INCLUDING THE TRAVELLING OF DENSITY WAVES IN THE ANALYTICAL MODELING OF BOILING CHANNEL STABILITY

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ABSTRACT

An existing analytical time-domain model to study Boiling Water Reactor (BWR) stability has been extended by implementing multiple nodes in the single-phase region and in the two-phase region of the core. In the single-phase region moving boundaries with fixed enthalpy values have been used. In the two-phase region the boundaries are equidistant, but move with the boiling boundary. The implementation of multiple nodes has been done to model the travelling of density waves in the core. In comparison with the existing model the new model predicts a less stable system, especially in the high-power and low-flow range. The model has been applied to study the nonlinear characteristics of the instabilities. Based on experiments with the test facility DESIRE, it is studied whether for specific operating conditions period doubling is predicted in the system. However, this is not the case. The amplitude of the limit cycle grows as the system becomes more unstable, until the model is no longer valid.

1. INTRODUCTION

Recently, a pure thermal-hydraulic period-doubling bifurcation has been observed in the test facility DESIRE, used to study BWR stability[1]. To our knowledge, this is the first experimental observation of this phenomenon in two-phase flow. Period doubling has been predicted for a boiling system with nuclear feedback[2,3,4], and in a boiling flow coupled with an adiabatic riser at high subcooling number and high Zuber number [5,6]. To understand the observed phenomena qualitatively, it is preferable to have an analytical model which grasps the physical essences. Therefore, the analytical model developed by Van Bragt and Van der Hagen[7] has been extended to model the travelling of density waves in a boiling channel. This phenomenon was not included originally since the core region in this model consists of one single-phase node and one two-phase node. The extension of the model to include the delay of travelling density waves is described in this paper and some results with the new model are given.

To study the stability of Boiling Water Reactors a wide range of analytical models has been developed because of their suitability to perform fast parameter studies, both in the linear and in the nonlinear domain. In this introduction we will focus on those analytical models which are able to model nonlinear phenomena. Furthermore, we will not consider state-of-the-art thermal-hydraulic system codes, which are of course also able to model these phenomena, but which should be used for quantitative studies.

A phenomenological model for BWR dynamics has been developed by March-Leuba, Cacuci and Perez[2]. Although this model is capable of predicting limit-cycle oscillations in a BWR due to neutronic feedback, one must be careful not to simplify the processes at issue too much, thereby distorting the physical understanding. This model cannot predict the thermal-hydraulic instabilities occurring in DESIRE. At the other end, Achard and Drew[8] developed an exact analytical model based on the homogeneous equilibrium model (HEM), resulting in a set of integrodifferential equations. Rizwan-uddin and Dorning developed a similar model based on the drift-flux model[9]. The disadvantage of these models is that it is necessary to solve a rather complicated set of Functional Differential Equations (FDEs); the advantage is that exact solutions are obtained which do not depend on the number of nodes used. Clausse and Lahey used HEM to develop a nodal method with moving boundaries in the single-phase region of the core, with one two-phase node in the core, and with a number of fixed riser nodes[5]. They assume a linear enthalpy profile within each single-phase node, and a linear mixture-enthalpy profile (linear quality profile) in the two-phase regions to arrive at a set of Ordinary Differential Equations (ODEs). Since the node boundaries are moving boundaries, these linear profiles vary with time. Chang and Lahey extended this model to include a variable number of moving nodes in the two-phase region of the core as well[3]. Van Bragt and Van der Hagen developed a similar model[7]. However, they only use one single-phase node and one two-phase node in the core. The extension of this model to multiple nodes is described in this paper. Karve et al.[10] also developed a model with one single-phase node and one two-phase node, but with a quadratic spatial approximation for the single-phase enthalpy and the two-phase quality. The expansion coefficients are time dependent. By comparing the stability boundary obtained with this model with the exact solution[11] it is concluded that the stability boundary agrees well when a quadratic-enthalpy/quadratic-quality approximation is used. However, this does not necessarily mean that this model achieves as well in the nonlinear analysis of density-wave oscillations. For this purpose a higher order spatial approximation could be necessary. Apart from these models with moving boundaries, analytical models with fixed nodes are also being applied. Ambrosini et al. show that a large number of nodes is then needed to model density-wave oscillations[12]. Podowski and Rosa show that the number of nodes needed drastically decreases as the moving interface is being modeled within the node containing this interface[13].

2. MODELING OF MULTIPLE NODES

2.1 INTRODUCTION

The model that has been used is described in detail by Van Bragt and Van der Hagen[7]. The governing equations for the thermal-hydraulic subsystem are derived on the basis of the one-dimensional HEM. The multiple parallel channels in a reactor core are approximated with one average channel. In this section we will focus on the changes needed to use multiple nodes in the single-phase region and multiple nodes in the two-phase region of the core.

2.2 SINGLE-PHASE REGION

The single-phase region of the core is divided into $N_{1\varphi}$ nodes with equal increase in enthalpy[5]. The equation for the dynamic behavior of these boundaries is derived by integrating the differential energy equation from the node inlet to the node outlet. The local enthalpy within a node is assumed to change simultaneously at all axial positions. Then, the dynamics of the single-phase node boundaries is given by:

$$P_{d2}\frac{dz_n(t)}{dt}=\frac{M_{C,i}(t)}{\rho_f}-P_s\frac{N_{1\varphi}\langle q'\rangle_n(t)}{\rho_f A_C(h_f-h_g)}(z_n(t)-z_{n-1}(t))-P_{d1}\frac{dz_{n-1}(t)}{dt}, \tag{1}$$

with:

$$z_0=0,\ \text{and}\ z_{N_{1\varphi}}(t)=Z_{bb}(t), \tag{2}$$

in which z is the axial position, t is the time, $M_{C,i}$ is the core inlet mass flow rate, ρ_f is the fluid density, q' is the linear power transferred from fuel rod to coolant, A_C the coolant-channel area, h_f and h_g are the liquid and gas enthalpy, and P_{d1}, P_{d2}, and P_s are factors determined by the power profile. The local quantities with a subscript $n-1$ are evaluated at the inlet of node n, while the local quantities with subscript n are evaluated at the outlet of node n.

2.3 TWO-PHASE REGION

The two-phase region in the core consists of $N_{2\varphi}$ nodes. The boundaries in the core region are equidistant and move with the boiling boundary:

$$\frac{dz_n(t)}{dt}=\frac{1}{N_{2\varphi}}\frac{dZ_{bb}(t)}{dt}\left(N_{2\varphi}+N_{1\varphi}-n\right), \tag{3}$$

with Z_{bb} the position of the boiling. This model differs from the model developed by Chang and Lahey[3], because they apply nodes with equal increase in enthalpy. In our case the node boundaries in the two-phase region move with the boiling boundary.

The equations for the core (and riser) void dynamics are derived by integrating the differential continuity and energy equations for the two-phase region nodes:

$$\frac{d<\alpha(t)>_n}{dt} = \frac{1}{(z_n(t)-z_{n-1}(t))}\left[\begin{array}{l}\frac{M_n(t)-M_{n-1}(t)}{(\rho_f-\rho_g)}+\alpha(z_n(t))\frac{dz_n(t)}{dt}+ \\ -\alpha(z_{n-1}(t))\frac{dz_{n-1}(t)}{dt}-<\alpha(t)>_n\frac{d(z_n(t)-z_{n-1}(t))}{dt}\end{array}\right], \tag{4}$$

$$\begin{aligned} M_{n-1}(t)\left[1+\frac{\rho_f-\rho_g}{\rho_g}\chi_{n-1}(t)\right] = M_n(t)\left[1+\frac{\rho_f-\rho_g}{\rho_g}\chi_n(t)\right] - \\ \frac{(\rho_f-\rho_g)}{\rho_g}\frac{(z_n(t)-z_{n-1}(t))2\pi r\gamma q''(t)}{A_C(h_g-h_f)}. \end{aligned} \tag{5}$$

in which α is the void fraction, χ is the flow quality, r the fuel-pin radius, and q'' the average heat flux in the channel.

The HEM is used to relate the void fraction in Eq.(4) with the quality in Eq.(5). The assumption is made that the local quality changes simultaneously at all axial positions within a node. The local quality is assumed to increase or decrease linearly between the node-inlet and the node-outlet quality. This leads to the following equation for the void fraction in node n:

$$<\alpha(t)>_n = \frac{1}{C_0}\frac{\rho_f}{\rho_f-\rho_g}\left\{1-\frac{\rho_g}{(\rho_f-\rho_g)(\chi_n(t)-\chi_{n-1}(t))}\ln\left[1+\frac{\rho_f-\rho_g}{\rho_g}\frac{(\chi_n(t)-\chi_{n-1}(t))}{1+\chi_{n-1}(t)\frac{\rho_f-\rho_g}{\rho_g}}\right]\right\}. \tag{6}$$

The concentration parameter C_0 is used to include the effect of a non-uniform radial void distribution and velocity profile.

The pressure drops in the single-phase region and in the two-phase region are modeled as in the existing model[7]. The assumption is made in the integration of the momentum equation that the mass flux density in the two-phase region is equal to $M_{C,e}$. The HEM two-phase friction multiplier used for the distributed friction pressure drop in the core is evaluated for $\frac{\chi_{C,e}}{2}$.

2.4 POWER PROFILE

A variable number of single-phase and two-phase nodes can be used for a flat power profile or in case of a power profile given by:

$$P(z) \equiv \frac{P}{L_C} f_p \sin\left(A + B\frac{z}{L_C} \right), \tag{7}$$

in which P is the power, f_p is the peaking factor, L_C is the length of the core, and the parameters A and B determine the position of the maximum power. Note that these factors have to be chosen such that the power distribution is normalized.

In case of a flat power profile the P-factors and the factor γ are given by:

$$P_{d1} = 0.5,\;\; P_{d2} = 0.5,\;\; P_s = 1,\;\; \gamma = 1. \tag{8}$$

In case of the power profile given by Eq. **(7)** the factors are given by:

$$P_{d1} = -\left\{ B\frac{(z_n - z_{n-1})}{L_C}\cos\left(A + B\frac{z_n}{L_C}\right) + \sin\left(A + B\frac{z_{n-1}}{L_C}\right) - \sin\left(A + B\frac{z_n}{L_C}\right) \right\} \frac{\sin\left(A + B\frac{z_{n-1}}{L_C}\right)}{\left(\cos\left(A + B\frac{z_{n-1}}{L_C}\right) - \cos\left(A + B\frac{z_n}{L_C}\right)\right)^2},$$

$$P_{d2} = \left\{ B\frac{(z_n - z_{n-1})}{L_C}\cos\left(A + B\frac{z_{n-1}}{L_C}\right) + \sin\left(A + B\frac{z_{n-1}}{L_C}\right) - \sin\left(A + B\frac{z_n}{L_C}\right) \right\} \frac{\sin\left(A + B\frac{z_n}{L_C}\right)}{\left(\cos\left(A + B\frac{z_{n-1}}{L_C}\right) - \cos\left(A + B\frac{z_n}{L_C}\right)\right)^2},$$

$$P_S = f_P \frac{L_C}{B}\left(\frac{\cos\left(A + B\frac{z_{n-1}}{L_C}\right) - \cos\left(A + B\frac{z_n}{L_C}\right)}{z_n - z_{n-1}} \right),$$

$$\gamma = f_P \frac{L_C}{B}\left(\frac{\cos\left(A + B\frac{z_{n-1}}{L_C}\right) - \cos\left(A + B\frac{z_n}{L_C}\right)}{z_n - z_{n-1}} \right). \tag{9}$$

3. MODEL CALCULATIONS

The extended model has been applied to a reference case of a natural-circulation cooled BWR, the Dodewaard reactor[14]. In the current study we have only considered the impact on the thermal-hydraulic stability, since this is the basic mechanism that changes when multiple nodes are used to describe the travelling density waves in the boiling channel. As a boundary condition the pressure drop along the channel and riser is kept constant. In this way the total recirculation flow is constant but individual channels might experience so-called parallel-channel instabilities. The axial power profile is assumed to be flat, and the influence of integral slip is neglected.

The model with one node in the single-phase region and with one node in the two-phase region of the core has been linearized to study the stability in the frequency domain. In the frequency-domain model the delay in the riser is treated exactly instead of a nodal approximation. However, Van Bragt et al.[4] have shown that the stability boundary obtained with 4 riser nodes is very similar to the boundary obtained with an exact delay treatment of the density wave in the riser. So, the difference in results will mainly be caused by the treatment of the core region, when 4 riser nodes are applied.

The frequency-domain model is much more suitable to determine a stability boundary. By considering specific points one can also get an idea of the stability boundary with the time-domain model, although this is much more time-consuming.

First the marginal stability boundary has been studied. In the current study we have applied 4 nodes in the single-phase region, 4 nodes in the two-phase region of the core, and 4 nodes in the riser. Figure 1 shows the stability of specific operating points as compared with the stability boundary determined by the frequency-domain model with one node in both core regions and an exact treatment of the delay in the riser region.

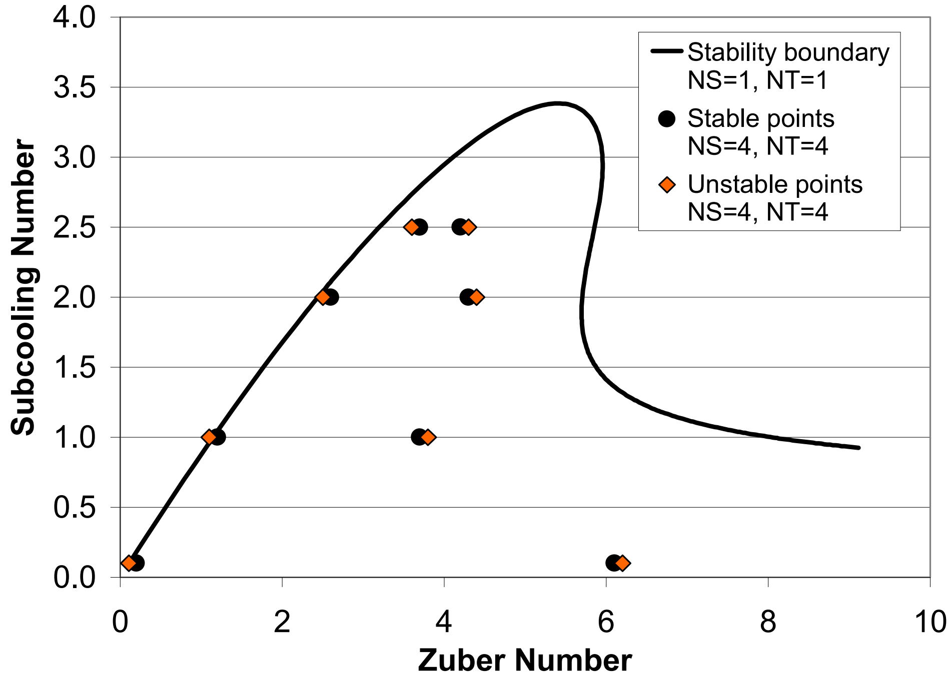

Figure 1. Marginal stability boundary of the boiling channel with riser determined with one node in the single-phase region (NS=1) and one node in the two-phase region (NT=1) compared with stable and unstable operating points according to the model with 4 nodes in both regions.

The figure shows that the stability of the boiling channel with riser is overestimated with one node in both regions. Especially for high-power and low-flow conditions, for which the interplay between the single-phase and two-phase friction pressure drops causes the instability, the system is predicted much more unstable with multiple nodes in both regions. This is in accordance with the results obtained by Karve et al.[10] The extra delay effect introduced by modeling multiple nodes destabilizes the boiling-channel model.

To show the travelling of the density waves in the boiling channel, some time traces of a limit-cycle oscillation are given in Figures 2 and 3. The limit cycle occurs close to the stability boundary at a subcooling number equal to 1 and a Zuber number equal to 3.8 (red diamond in Figure1).

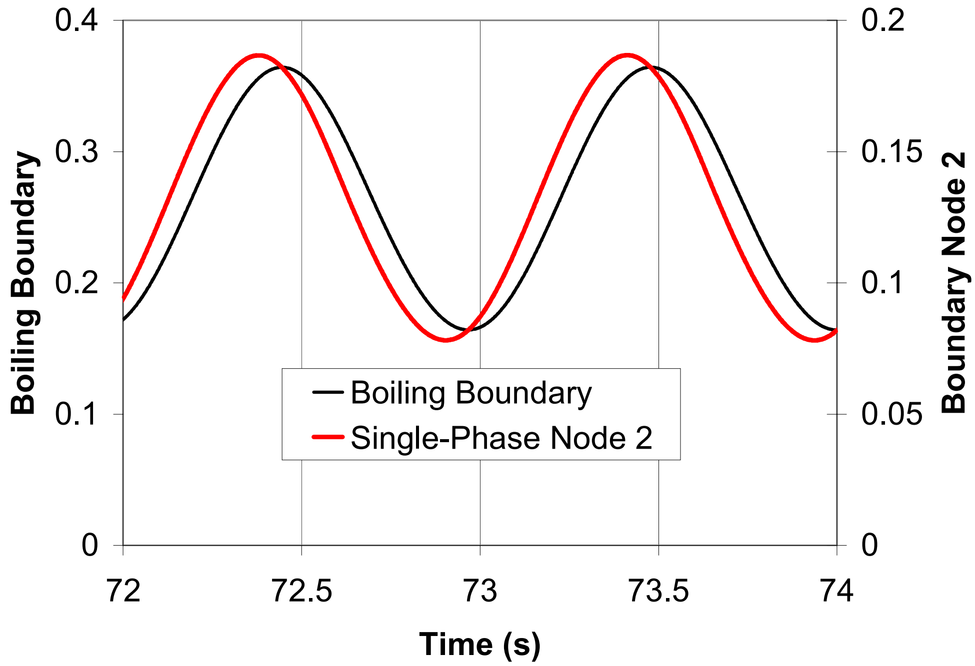

Figure 2. Dimensionless boiling boundary and single-phase boundary of node 2 (both normalized to core length) during limit cycle oscillation.

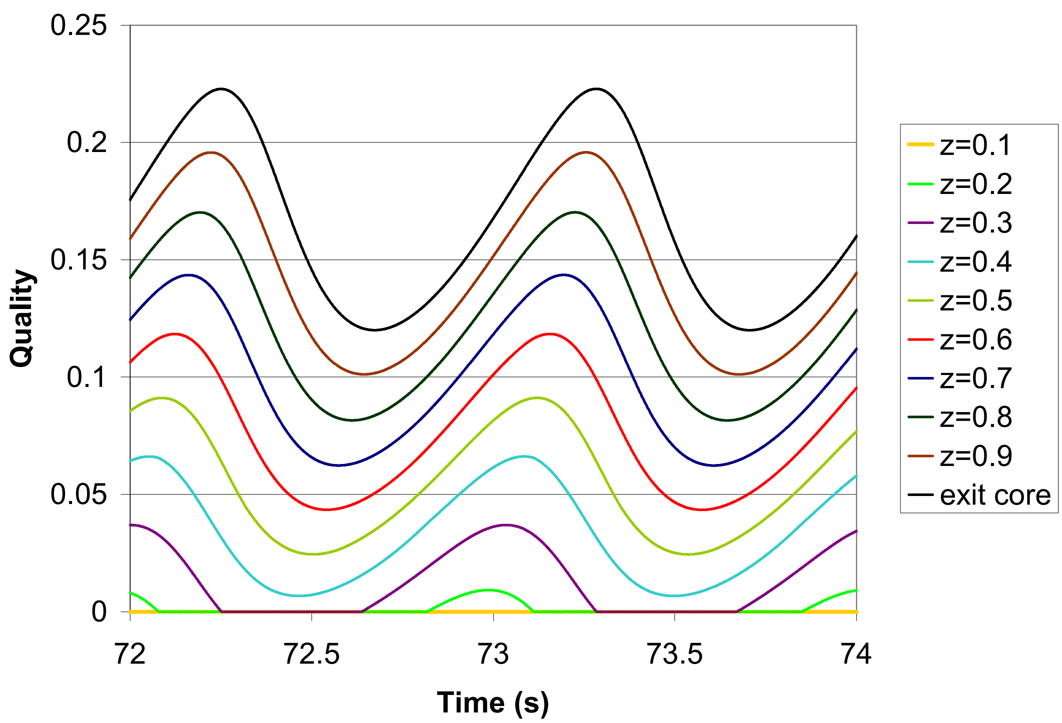

Figure 3. Quality at different axial positions during limit cycle oscillation. The quality increases with the axial position. The line for z=0.1 coincides with the x-axis.

These figures clearly show the delay effect introduced by modeling multiple nodes in both regions. Because of the chosen scales of the y-axes, the curves in Figure 2 would coincide when only one node in each region is modeled. With multiple nodes, the travelling of the density waves can be seen in the time traces of the node boundaries in the single-phase region and the quality in the two-phase region.

Our aim with these models is to obtain a better understanding of the phenomena that occur in the non-linear regime, as is measured in our test facility DESIRE[1]. The idea is to have a qualitative model that describes the phenomena that occur; for a quantitative comparison advanced thermal-hydraulic codes should be used.

As a first application it is investigated whether period doubling is predicted by our model of a boiling channel. For this reason the time traces of operating points with subcooling number equal to 1 and increasing Zuber number have been studied. As is shown in figure 4 no evidence of period doubling is found in this way. When the Zuber number is further increased the analytical model is no longer valid, since non-physical values of quality and boiling-boundary position are predicted.

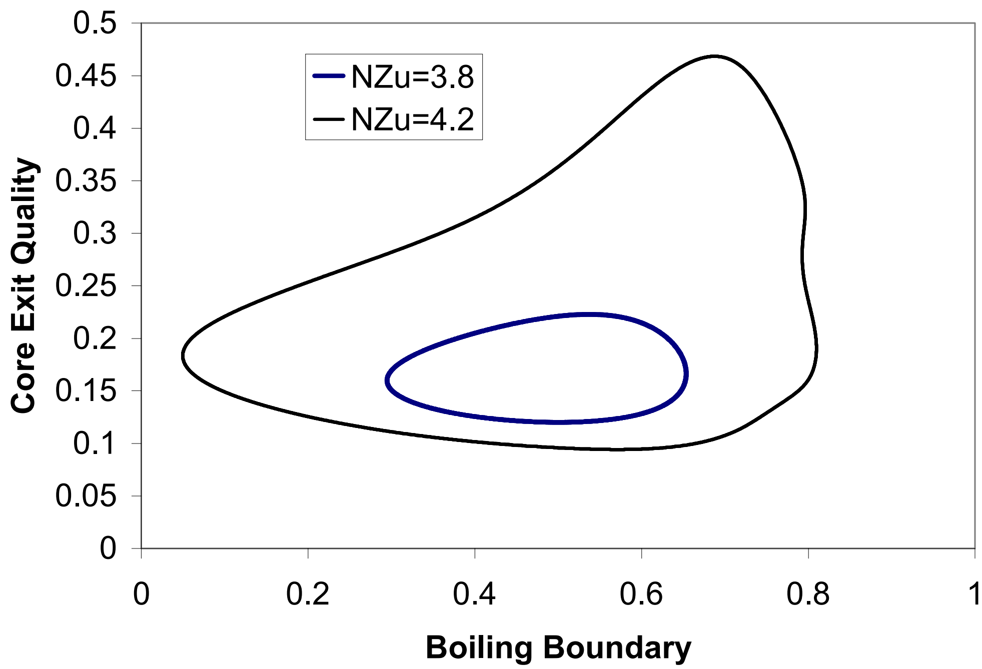

Figure 4. Limit cycle oscillations shown in the boiling boundary - core exit quality parameter space. As the Zuber number increases no sign of period doubling is observed.

The absence of period doubling for this specific case is of course only an indication, because we have not extensively searched the whole parameter space. More studies are needed to reveal the mechanism that causes the period-doubling bifurcations in boiling channels such as DESIRE.

A difference between the boundary condition in the experiments and the boundary condition in the current analysis is that in the experiment the pressure drop along the loop is zero whereas in the analysis a constant pressure drop along the channel and riser is used. Instead of using the subcooling at the core inlet as an independent variable, the feed-fluid temperature and feed-fluid flow should be used as independent variables in the natural-circulation model of a boiling channel to study loop instabilities.

CONCLUSIONS

The analytical model to study Boiling Water Reactor stability[7] has been extended by implementing multiple nodes in the single-phase region and in the two-phase region of the core. In this way the travelling of density waves in the core can be modeled. In comparison with predictions by the original model the stability of a reference boiling channel with riser decreases a lot, especially at high-power and low-flow conditions. In the unstable region, the model predicts the occurrence of limit-cycle oscillations. However, no period doubling was predicted at a specific operating condition.

NOMENCLATURE

List of Symbols:

A	area [m^2]
A	factor characterizing power profile
B	factor characterizing power profile
f_p	peaking factor
h	enthalpy [J/kg]
L	length [m]
M	mass flux density
$N_{1\varphi}$	number single-phase nodes
$N_{2\varphi}$	number two-phase nodes
P	power [W]
P(z)	axial power profile [W/m]
P_{d1}	factor determined by power profile
P_{d2}	factor determined by power profile
P_s	power fraction in single-phase node
q'	linear power density [W/m]
q''	heat flux density [W/m^2]
t	time [s]
z	axial position [m]
Z_{bb}	boiling boundary [m]
α	void fraction
γ	power fraction in two-phase node
ρ	density [kg/m^3]
χ	flow quality

Subscripts:

C	core
e	exit
f	fluid
g	gas
i	inlet
n-1	node n inlet
n	node n outlet or average in node n

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