\underline{V}} \underline{S}_E = \Delta E^2$ represent respectively the *a priori* variances on R and E due to the nuclear data uncertainties, propagated by the classical « sandwich » rule.

The numerator $\underline{S}_R^+ \underline{\underline{V}} \underline{S}_E$ of Eq. (3) represents formally the covariance between the experiment and the reactor responses, while the denominator is simply the squared product of the variances of R and E. The larger the magnitude of r_{RE} , the higher the information transferability from the EOLE mock-ups to the AAC assembly designs. When the similarity between $\underline{S}_R$ and $\underline{S}_E$ increases, r_{RE} reaches an optimum value $r_{RE} = 1$, indicating fully correlated neutronic systems (with respect to the variance-covariance matrix). This representativity approach enables also to predict an *a posteriori* reduction in the reactor response uncertainty ΔR_1 after having injected the experimental information. This reduction factor is given by the expression :

$$\Delta R_1^2 = \Delta R_0^2 \left(1 - \frac{r_{RE}^2}{1 + dE^2 \Big/ \underline{S}_E^+ \underline{\underline{V}} \underline{S}_E}\right) = \Delta R_0^2 \left(1 - \frac{r_{RE}^2}{1 + dE^2 / \Delta E^2}\right) \tag{4}$$

where dE is the *experimental* uncertainty on the response E.

Within the limit of $r_{RE} = 1$ and a ratio $dE^2 / \Delta E^2 \to 0$, the reduction factor can vanish. Figure 1 reproduces the variation of this reduction factor as a function of the ratio $dE / \Delta E$.

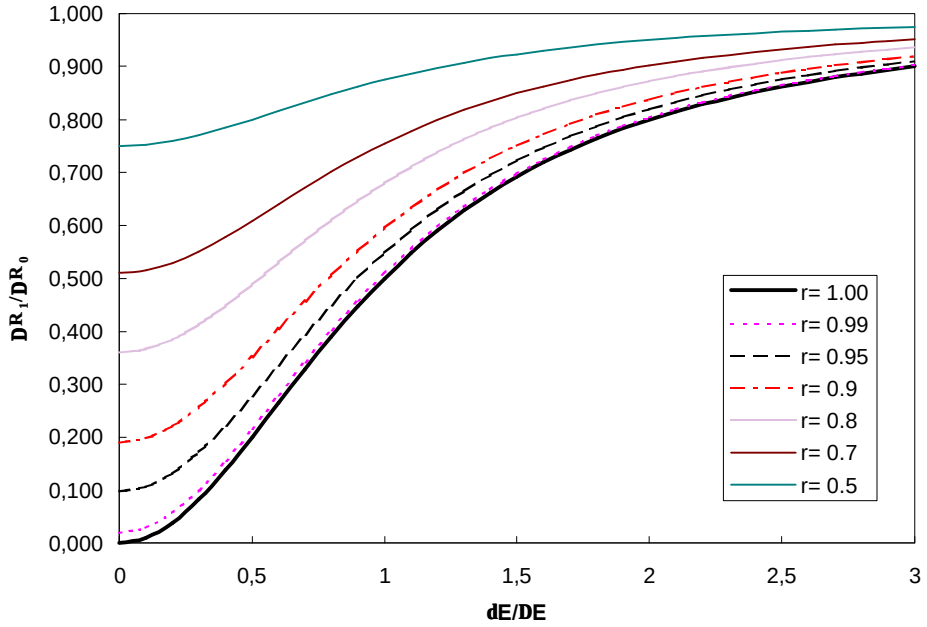

Figure 1. Uncertainty reduction factor versus $dE/\Delta E$, as a function of the representativity value (the perfect representativity of 1.00 has been represented for comparison).

The methodology, described in part 3, has been tested on MOX-zoned assemblies devoted to Pu recycling in thermal reactors. Such mock-up experiments have been performed in the EOLE critical Facility at Cadarache, as part of the French EPICURE Programme (ref.7), detailed in part 4.

3. REPRESENTATIVITY APPROACH

The representativity approach has already been used in previous studies, mainly for design purposes in fast reactors (ref.8). An application had been foreseen for thermal reactors, and in particular for EPICURE and Saint Laurent B1 MOX recycling (ref.9), but only sensitivity profile similarities were analyzed.

We propose here an original approach to investigate the representativity of Advanced Assembly Concepts, starting from its elementary pin-cells.. If a sufficient representativity between these pin-cells can be achieved and demonstrated, at least on the neutron balance (for example the pin-cells multiplication factor in an infinite medium), a further and more fundamental investigation can be envisaged on different integral parameters associated to the assembly itself. We then distinguished the « global » effect or behaviour (neutron balance of the assembly, average reactions rates - spectral indices -) from the « local » effect (such as the fission rate distributions pin-by-pin). If the first type of effect gives a general information on the neutron spectrum averaged over the entire assembly, the second one permits to study more rigourously the power distribution and the nearly pointwise effects that are « smoothed » by a global study. The parameters retained for the study are the following ones:

for global parameters : - k_{eff} (for both pin-cell and assemblies)
- spectral indices in thermal, epithermal and fast energy range (assemblies)

for local parameters : - pin-by-pin fission rate distributions, subdivided in
- P_{max}/P_{av}

- P_{min}/P_{av}
- P_{min}/P_{max}

4. DESCRIPTION OF CELLS AND ASSEMBLIES

An already existing programme has been retained for the study, being evident that the methodology would apply exactly the same manner to new concepts. As an example,, a particular configuration of the EPICURE prgramme and its representativity versus the first SLB-1 MOX loading has been retained. This programme had been undertaken at the beginnning of the Nineties, within the framework of a collaboration between CEA, EdF and FRAMATOME, whose aim was, for one part, to validate power maps within multi-zoned MOX assemblies, and MOX/UOx interface calculational schemes and associated nuclear data.

The UM-ZONE configuration was built at that time to improve the knowledge of MOX assemblies behavior in UOx environment, as an experimental support to the first 30% MOX loading in the Saint Laurent B1 French PWR, and also to predict the efficiency of several clusters made of various absorbing materials (B_4C and AIC).

The pin-cells of the experiment are made of three different Pu enrichment, close to the ones used in the SLB-1 MOX assemblies, and typicaly issued from UO_2 assemblies with BU of about 33GWd/T. The dimensions of the fuel pins are identical to that of a 900 Mwe PWR, except for the lenght. The available pins are UO2 at 3,7% enrichment, 4,3% MOX, 7% MOX and 8,7% MOX.
These Pu vectors are close to the ones used in the SLB-1 assembly, denoted respectively Pu-type 1, type2 and type 3 in TABLE 1.

Furthermore, in order to simulate in the experiment the nominal operating conditions of a power reactor, and to respect the H/HM ratio, an aluminium overcladding of these pins have been introduced. The description of the EPICURE cells and assemblies is represented on figures 2 and 3, as well as the SLB-1 description

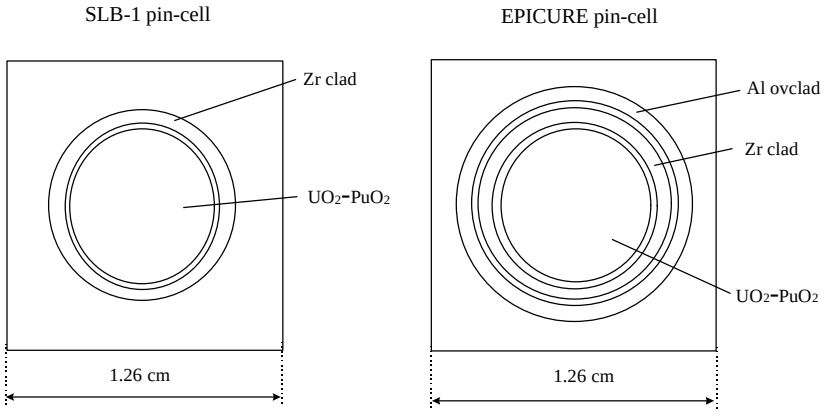

Figure 2. *Pin-cell description of EPICURE and SLB-1 fuels*

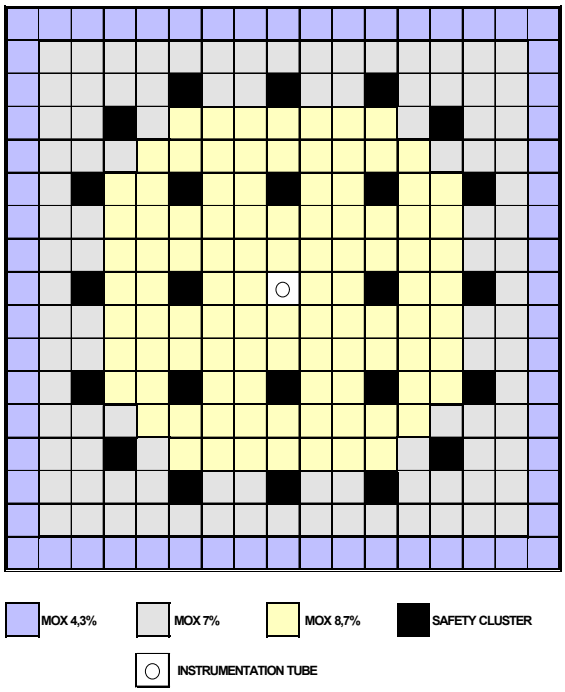

Figure 3.*multi-zoned MOX Assembly*

Two configurations of both assemblies (UM-ZONE and SLB-1) have been analyzed for the present study : 2 multi-zoned MOX without cluster (and comprizing 24 guide-tubes) and 2 MOX with AIC cluster (24 absorber pins). Each EPICURE assembly has been compared versus its corresponding pattern in SLB-1.

5. SENSITIVITY CALCULATIONS

The neutronic and perturbation calculational schemes used in the current work are detailed hereafter. All the modelizations are based on an infinite medium representation of the cell or of the assembly and by critical buckiling search. The calculational scheme is decomposed into two distinct steps (figure 4) :

a. cell or assembly calculation by using the french APOLLO-1 P_{ij} transport code (ref. 10) and its 99 groups JEF-1 based CEA-86 library (ref. 11). The effective cross sections produced are condensed and homogeneized using equivalence procedures on each cell in 15 groups whose limits enable a good representation of the different spectral regions.

b. S_8-P_1 transport calculation by using the ERANOS system of codes (ref. 12) on homogeneized (for global parameters) or heterogeneous geometries (for power distribution) both for direct flux and (standard or generalized) adjoint

5.1.Sensitivity of the global integral parameters to the nuclear data

The sensitivity to the different global integral parameters are calculated by using classical Standard (k_{eff}) and Generalized (spectral indices) Perturbation Theories, implemented in various modules of the ERANOS system. The various expressions of the sensitivity coefficients are well known and will not be reproduced here. They can be found elsewhere, for example in (ref.5 and 13).

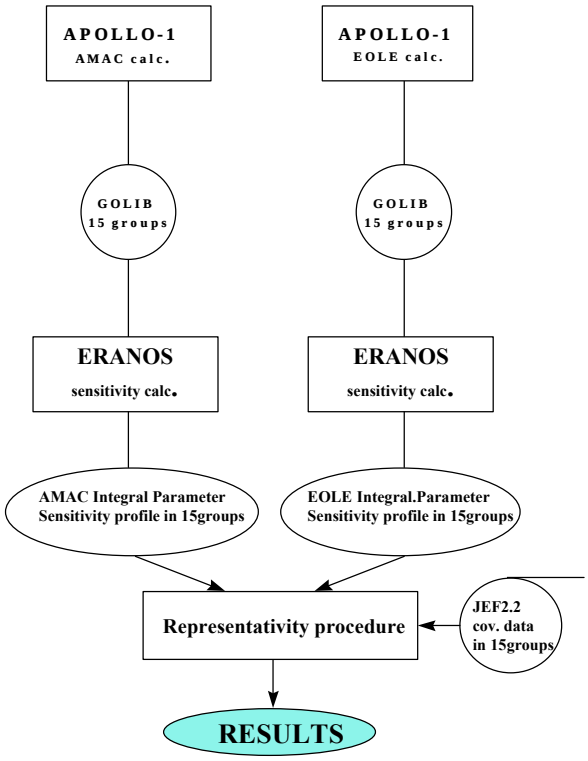

Figure 4. *Calculational scheme used for representativity study*

5.2. Sensitivity of the pin-by-pin power distribution to the nuclear data

The treatment of the pin-by-pin power distribution of the assemblies is derived from the GPT expression of a reaction ratio variation to input data. The general expression of the relative power of a particular pin to the overal power of the assembly can be expressed as :

$$R = \frac{\left\langle \Sigma_f , j \right\rangle_{cell}}{\left\langle \Sigma_f , j \right\rangle_{assembly}} = \frac{\left(\Sigma_{fc} , j_c \right)}{\left\langle \overline{\Sigma_f , j} \right\rangle} \tag{5}$$

where the (...) term denotes integration on the individual pin-cell and $< \bar{} >$ denotes integration on the whole assembly.
The perturbation of both numerator and denominator leads to a 4-terms relation corresponding to an entire sensitivity calculation on both the cell of interest and the homogeneized assembly.

$$\frac{dR}{R} = \underbrace{\frac{\left(d\Sigma_{fc} , j_c \right)}{\left(\Sigma_{fc} , j_c \right)} - \frac{\left\langle \overline{d\Sigma_f , j} \right\rangle}{\left\langle \overline{\Sigma_f , j} \right\rangle}}_{\text{Total direct effect}} + \underbrace{\left[\left(\frac{\Sigma_{fc}}{\left(\Sigma_{fc} , j_c \right)} , dj_c \right) - \left\langle \frac{\overline{\Sigma_f}}{\left\langle \overline{\Sigma_f , j} \right\rangle} , dj \right\rangle \right]}_{\text{Total indirect effect}} \tag{6}$$

The last term is called indirect effect because it represents the impact of the flux perturbation (caused by a modification of an input variable) on the response· Neglecting higher order effects, this flux perturbation can be determined by by defining formally a generalized inhomogeneous adjoint equation as

$$\begin{cases} (L^{+} - lP^{+})_c \Psi_c^{+} = \dfrac{1}{R}\dfrac{dR}{dj_c} = \dfrac{\Sigma_{f,ci}}{\langle \Sigma_{fc}, j_c \rangle} = S_c^{+} & \text{for the cell} \\ (\overline{L^{+} - lP^{+}})\overline{\Psi}^{+} = \dfrac{1}{R}\dfrac{dR}{d\overline{j}} = \dfrac{\overline{\Sigma_f}}{\langle \overline{\Sigma_f}, \overline{j} \rangle} = S & \text{for the assembly} \end{cases} \quad + \text{ constraints for uniqueness of the solutions} \tag{7}$$

Having imposed the « fundamental mode decontamination » constraints on the generalized adjoints, these last ones can be calculated. The perturbation on the integral parameter is rewritten

$$\frac{dR}{R} = \underbrace{\frac{(d\Sigma_{fc}, j_c)}{(\Sigma_{fc}, j_c)} - \frac{\langle \overline{d\Sigma_f}, \overline{j} \rangle}{\langle \overline{\Sigma_f}, \overline{j} \rangle}}_{\text{Total direct effect}} + \underbrace{\left[\left(\Psi_c^{+}, (dL - l\,dP)_c j_c \right) - \left\langle \overline{\Psi^{+}, (dL - l\,dP) j} \right\rangle \right]}_{\text{Total indirect effect}} \tag{8}$$

The sensitivities coefficients of the power ratio to the nuclear data will then be given by the expressions:

$$\begin{aligned} \left(S_{a_g}^{R} \right)_{\text{indirect effect}} &\equiv \left(\frac{dR}{R} \right)_{\text{indirect effect}} \Big/ \frac{da_g}{a_g} \\ &= a_g \left[\left(\Psi_c^{+}, \left(\frac{\partial L}{\partial a_g} - l \frac{\partial P}{\partial a_g} \right)_c j_c \right) - \left\langle \overline{\Psi^{+}, \left(\frac{\partial L}{\partial a_g} - l \frac{\partial P}{\partial a_g} \right) j} \right\rangle \right] \end{aligned} \tag{9}$$

for the indirect effects
and

$$\left(S_{a,g}^{R} \right)_{\text{direct effect}} = \left(\frac{d\Sigma_{fg} j_g}{(\Sigma_f j)} \Big/ \frac{da_g}{a_g} \right)_c - \left(\frac{d\overline{\Sigma_{fg} j_g}}{\langle \overline{\Sigma_f j} \rangle} \Big/ \frac{d\overline{a_g}}{\overline{a_g}} \right) = \left. \frac{\Sigma_{fg} j_g}{(\Sigma_f j)} \right|_c - \frac{\overline{\Sigma_{fg} j_g}}{\langle \overline{\Sigma_f j} \rangle} \tag{10}$$

for the direct ones.

These last expressions can be easily extended to any desired pin-by-pin power ratio. As an example, we treated in the same manner the P_{max}/P_{av}, the P_{min}/P_{av} and the P_{max}/P_{min} reaction rate ratios. A procedure has been especially developped within the ERANOS code to treat these new perturbation problems. It has also to be noted that in the particular case where the average power appears in the expression, it contains implicitely the individual pin-cell studied and a cross term effect (in term of perturbation) can be present because the perturbation of the pin-cell cross sections modifies also the average distribution. The simplified first order approximation chosen in this study avoids to treat the cross term. This situation is detailed in Appendix.

6. RESULTS

Some preliminary constatations have to be mentionned before going on the representativity results. Supplementary investigations on the profiles have shown that a discrepancy of about 10% in the most important sensitivity coefficients (^{239}Pu fission, ^{238}U capture, etc..), that we took as the maximum admissible discrepancy, leads to correlation coefficients of the ordrer of 0,9. This last value has then been chozen as the lowest acceptable in the present study.

6.1. Pin-cell representativities

As mentionned, only representativity of the k_{eff} to the nuclear data has been processed. We have tested the influence of the presence of overcladding on the result. These ones are resumed in TABLE 1.

	conditions	H/HM	SLB-1 Pu-type 1 (H/HM ~ 4.5)	SLB-1 Pu-type 2 (H/HM ~ 4.6)	SLB-1 Pu-type 3 (H/HM ~ 4.6)
EPICURE MOX 4,3%	without overclad	4.6	0.93	0.92	0.90
	with overclad	4.9	~1.00	0.97	0.95
EPICURE MOX 7%	without overclad	4.6	0.92	0.94	0.93
	with overclad	4.9	0.98	0.99	0.98
EPICURE MOX 8,7%	without overclad	4.6	0.89	0.92	0.94
	with overclad	4.9	0.95	0.97	0.99

TABLE 1. *Representativity factors between the pin-cell k_{eff}*

The similarity profiles of these differents cells are reproduced, for important cross sections, on figures 5 and 6.

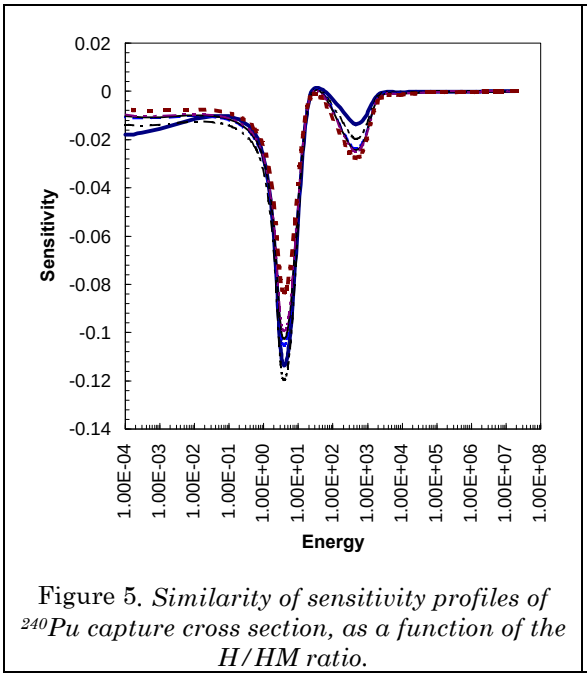

Figure 5. *Similarity of sensitivity profiles of ^{240}Pu capture cross section, as a function of the H/HM ratio.*

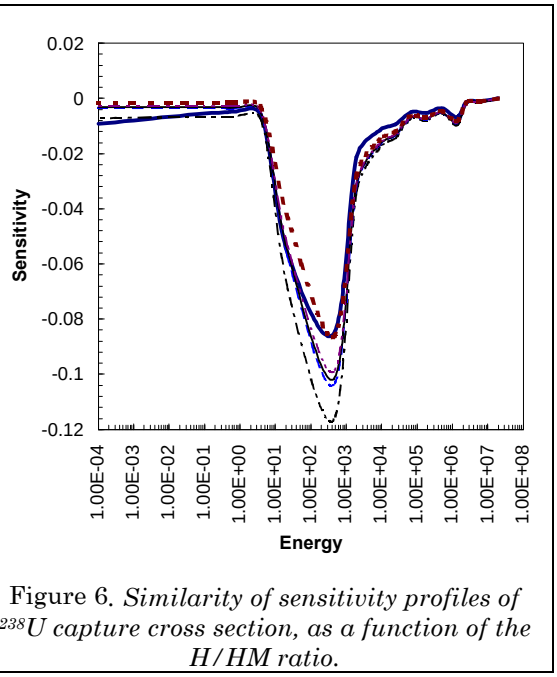

Figure 6. *Similarity of sensitivity profiles of ^{238}U capture cross section, as a function of the H/HM ratio.*

The presence of the overclad is evident, it increases strongly, by modifying the H/HM ratio, the representativity of the cells. Its influence can be seen on the sensitivty profiles of the above figures. The closest tthe H/HM ratio and the Pu-vector, the highest the representativity.

6.2. Assembly representativities

The H/HM ratios have been computed for the 4 configurations retained.

Assembly	H/HM
UM-ZONE W.H	5.5
UM-ZONE AIC	5.1
SLB-1 WH	5.4
SLB-1 AIC.	5.1

TABLE 2. *H/HM ratio of the assemblies (W.H. : Water Hole, AIC : absorber cluster in Ag-In-Cd)*

The representativity results obtained for all the integral parameters are reproduced in TABLE 3.

Integral parameter	UM-ZONE Config..	SLB-1 W.H	SLB-1 AIC
k-infinite	UM-ZONE W.H	0.96	
	UM-ZONE AIC		0.91
Fission ^{239}Pu/ Fission ^{235}U	UM-ZONE W.H	0.92	
	UM-ZONE AIC		0.87
Capture ^{238}U/ Fission ^{235}U	UM-ZONE W.H	0.96	
	UM-ZONE AIC		0.88
Fission ^{240}Pu/ Fission ^{239}Pu	UM-ZONE W.H	~1.00	
	UM-ZONE AIC		~1.00
P_{max}/P_{av}	UM-ZONE W.H	0.98	
	UM-ZONE AIC		0.98
P_{min}/P_{av}	UM-ZONE W.H	0.97	
	UM-ZONE AIC		0.96
P_{max}/P_{min}	UM-ZONE W.H	0.95	
	UM-ZONE AIC		0.94

TABLE 3. *Representativity factors for global and local integral parameters on the assemblies in infinite medium*

A very good agreement is reached for all the integral parameters retained. The propagated uncertainty enables us to calculate the reduction uncertainty factor. An experimental uncertainty of 1,5% (integral gamma-scanning measurement) has been chosen. The results obtained are reproduced TABLE 4 :

Integral param	EOLE config.	SLB-1 W.H	SLB-1 AIC
Fission ^{239}Pu/ Fission ^{235}U	UM-ZONE W.H	0.79	-
	UM-ZONE AIC	-	0.85
Capture ^{238}U/ Fission ^{235}U	UM-ZONE W.H	0.84	-
	UM-ZONE AIC	-	0.86
P_{max}/P_{av}	UM-ZONE W.H	0.68	-
	UM-ZONE AIC	-	0.73
P_{min}/P_{av}	UM-ZONE W.H	0.63	-
	UM-ZONE AIC	-	0.68
P_{max}/P_{min}	UM-ZONE W.H	0.91	-
	UM-ZONE AIC	-	0.95

TABLE 4. *Uncertainty reduction factors for spectral indices and pin-by-pin fission rates distribution*

The uncertainty reduction factor is lower for power distribution, due to a lower $\delta E/\Delta E$ value, and in particular for Pimn/Pmax.

6. CONCLUSIONS

The study of new advanced assembly designs to optimize Pu-burning in current reactor cores is, more than ever, an exiting challenge. If the purely conceptual design of such assembly doesn't pose untractable problems in terms of heavy costs other than calculational ones, the interest of the work remains quite limited if an experimental programme is not built to validate (or infirmate) the theoretical results and options. This (compelled) step is often done through huge expenses concerning new fuel pins design and composition. The proposed approach enables a neutronic optimization for both pin cell and assembly design to be conducted in the critical facility, by proposing some modifications to the *already existing* materials, in order to increase the representativity with minimized cost expenses.

The study shows that two parameters are of the utmost importance on the results :

- the Pu-vector
- the H/HM ratio.

The closest these values the better the result. The representativity approach proposed enables an optimization of both parameters.

Complementary studies, not presented here, have shown that neither the fuel pin diameter nor the effect of Pu-aging of the mock-up fuel pins influence the result if both H/HM and Pu-vectors are simultaneously fullfilled. This means that assemblies of different types can be modelized with « standard » PWR pin-cells. The method has been successfully applied to the definition of a new experimental programme called BASALA (ref 14) devoted to 100% MOX loading in Advanced BWR. It was shown that the local pin-by-pin representativity approach brought the necessary and complementary information to the global parameters. In particular, it was shown that care must be taken if the representativity approach is restricted only to the for global spectral integral parameters (k_∞ and spectral indices), even if good (preliminary) results can be achieved. This point enabled us to optimize the initial designs with minimal cost expenses.

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APPENDIX

The treatment of the pin-power distribution of the assemblies as we neglects a correction factor due to the cross term between the numerator and the denominator when the average power is taken into account in the ratio. We will firstly recall equations (5), and rewrite it as:

$$R = \frac{\left\langle \Sigma_f , j \right\rangle_{cell}}{\left\langle \Sigma_f , j \right\rangle_{assembly}} = \frac{\left\langle \Sigma_{fi} , j_i \right\rangle}{\left\langle \Sigma_{fi} , j_i \right\rangle + \sum_{j \neq i} \left\langle \Sigma_{fj} , j_j \right\rangle} \tag{A.1}$$

Here, the <...> notation remains since it indicates an integration on individual pin-cells.
We showed that, when the cross term is neglected, the perturbation of both numerator and denominator leads to the 4-terms relation:

$$\frac{dR}{R} = \underbrace{\frac{\left\langle d\Sigma_{fi} , j_i \right\rangle}{\left\langle \Sigma_{fi} , j_i \right\rangle} - \frac{\sum_j \left\langle d\Sigma_{fj} , j_j \right\rangle}{\sum_j \left\langle \Sigma_{fj} , j_j \right\rangle}}_{\text{Total direct effect}} + \underbrace{\left[\left\langle \Psi_i^+ , (dL - l\, dP)_i j_i \right\rangle - \sum_j \left\langle \Psi_j^+ , (dL - l\, dP)_j j_j \right\rangle \right]}_{\text{Total indirect effect}} \tag{A.2}$$

To evaluate the correction term due to the taking into account of the local pin-cell power in the average power over the assembly, we havel proceed by analogy: Without going into details, the correction factor to apply to relation () is expressed as :

$$f_i = \sum_{j \neq i} \left\langle \Sigma_{fj} , j_j \right\rangle \Big/ \sum_j \left\langle \Sigma_{fj} , j_j \right\rangle \tag{A.3}$$

As it can be seen, this factor is in general very close to 1, and can be usually neglected in the present calculations. We will see further that in fact, it doesn't have to be taken into account in the representativity calculation. The « exact » perturbation of the response function becomes (still to first order with respect to the perturbation products):

$$\frac{dR}{R} = f_i \times \left\{ \underbrace{\frac{\left\langle d\Sigma_{fi} , j_i \right\rangle}{\left\langle \Sigma_{fi} , j_i \right\rangle} - \sum_j \frac{\left\langle d\Sigma_{fj} , j_j \right\rangle}{\left\langle \Sigma_{fj} , j_j \right\rangle}}_{\text{Total direct effect}} + \underbrace{\left[\left\langle \Psi_i^+ , (dL - l\, dP)_i j_i \right\rangle - \sum_j \left\langle \Psi_j^+ , (dL - l\, dP)_j j_j \right\rangle \right]}_{\text{Total indirect effect}} \right\} \tag{A.4}$$

The sensitivities coefficients will then simply be given by the expressions

$$\left(S_{a_g}^R \right)_{\text{indirect4}} \equiv f_i \times a_g \left[\left\langle \Psi_i^+ , \left(\frac{\partial L}{\partial a_g} - l \frac{\partial P}{\partial a_g} \right)_i j \right\rangle - \sum_j \left\langle \Psi_j^+ , \left(\frac{\partial L}{\partial a_g} - l \frac{\partial P}{\partial a_g} \right)_j j_j \right\rangle \right] \tag{A.5}$$

$$\left(S_{a_g}^R \right)_{direc} = f_i \times \left\{ \left. \frac{\Sigma_{fg} j_g}{\left\langle \Sigma_f j \right\rangle} \right|_i - \sum_j \left. \frac{\Sigma_{fg} j_g}{\left\langle \Sigma_f j \right\rangle} \right|_j \right\} \tag{A.6}$$

for the indirect and the direct effect respectively.

Going on to the representativity, we can easily show that this correction doesn't influence the result. Rewritting the representativity more exactly as

$$r_{RE} \equiv \frac{(f_R \underline{S}_R)^+ \underline{\underline{V}} (f_E \underline{S}_E)}{\left[\left((f_R \underline{S}_R)^+ \underline{\underline{V}} (f_R \underline{S}_R) \right) \left((f_E \underline{S}_E)^+ \underline{\underline{V}} (f_E \underline{S}_E) \right) \right]^{1/2}} \tag{A.7}$$

and knowing that the corrections factors f_R and f_E are constants, they can be simplified in eqn (A.7), as symbolized by the arrows, and the representativity factor remains unchanged. These correction factors will play a role in the calculation of the uncertainty propagation from the cross sections to the integral parameters.