COMPARISON OF RESONANCE SHIELDING METHODS FOR APPLICATION TO PWR CORE DESIGN

Hideki Matsumoto and Yoshihisa Tahara
Mitsubishi Heavy Industries, Ltd.,
3-1, Minatomirai 3-Chome, Nishi-ku,
Yokohama, 220-8401, Japan
matsu@atom.hq.mhi.co.jp and tahara@atom.hq.mhi.co.jp

ABSTRACT

Resonance shielding methods to treat radially non-uniform shielding effect within fuel pellet have been compared with continious energy Monte Carlo calculations for application to PWR core design code. To evaluate the power distribution within the pellet, the pellet is modeled as concentric cylinders. However, the rigorous effective resonance cross sections for each fuel cylinder cannot be obtained with the conventional Dancoff correction method. We need some additional correction or more detailed method such as Multiband method. The method introduced by Stoker and Weiss in 1996 that generates the spatially dependent resonance cross sections based on the calassical theory such as the extention of the well-known Dancoff method and the multiband method have been implemented into PWR design code PHOENIX-P for comparison with continuous energy Monte Carlo calculations.
The Stoker-Weiss method gives k_{inf} values agree well with those from continuous energy Monte Carlo calculations and the multiband method. The spatially dependent absorption rate within a pellet also agrees well. Thus it is found that the Stoker-Weiss method is practically useful and apprpopriate for the design calculation.

1. INTRODUCTION

Today's fuel integrity evaluation must take into account for the power distribution within pellet. To evaluate radial power distribution within the pellet, the pellet is usually modeled as concentric cylinders to treat the non-uniform nuclear transmutation effect. However, the rigorous effective resonance cross sections for each fuel ring cannot be obtained with the conventional Dancoff correction method[1] We need some additional correction or more detailed method such as Multiband Method[2]. With multiband method we can obtain the appropriate resonance cross sections for each ring[3]. However it takes very large computing time and memory to get the effective cross sections in an assembly calculation because the back and forth flux iterations are needed between the generation of effective cross sections and the calculation of the micro-region flux. The Stoker-Weiss method[4] can generate the spatially dependent resonance cross sections

much faster, because it dose not need such iterations. We have compared the methods for application to PWR core design code.

2. RESONANCE SHIELDING METHODS FOR SUB-DIVIDED FUEL REGIONS

2.1 MULTIBAND METHOD AND STOKER/WEISS METHOD

The spatially dependent resonance cross section of nuclide *k*, region *i* and energy *g* is obtained from multiband method, using the following Eq.(1) [2,3],

$$\sigma_{ig}^{k} = \frac{\sum_{B} P_{B,g}^{k} \cdot \sigma_{B,g}^{k} \cdot \phi_{i,g}^{B}}{\sum_{B} P_{B,g}^{k} \cdot \phi_{i,g}^{B}}, \tag{1}$$

$P_{B,g}^{k}$ and $\sigma_{B,g}^{k}$ are multiband probabilities and multiband cross sections for band *B* obtained from resonance integrals and $\phi_{i,g}^{B}$ is multiband flux obtained from the neutron transport calculation.

On the other hand, the Stoker-Weiss method gives the spatially dependent resonance cross-section, σ_{zig}^{k}, of nuclide *k*, region *i*, energy *g* and Dancoff factor *z* as the following Eq.(2) [4],

$$\sigma_{zig}^{k} = \frac{(1-z) I_{xg}^{k}\left(\sigma_{b}^{k}\right) + z\sum_{m=1}^{4} F_{m} \sum_{n=1}^{2} a_{n} I_{xg}^{k}\left(\sigma_{b,i}^{nmk}\right)}{1-(1-z)\frac{I_{ag}^{k}\left(\sigma_{b}^{k}\right)}{\sigma_{b}^{k}} - z\sum_{m=1}^{4} F_{m} \sum_{n=1}^{2} a_{n} \frac{I_{ag}^{k}\left(\sigma_{b,i}^{nmk}\right)}{\sigma_{b,i}^{nmk}}}, \tag{2}$$

where

$$\begin{cases} \sigma_{b}^{k} = \dfrac{\Sigma_{pf}}{N_{k}} \\ \sigma_{b,i}^{nmk} = \dfrac{\Sigma_{pf} + b_{n} / Y_{m}^{i}}{N_{k}} \end{cases}. \tag{3}$$

I_{xg}^{k} means a resonance integral of nuclide *k*, energy *g* and reaction *x*. Σ_{pf} is a background cross section in a fuel lump. The coefficient a_1, a_2, b_1 and b_2 are 2, -1, 2 and 3, respectively. N_k is the number density of nuclide *k*. The coefficients F_m are the functions of the mean chord length Y_m^i for ring *i*. The Eq.(2) corresponds to the extension of the well-known Dancoff equation to multi-region problems.

Those methods have been implemented into the two dimensional neutron transport code PHOENIX-P[5] and some benchmark calculations have been done using PHOENIX-P.

2.2 NUMERICAL RESULTS

Non-uniform shielding effect is present, even when the pellet is physically uniform, because the distance from the moderator region affects the resonance escape probability.
The OECD/NEA Pu recycle benchmark problem[6] was slightly modified for the comparison of resonance shielding methods from the restriction of nuclides available in the PHOENIX-P library, that is, Pu-238 was eliminated and temperatures are changed to 900(k) for pellet and 600(K) for both cladding and moderator regions. The cell calculations have been performed using the multiband method, the Stoker/Weiss method and the continuous energy Monte Carlo code MVP[7] with JENDL3.2[8]. To allow direct comparison of k_{inf} values, the multi-group cross section library for PHOENIX-P was generated from the same JENDL3.2 using the NJOY[9] code. The results are shown in Table I.

Table-I Comparison of k_{inf} values among Dancoff, Multiband and Stoker/Weiss Methods

	MVP (Reference)	PHOENIX-P		
		Dancoff (One region)	**Multiband (10 region)**	**Stoker/Weiss (10region)**
UO2 (4.1wt%)	**1.35067** **Std.0.0005**	1.34924 -0.11%	**1.35169** **+0.08%**	**1.35085** **+0.01%**
Better	**1.19059** **Std.0.0007**	1.19380 +0.27%	**1.19119** **+0.05%**	**1.19068** **+0.01%**
Dirty	**1.13917** **Std.0.0007**	1.13710 -0.18%	**1.13828** **-0.08%**	**1.13799** **-0.10%**

Note: All the calculations are based on JENDL3.2
The k_{inf} values are shown in upper column.
The standard deviation is shown in lower column of MVP.
The difference from MVP is shown in lower column of PHOENIX-P

From the Table I, it can be seen that k_{inf} values of the Stoker-Weiss method agree those of MVP and multiband methods. Figures 1 and 2 show comparisons of U-238 and U-235 absorption rate distribution in a UO_2 pellet among MVP, Multiband and Stoker-Weiss methods, respectively. From the figures, it can be seen that the multiband and Stoker-Weiss methods agree well with MVP results. Figure 3 shows comparison of resonance cross section of U-238 in outermost cylinder region of UO_2 pellet. Self-shielding effect in the outer region is lower and the effective cross section is larger than inner region. From the figure 3, it can be found that the Stoker-Weiss method generates energy-dependent effective cross section accurately in that region. Regarding CPU time, Stoker/Weiss method is 3 times faster than multiband method in the pin cell cases. The difference of CPU time between the multiband method and the Stoker-Weiss method would be much larger in multi-cell calculation because the multiband method needs more flux iterations for each region within the multi-cell model. For practical design calculations, the Stoker-Weiss

method seems to be accurate enough and useful. The Stoker-Weiss equation, Eq.(2), reduces to the well-known Dancoff equation in a case of one-region pellet. However, it can be seen from Table I that the k_{inf} values of the Stoker-Weiss method agree more with MVP than those of the Dancoff method.

With deplete fuel, non-uniform shielding effect is enhanced by non-uniform transmutation of heavy nuclides. To validate the method through depletion, a fuel cell with the spatially dependent isotopic concentration was employed for comparison with MVP. The specification of the calculation model is shown in Table II and Figure 4. The pellet was divided into 17 concentric cylinders. First, the whole pellet was divided into 10 equal volume rings. Then the outermost ring was again divided into 8 equal volume rings. The peripheral rings represent very thin region called rim. The k_{inf} value obtained from MVP calculation with 5 million histories is 1.04653±0.02%. That of Stoker/Weiss method is 1.04543. The difference from the MVP result is 0.11%. Figure 5 shows the comparison between the Stoker/Weiss and MVP with respect to the absorption rate distribution by main heavy nuclides. It can be seen that good agreement has been obtained for all nuclides. So, the Stoker/Weiss method can be applied to depletion calculation.

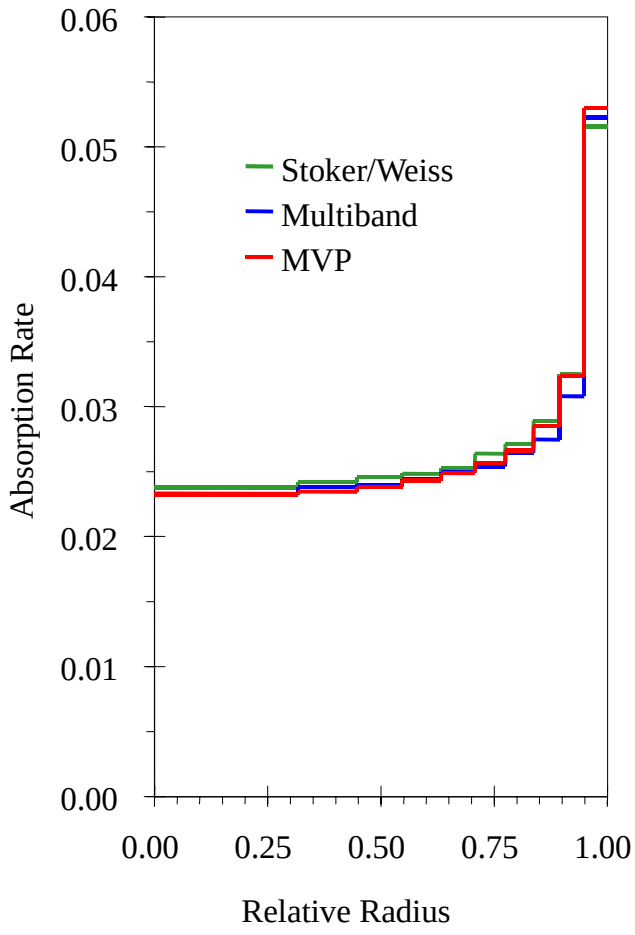

Figure 1 Comparison of U-238 Absorption Rate Distribution in a UO_2 pellet.

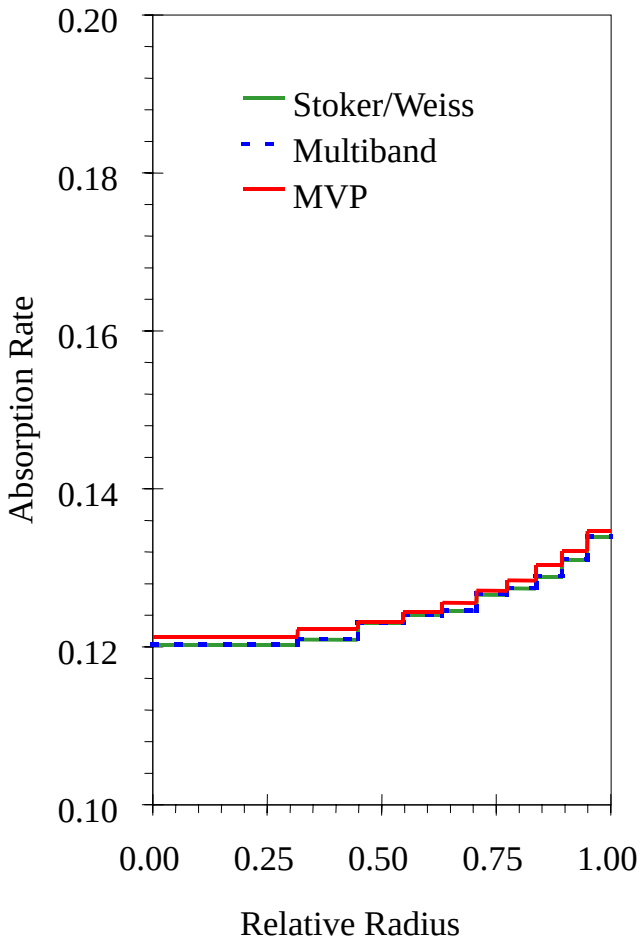

Figure 2 Comparison of U-235 Absorption Rate Distribution in a UO_2 pellet.

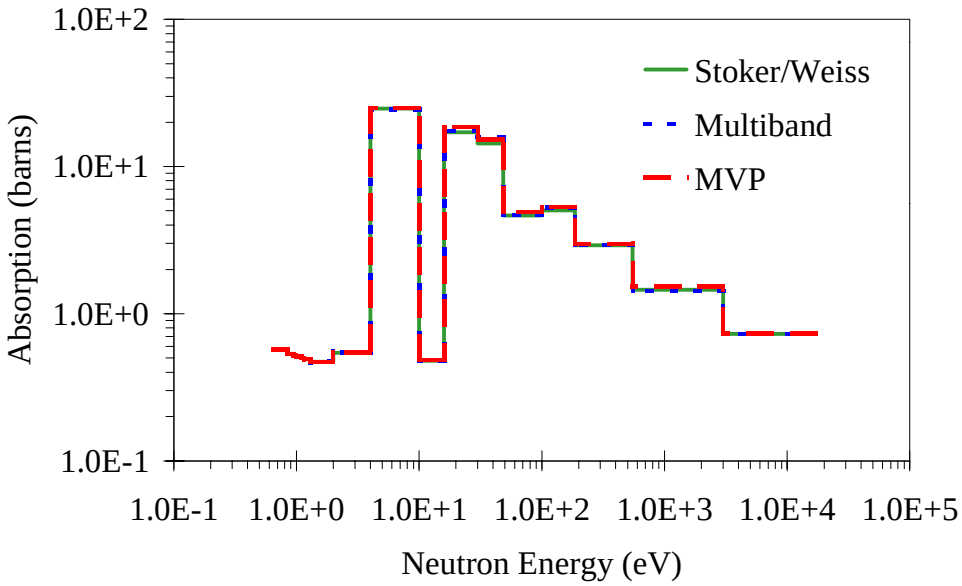

Figure 3 Comparison of U-238 absorption cross section in outer cylinder region of UO_2 pellet

Table II Specification of the test model for depletion calculation

Cell pitch	1.265 cm
Fuel	UO_2
Radius	0.412 cm
Enrichment(BOL)	4.1wt%U-235
Temperature	900 K
Burnup(Average)	62 GWd/t
Cladding	Zr-4
Outer Radius	0.476 cm
Thickness	0.064 cm
Temperature	600 K
Moderator	H_2O
Temperature	600 K
Boron concentration	500 ppm

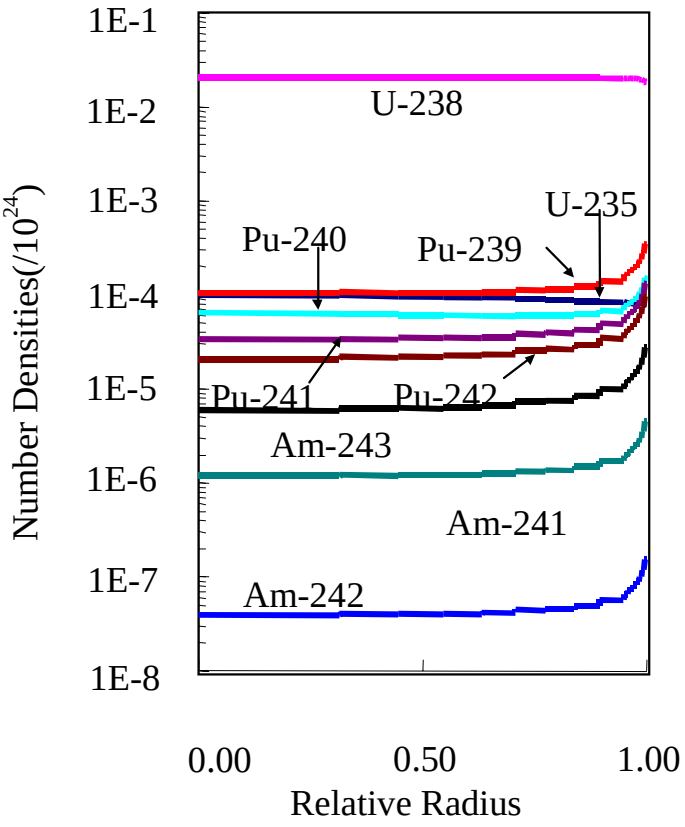

Figure 4 Number densities in a test model for depletion calculation

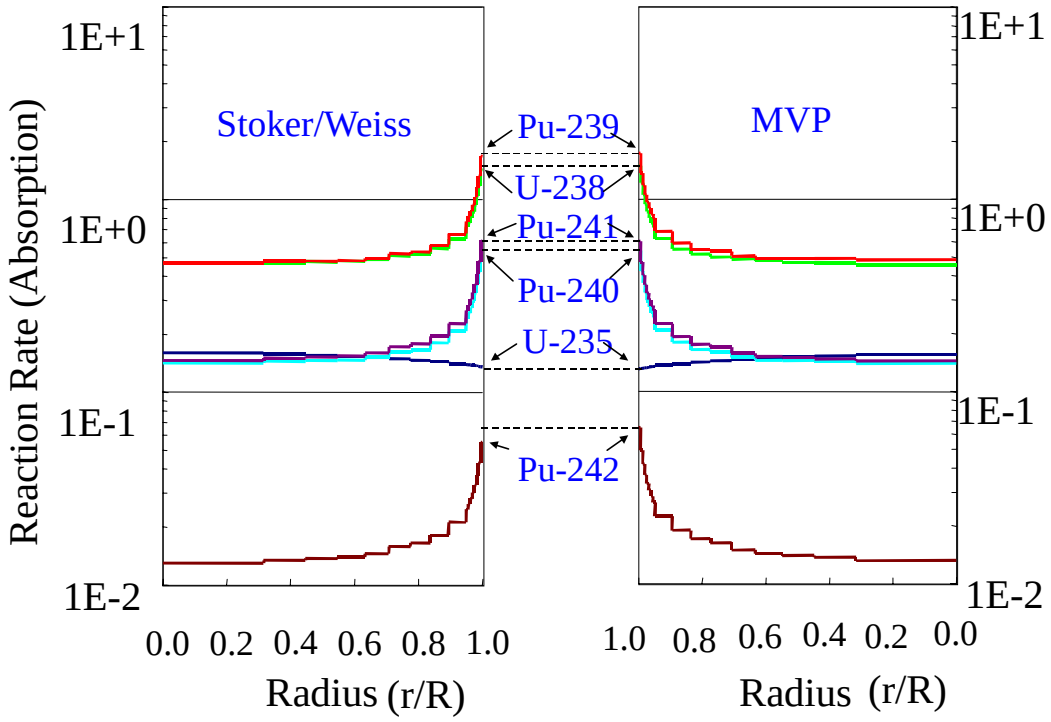

Figure5 Comparison of microscopic absorption rate distribution in a 62 GWd/t UO_2 fuel.

2.3 IMPROVEMENT AND IMPLEMENTATION

In the conventional Dancoff method the Bell factor is usually applied to correct the transmission probabilities between fuel to fuel. Because the Stoker-Weiss method is based on similar rational approximation in calculating transmission probability, we introduced the Bell factor of 1.06 to the Dancoff factor *z* in Eq.(2). It is determined by the comparison of k_{inf} values and U-238 absorption rate between the one region Stoker-Weiss case and the conventional Dancoff case with well-established Bell correction

For PWR design calculation, we have developed a new 70-group library based on ENDF/B-VI[10]. And a two-dimensional transport code PHOENIX-CP has been also developed. PHOENIX-CP is a current coupling collision probability (CCCP) code. It employs Discrete Angular Flux Method[11] based on collision probability. It has been validated with the ENDF/B-VI library as described in Ref.(10).

The Stoker-Weiss method with the 1.06 Bell factor has been implemented into PHOENIX-CP. Some benchmarks for application to PWR core design have been performed using PHOENIX-CP with the Stoker/Weiss method including the 1.06 Bell factor. Those results are shown in the next section.

2.4 BENCHMARKS FOR APPLICATION TO PWR CORE DESIGN

To validate the Stoker/Weiss method for PWR design use, following cold benchmarks were selected. Monte Carlo models for those sets of benchmarks had been set up and descried in Ref.(12).
(1)17X17 PWR assembly benchmarks
(2)14X14 PWR assembly benchmarks
(3)Numerical benchmarks with high plutonium contents and high U-235 enrichment
Comparison of k_{inf} values between PHOENIX-CP with Stoker/Weiss and the continuous energy Monte Carlo code MCNP are shown in Table III through Table V. In those benchmarks, MCNP was selected as same as in Ref.(10) and Ref.(12).

The brief description of the 17X17 PWR assembly benchmarks shown in Table III is given below:
(1)MOX assembly with typical Pu isotopic composition(6.1w/o Pu-fissile content)
(2)MOX assembly with worst Pu isotopic composition(6.0w/o Pu-fissile content)
(3)UO_2 assembly with 4.1w/oU-235 and 16 rods of 6w/oGd_2O_3
(4)UO_2 assembly with 4.8w/oU-235 and 24 rods of 9w/oGd_2O_3

The typical Pu isotopic composition corresponds to the following Pu vector(w/o); ^{238}Pu: ^{239}Pu: ^{240}Pu: ^{241}Pu: ^{242}Pu: ^{241}Am =1.9: 57.5: 23.3: 10.0: 5.4: 1.9. The ^{240}Pu content of the worst case is about twice as large as that of the typical case. The results are shown in Table III. It can be seen that the Dancoff case and the one region Stoker/Weiss case agree very well, the differences between one region case and 10 region case of Stoker/Weiss method exist but are small from practical point of view. It can be also seen from Table III that the each case of PHOENIX-CP agrees well with MCNP result.

Table III k_{inf} Benchmarks for 17X17 assembly

	PHOENIX-CP			MCNP
	Dancoff (One region)	**Stoker/Weiss (One Region)**	**Stoker/Weiss (10region)***	
6.1w/o Pu Typical	**1.20074 (-0.01%)**	**1.20125 (+0.03%)**	**1.20075 (-0.01%)**	**1.20088 Std.0.0007**
6.0w/o Pu Worst	**1.14038 (-0.07%)**	**1.14051 (-0.06%)**	**1.13986 (-0.11%)**	**1.14117 Std.0.0006**
UO_2 with 6w/oGd_2O_3	**1.17899 (-0.18%)**	**1.17924 (-0.16%)**	**1.17957 (-0.14%)**	**1.18117 Std.0.0006**
UO_2 with 9w/o Gd_2O_3	**1.13788 (-0.24%)**	**1.13784 (-0.25%)**	**1.13820 (-0.21%)**	**1.14065 Std.0.0007**

* In MOX cases, all fuel pins are divided into concentric 10 cylinders.
In UO_2 cases, fuel rods including Gd_2O_3 are divided into concentric 10 cylinders.
Upper values denote k_{inf} and lower values denote relative difference from MCNP result.

A 14X14 assembly calculation was also done and good agreement among the methods can be seen in Table IV. We performed the numerical cell benchmarks with high Plutonium contents

proposed by OECD/NEA Pu recycle cell. The result is shown in Table V. The agreement among both the methods of PHOENIX-CP and MCNP is good. And the good agreement between the conventional Dancoff and the one region Stoker-Weiss method also can be seen.
It is found that the Stoker-Weiss method with the 1.06 Bell factor works well and the method is adequate for the generation of the effective resonance cross sections.

Table IV k_{inf} Benchmarks for 14X14 assembly

	PHOENIX-CP			MCNP
	Dancoff (One region)	**Stoker/Weiss (One Region)**	**Stoker/Weiss (10region)***	
4.0w/o	**1.29520 (-0.11%)**	**1.29447 (-0.16%)**	**1.29356 (-0.23%)**	**1.29659 Std.0.0006**

* All fuel pins are divided into concentric 10 cylinders.
Upper values denote k_{inf} and lower values denote relative difference from MCNP result.

Table V k_{inf} Additional Cell Benchmarks

	PHOENIX-CP			MCNP
	Dancoff (One region)	**Stoker/Weiss (One Region)**	**Stoker/Weiss (10region)***	
Better	1.22415 (+0.11%)	**1.22335 (+0.04%)**	**1.22495 (+0.18%)**	**1.22281 Std.0.0008**
Dirty	1.17441 (+0.05%)	**1.17239 (-0.12%)**	**1.17436 (+0.04%)**	**1.17384 Std.0.0008**
6.1w/o Pu	1.20832 (-0.12%)	**1.20710 (-0.22%)**	**1.20900 (-0.06%)**	**1.20975 Std.0.0007**
4.1w/o U	1.36890 (-0.19%)	**1.36880 (-0.20%)**	**1.36822 (-0.24%)**	**1.37152 Std.0.0006**

* Each pin is divided into concentric 10 cylinders.
Upper values denote k_{inf} and lower values denote relative difference from MCNP result.

2.4 VALIDATION FROM ISOTOPICS

The UO_2 and MOX cell depletion calculations were compared with destructive measurement data.[13] The burnup distributions are shown in Figure 6 with measurements. For UO_2 cell calculation, the pellet was divided into 17 concentric cylinders in the same manner as in 2.2. The peripheral rings represent very thin region called rim. For MOX cell calculation, the pellet was divided into 10 regions. Very good agreement can be seen in both UO_2 and MOX cases.
With the Stoker-Weiss method with Bell correction the power distribution within fuel pellet can be predicted accurately.

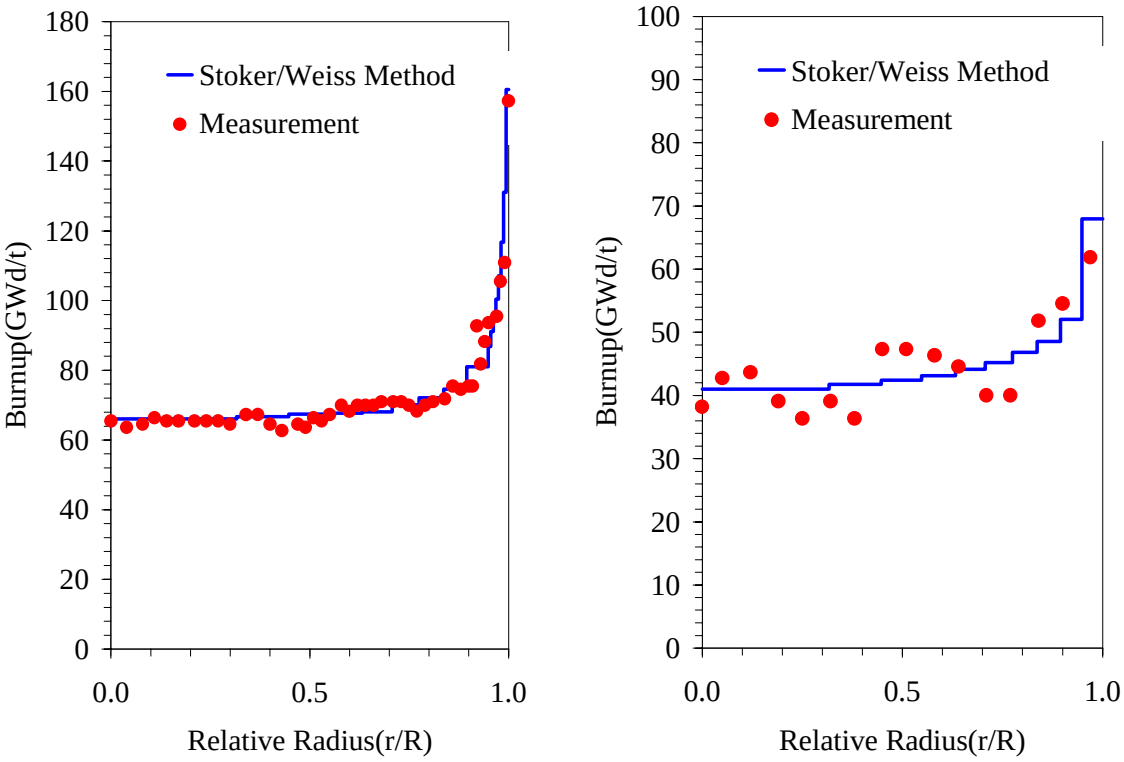

Figure 6 Comparison of burnup distribution in a UO_2 and MOX fuel pellets

CONCLUSIONS

Spatially dependent resonance shielding methods: multiband method; Stoker-Weiss method, have been compared with MVP and MCNP from the viewpoint of accuracy of reactivity calculation and the radial distribution of reaction rates within fuel pellets. Theoretically, the multiband method is superior to the conventional Dancoff method because the multiband method can treat any regions as resonant region and also can treat the heterogeneity directly through the multiband neutron flux. The Stoker-Weiss method treats fuels as black region such as the conventional Dancoff method. So, it is thought that the accuracy and applicability of the method is of same grade as the conventional Dancoff method. We employed Bell correction in Stoker-Weiss method. The present method can generate the spatially dependent resonance cross section very faster without large amount of computer memory. Its k_{inf} values agree well with those of MVP, MCNP and the multiband method. The spatially dependent absorption rates in a pellet also agrees very well and the prediction of burnup distribution in a fuel pellets are excellent. So, it can be said that the Stoker-Weiss method is a practical method for design. To improve the calculation accuracy of the Stoker-Weiss method, we will try to study the way further to sophisticate Bell correction method.

ACKNOWLEDGEMENTS

The authors would like to thank Dr. Y.A.Chao and Dr.M.Ouisloumen for their helpful suggestions and valuable advice.

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