APPLICATION OF THE SURFACE PSEUDOSOURCES METHOD TO MULTIGROUP CALCULATION OF THE BWR REACTOR CELL WITH A MOX FUEL

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ABSTRACT

The spliting operation method has been applied for solving the multigroup transport equation with the general anisotropic scattering with solving the spatial-angular part of a problem by the surface pseudosources method. Using the approximation of the "plane" source for solving onegroup problems has allowed to obtain the resulting algebraic equation of a small order for the nodes (the one node on the cell homogeneous zone of any optical thickness). Therefore calculation time of a whole cell has been little. The calculations of the BWR typical cells with the MOX fuel show that the difference in the K_∞ value between the transport and linearly anisotropic approximations can achieve 0,5%. The errors introduced by replacing actual (square) cell boundary by a cylindrical one can also achieved 0,5%. These two effects have contrary signs. Thus, the difference in the K_∞ value calculated, firstly, in the transport approximation with the white boundary condition in the cylindicalized cell and, secondly, in the linealy anisotropic approximation in the square cell becomes 0,1-0,2%.

1. INTRODUCTION

In actual reactors the lattice is usually square, hexagonal or triangular; however, most practical methods do not solve the actual cell but, instead, solve the equivalent cylindricalized cell under the condition: their volumes are the same. Such replacement allows essentially to simplify a problem. The errors introduced by replacing the actual cell boundary by a cylindrical one is subject of recurring interest that has been studied by many authors (see, for example, [1,2]). An outcome of this has been the wide acceptance that an isotropic boundary condition is to be preferred in the cylindricalized cell. This is really so for the pin cells with the UO_2 fuel [1-4]. The calculations of the pin cells with the MOX fuel shown that the errors introduced by replacing the actual cell boundary by a cylindrical one can achieve 0,5-0,6% in the K_∞ value [4-7]. Therefore in these pin cells it is necessary to take into account the actual cell boundaries. This makes formidable demands on computer time for many methods [2]. A computer time does not practically increase in the surface pseudosources method which can take into account the actual cell boundary because the large analytic work has been made before [6].

In pin cell calculations the anisotropic scattering is usually taken into account in the transport and sometimes in the linearly anisotropic approximations (see, for example, [8-12]). The transport approximation allows to solve a problem in the isotropic approximation with a corresponding correction of the scattering cross-section. The use of the linearly anisotropic approximation even in the onegroup case (for example, in the first collision probability method, often being used in cell calculations) reduces to complication of a problem and to an increase of calculation time. The surface pseudosources method can enough simply take into account the anisotropic scattering without an increase of calculation time [12].

The aim of this paper has been more accurate to define of the mathematical model for the multigroup calculation of the transport equation with the general anisotropic scattering by the spliting operation method with solving the spatial-angular part of a problem by the surface pseudosources method. Having elaborated method has been realized in the option RATIA of the complex WIMS-SH-2.0 [21]. The calculations of the BWR typical cells with the MOX fuel show that the difference in the K_∞ value between the transport and linearly anisotropic approximations can achieve 0,5%. The errors introduced by replacing actual (square) cell boundary by a cylindrical one can also achieved 0,5%. These two effects have contrary signs. In result they are essentially compensated each other. Thus, the difference in the K_∞ value calculated, firstly, in the transport approximation with the white boundary condition in the cylindicalized cell and, secondly, in the linealy anisotropic approximation in the square cell becomes 0,1-0,2%. It should be noted that the calculations in the transport approximation with the white boundary condition in the cylindicalized cell are widely used in the modern codes (see, for example, [4]). The calculations in the linealy anisotropic approximation and in the square cell are seldom used (see, for exaple, [7]).

In pin cell calculations the spliting operation method with solving the spatial-angular part of a problem by the surface pseudosources method was used in papers [3,6,10-12]. The advantages of this approach over the other transport methods (for example, over the spherical harmonic method in the P_N-approximation, over the discrete ordinate method in the S_n-approximation, over the characteristic method, over the first collision probability method and over others) are shown in paper [13]. The surface pseudosources method is with the integral method, therefore below it will be compared with the first collision probability method.

2. THE SPLITING OPERATION METHOD

A multizone cell is given. An each zone of this cell is described with the differential scattering cross sections $\Sigma_{sg\leftarrow g'}\left(\vec{\Omega}'\rightarrow\vec{\Omega}\right)$, with the group cross sections of absorption Σ_{ag}, of scattering Σ_{sg}, and of fission Σ_{fg}. These group cross sections are constant in an each zone of a cell. Inside of a cell the multigroup neutron transport equation is solved:

$$\vec{\Omega}\vec{\nabla}\Psi_g\left(\vec{r},\vec{\Omega}\right)+\Sigma_{t,g}\left(\vec{r}\right)\Psi_g\left(\vec{r},\vec{\Omega}\right)=\sum_{g'=1}^{G}\int_{\vec{\Omega}'}\Sigma_{s,g\leftarrow g'}\left(\vec{\Omega}'\rightarrow\vec{\Omega}\right)\Psi_{g'}\left(\vec{r},\vec{\Omega}'\right)d\vec{\Omega}'+\frac{\chi_g\left(\vec{r}\right)}{K_\infty}\sum_{g'=1}^{G}\nu\Sigma_{fg'}\Psi_{g'0}\left(\vec{r}\right),\quad(1)$$

where χ_g is the neutron spectrum of fission,
with the boundary condition:

$$\int \left(\vec{\Omega}\vec{n}\right)\Psi_g\left(\vec{r}_S,\vec{\Omega}'\right)d\vec{\Omega}' = \text{tok}_g \qquad (2)$$

When $\text{tok}_g = 0$ for $1 \le g \le G$ in the boundary condition (2) we will determine the eigenvalue K_∞ in equation (1). When $\chi_g = 0$ for $1 \le g \le G$ in equation (1), we will solve the problem with the external sources (there are either slowing-down sources or neutron group currents ($\text{tok}_g \neq 0$ for $1 \le g \le G$ in equation (2)), coming in the cell through an external surface, or both simultaneously). We will consider the case when the differential scattering cross-section $\Sigma_{sg\leftarrow g'}\left(\vec{\Omega}' \to \vec{\Omega}\right)$ may be written as

$$\Sigma_{sg\leftarrow g'}\left(\vec{\Omega}' \to \vec{\Omega}\right) = \Sigma_{sg\leftarrow g'}\left(\vec{\Omega}' * \vec{\Omega}\right) \qquad (3)$$

Expanding $\Sigma_{sg\leftarrow g'}\left(\vec{\Omega}' * \vec{\Omega}\right)$ in series in the Legendre polynomials, we obtain

$$\Sigma_{sg\leftarrow g'}\left(\vec{\Omega}' * \vec{\Omega}\right) = \sum_{p=0}^{\infty} \frac{2p+1}{4\pi} \Sigma_{sg\leftarrow g' p} P_p\left(\vec{\Omega}' * \vec{\Omega}\right) \qquad (4)$$

where $\Sigma_{sg\leftarrow g' p} = 2\pi \int_{-1}^{1} \Sigma_{sg\leftarrow g'}\left(\vec{\Omega}' * \vec{\Omega}\right) P_p\left(\vec{\Omega}' * \vec{\Omega}\right) d\mu$

From the theorem of the addition for the Legendre polynomials it is known the following relation:

$$P_p\left(\vec{\Omega}' * \vec{\Omega}\right) = P_p^0(\mu) P_p^0(\mu') + 2\sum_{k=1}^{p} \frac{(p-k)!}{(p+k)!} P_p^k(\mu) P_p^k(\mu') \cos k(\alpha - \alpha')$$

where μ_0, μ, μ' are the cosines: $\mu_0 = \left(\vec{\Omega}' * \vec{\Omega}\right)$, $\mu = (\vec{\Omega} * \frac{\vec{r}}{r})$, $\mu' = (\vec{\Omega}' * \frac{\vec{\rho}}{\rho})$; α, α' are the azimuthal angles; $P_p^k(\mu)$ are the associated Legendre polynomials. These angles must satisfy to the relation:

$$\mu_0 = \mu\mu' + \sqrt{1-\mu^2}\sqrt{1-\mu'^2}\cos(\alpha - \alpha').$$

We restrict our attention to the case where the differential scattering cross-section (4) will have the p terms. Then the expression (4) becomes:

$$\Sigma_{sg\leftarrow g'}\left(\vec{\Omega}' * \vec{\Omega}\right) =$$

$$\sum_{p=0}^{P} \frac{2p+1}{4\pi} \Sigma_{sg\leftarrow g' p} \left[P_p^0(\mu_1) P_p^0(\mu_2) + 2\sum_{k=1}^{p} \frac{(p-k)!}{(p+k)!} P_p^k(\mu) P_p^k(\mu') \cos k(\alpha - \alpha') \right] \qquad (5)$$

Substituting this expression into equation (1) and integrating over all directions $\vec{\Omega}'$, the multigroup equation may be written as:

$$\vec{\Omega}\vec{\nabla}\Psi_g\left(\vec{r},\vec{\Omega}\right) + \Sigma_{t,g}\left(\vec{r}\right)\Psi_g\left(\vec{r},\vec{\Omega}\right) = \frac{\chi_g\left(\vec{r}\right)}{K_\infty} \sum_{g'=1}^{G} \nu\Sigma_{fg'} \Psi_{g'00}\left(\vec{r}\right) +$$

$$\sum_{g'=1}^{G}\sum_{p=0}^{P} \frac{(2p+1)(1+\delta_{0p})}{2} \Sigma_{sg\leftarrow g' p0} \left[\Psi_{g' p0}(\vec{r}) Y_p^0(\vec{\Omega}) + 2\sum_{k=1}^{p} \frac{(p-k)!}{(p+k)!} \Psi_{g' pk}(\vec{r}) Y_p^k(\vec{\Omega}) \right] \qquad (6)$$

To solve equation (6) with boundary condition (2), the spliting operation method is applied. The essence of this method is reduced to the following way. Equation (6) is reorganized to the equivalent system of equations:

$$\begin{cases} \vec{\Omega}\vec{\nabla}\Psi_g\left(\vec{r},\vec{\Omega}\right)+\Sigma_{t,g}\left(\vec{r}\right)\Psi_g\left(\vec{r},\vec{\Omega}\right)=\sum_{p=0}^{P}\frac{(2p+1)(1+\delta_{0p})}{2}\Sigma_{sg'p}\left[\Psi_{g'p0}(\vec{r})Y_p^0(\vec{\Omega})+\right. \\ \left.+2\sum_{k=1}^{p}\frac{(p-k)!}{(p+k)!}\Psi_{g'pk}(\vec{r})Y_p^k(\vec{\Omega})\right]+S_g\left(\vec{r},\vec{\Omega}\right) \\ S_g\left(\vec{r},\vec{\Omega}\right)=\sum_{p=0}^{P}\frac{(2p+1)(1+\delta_{0p})}{2}\left\{\left[\sum_{g'=1}^{G}\Sigma_{sg\leftarrow g'p}\Psi_{g'p}(\vec{r})-\Sigma_{sgp}\left(\vec{r}\right)\Psi_{gp}\left(\vec{r}\right)\right]Y_p^0(\vec{\Omega})+.....(7)\right. \\ \left.+2\sum_{k=1}^{p}\frac{(p-k)!}{(p+k)!}\left[\sum_{g'=1}^{G}\Sigma_{sg\leftarrow g'p0}\Psi_{g'pk}(\vec{r})-\Sigma_{sgpo}\left(\vec{r}\right)\Psi_{gpk}\left(\vec{r}\right)\right]Y_p^k(\vec{\Omega})\right\}+ \\ +\frac{\chi_g(\vec{r})}{K_\infty}\sum_{g'=1}^{G}\nu\Sigma_{fg'}\Psi_{g'0}\left(\vec{r}\right) \end{cases}$$

As you can see from the equation system (7), the first equation is responsible for spatial-angular part of a problem (the group index g enters in this equation as a parameter, therefore all the onegroup equations may be solved separately). The second equation is responsible for the energetic part of a problem because of the elastic isotropic and anisotropic scattering and because of a fission of the nucleuses in the fuel zone. Thus, the problem (7) - (2) is splited on two relatively independent problems. The surface pseudosources method (see, for example, [15]) is used for solving the onegroup equations. Then the equation system (7) may be written as:

$$\begin{cases} \Psi_g^z\left(\vec{r},\vec{\Omega}\right)=\int_{\vec{\Omega}'}\int_{V}S_g^z\left(\vec{r}'\right)G^z\left(\vec{r},\vec{\Omega}/\vec{r}',\vec{\Omega}'\right)d\vec{\Omega}'dV+\int_{\Omega}\int_{s}g_s^z\left(\vec{r}',\vec{\Omega}'\right)G^z\left(\vec{r},\vec{\Omega}/\vec{r}',\vec{\Omega}'\right)d\vec{r}_s'd\vec{\Omega}' \\ S_g\left(\vec{r},\vec{\Omega}\right)=\sum_{p=0}^{P}\frac{(2p+1)(1+\delta_{0p})}{2}\left\{\left[\sum_{g'=1}^{G}\Sigma_{sg\leftarrow g'p}\Psi_{g'p}(\vec{r})-\Sigma_{sgp}\left(\vec{r}\right)\Psi_{gp}\left(\vec{r}\right)\right]Y_p^0(\vec{\Omega})+\right. \\ \left.+2\sum_{k=1}^{p}\frac{(p-k)!}{(p+k)!}\left[\sum_{g'=1}^{G}\Sigma_{sg\leftarrow g'p0}\Psi_{g'pk}(\vec{r})-\Sigma_{sgp0}\left(\vec{r}\right)\Psi_{gpk}\left(\vec{r}\right)\right]Y_p^k(\vec{\Omega})\right\}+ \\ +\frac{\chi_g(\vec{r})}{K_\infty}\sum_{g'=1}^{G}\nu\Sigma_{fg'}\Psi_{g'0}\left(\vec{r}\right) \end{cases} .....(8)$$

where $\Psi_g^z\left(\vec{r},\vec{\Omega}\right)$ is the neutron distribution function in the z zone (z≤Z); $G^z\left(\vec{r},\vec{\Omega}/\vec{r}',\vec{\Omega}'\right)$ is the Green function for the homogeneous infinite medium, consisting of a z zone material; $g_s^z\left(\vec{\mathbf{r}}_s,\vec{\Omega}'\right)$ is a capacity of the surface pseudosources, located on the boundaries of the z zone; they are determined on each interior

surface of the z zone and are antisymmetric with respect to angular variable $\vec{\Omega}$, i.e. $g_s\left(\vec{r}_s,-\vec{\Omega}\right)=-g_s\left(\vec{r}_s,\vec{\Omega}\right)$; $s_g^z\left(\vec{r},\vec{\Omega}\right)$ is a capacity of the volume sources, coinciding within the z zone with the volume sources of this zone and continued without the z zone to an infinity with some way (see, for example, [15]); S, V is the surface and the volume of the z zone accordingly.

We will begin to solve the equation system (8) from the second equation:

$$S_g\left(\vec{r},\vec{\Omega}\right)=\sum_{p=0}^{P}\frac{(2p+1)(1+\delta_{0p})}{2}\left\{\left[\sum_{g'=1}^{G}\Sigma_{sg\leftarrow g'p}\Psi_{pg'}(\vec{r})-\Sigma_{sgp}\left(\vec{r}\right)\Psi_{gp}\left(\vec{r}\right)\right]Y_p^0(\vec{\Omega})+\right.$$

$$\left.+2\sum_{k=1}^{p}\frac{(p-k)!}{(p+k)!}\left[\sum_{g'=1}^{G}\Sigma_{sg\leftarrow g'p0}\Psi_{g'pk}(\vec{r})-\Sigma_{sgp0}\left(\vec{r}\right)\Psi_{gpk}\left(\vec{r}\right)\right]Y_p^k(\vec{\Omega})\right\}+\text{.....}\quad\text{.....}\quad\text{.....}(9)$$

$$+\frac{\sigma_g(\vec{r})}{K_\infty}\sum_{g'=1}^{G}\nu\Sigma_{fg'}\Psi_{g'0}\left(\vec{r}\right)$$

It should be noted that the source $s_g\left(\vec{r},\vec{\Omega}\right)$ is anisotropic with respect to the angular variable $\vec{\Omega}$.

The expression in the quadratic brackets of equation (9) is the derivative of the angular moment of the neutron current on the energetic axis. We remind that the angular moment from the neutron current on the energetic axis has the following form:

$$Q_{pk}^z\left(\vec{r},E\right)=\int_0^E\left[\int_E^\infty\Sigma_{sp0}^z(E'\to E'')\Psi_{pk}^z(\vec{r},E')dE'\right]dE''-$$

$$-\int_E^\infty\left[\int_0^E\Sigma_{sp0}^z(E'\to E'')\Psi_{pk}^z(\vec{r},E')dE'\right]dE''$$

The derivative of the angular moment of the neutron current on the energetic axis may be written as:

$$\frac{dQ_{pk}^z\left(\vec{r},E\right)}{dE}=\int_0^\infty\Sigma_{sp0}^z(E'\to E)\Psi_{pk}^z(\vec{r},E')dE'-\Sigma_{sp0}^z(E)\Psi_{pk}^z(\vec{r},E)$$

The approximations of the "plane" flux and of the "plane" current are usually used in the first collision probability method. If these approximations are substituted in equation (9), we obtain the approximation of the "plane" sources. Below we will use the approximation of the "plane" sources which do not reduces to the approximations of the "plane' flux and of the "plane" current. For example, the "plane" sources are in the each zone of a cell. We solve the onegroup transport equation by the surface pseudosources method. The flux and the current are changed continuously from the one spatial point to the other one. We note that though the surface pseudosources method is with the integral method, using it we can determine a flux, a current and other angular moments of the neutron distribution function at any spatial point of a cell. The derivative from the energetic current (expressions in the quadratic brackets of equation (9)) appears to be more smooth function from the spatial variable than the neutron distribution function $\Psi_g\left(\vec{r},\vec{\Omega}\right)$ [15]. Calculations of cells both with the optical thin zones and with the optical thick (up to the few optical lengths of the free neutron path) zones shown that we can keep only

the one function of the simple form in the cell zone. For example, one constant for any zone was kept in calculations both of the VVER-1000 pin cells [14,16] and of the RBMK cluster cells [14,17]. Thus, using the approximation of the "plane" sources allow to obtain the resulting equation with respect to the nodes. It is with the algebraic equation of a few order, but time for its calculation will be little [14]. Therefore below we how in case of the isotropic scattering prefer the approximation of "plane" sources. As a consequence we will approximate the current with one simple function.

We introduce the function $j_j^z\left(\vec{r},\vec{\Omega}\right)$ for the approximation of the neutron sources (9). As you can see from equation (9), the spherical harmonics are used for expanding the neutron sources with respect to the angular variable $\vec{\Omega}$. The obvious form of function, connected with the spatial variable $\vec{r}$, will be determined below. We note that this function from $\vec{r}$ will be finitary: it will be different from zero only inside of the cell z zone; and it will not depend on the group number. So we expand the neutron source (9) $S_{gp}^z(\vec{r},\vec{\Omega})$ in the spatial and angular functions in the form:

$$S_{gpk}^z(\vec{r},\vec{\Omega}) = \sum_{j=1}^{J} S_{gpkj}^z \, j_{jpk}^z(\vec{r}) Y_p^k(\vec{\Omega}) \tag{10}$$

here $S_{gpkj}^z = \int_{v_z}\int_{\vec{\Omega}} S_{gpk}^z\left(\vec{r},\vec{\Omega}\right) j_{jpk}^z\left(\vec{r}\right) Y_p^k(\vec{\Omega})\, dn d\vec{\Omega} \Big/ \int_{v_z}\int_{\vec{\Omega}} \left[j_{jpk}^z\left(\vec{r}\right) Y_p^k(\vec{\Omega})\right]^2 dn d\vec{\Omega}$; j is the function number (j≤J).

Using the obvious form of the source (9), we obtain its expansion in the spatial and angular functions in the obvious form:

$$\begin{cases} S_{g0J}^z(\vec{r}) = \sum_{j=1}^{J}\left[\sum_{g'=1}^{G}\Sigma_{sg\leftarrow g'0}^z \Psi_{g'0j}^z - \Sigma_{sg0}^z \Psi_{g0j}^z + \dfrac{c_g^z}{K_\infty}\sum_{g'=1}^{G} n\Sigma_{fg'}^z \Psi_{g'0j}^z\right] j_{j0}^z(\vec{r}) Y_0^0(\vec{\Omega}) \\ S_{g1J}^z(\vec{r},\vec{\Omega}) = \dfrac{3}{2}\sum_{j=1}^{J}\left[\sum_{g'=1}^{G}\Sigma_{sg\leftarrow g'1}^z \Psi_{g'1j}^z - \Sigma_{sg1}^z \Psi_{g1j}^z\right] j_{j1}^z(\vec{r}) Y_1^0(\vec{\Omega}) \quad \text{..... ....(11)} \\ S_{gpkJ}\left(\vec{r},\vec{\Omega}\right) = \sum_{j=1}^{J}\sum_{p=2}^{P}\dfrac{(2p+1)(1+\delta_{0p})}{2}\left\{\left[\sum_{g'=1}^{G}\Sigma_{sg\leftarrow g'p0}\Psi_{g'p0j} - \Sigma_{sgp0}(\vec{r})\Psi_{gp0j}\right] j_{jp}^z(\vec{r}) Y_p^0(\vec{\Omega}) + \right. \\ \left. + 2\sum_{k=1}^{p}\dfrac{(p-k)!}{(p+k)!}\left[\sum_{g'=1}^{G}\Sigma_{sg\leftarrow g'p0}\Psi_{g'pkj} - \Sigma_{sgp0}(\vec{r})\Psi_{gpkj}\right] j_{jpk}^z(\vec{r}) Y_p^k(\vec{\Omega})\right\} \end{cases}$$

where $\Psi_{gpkj}^z = \int_{v_z}\Psi_{gpk}^z\left(\vec{r}\right) j_{jpk}^z\left(\vec{r}\right) dr \Big/ \int_{v_z}\left[j_{jpk}^z\left(\vec{r}\right)\right]^2 d\vec{r}$ is the spatial moment of the pk-angular moment of the neutron distribution function.

As you can see from equation (11), the expansion of the neutron sources (10) in the spatial and angular functions $j_j^z(\vec{r})Y_p^k(\vec{\Omega})$ result in the end finally to the expansion in the same spatial and angular functions of the neutron distribution function.

Multiplying equation (11) by $j_j^z(\vec{r})Y_p^k(\vec{\Omega})$ and integrating over the volume of the z zone and over all the directions $\mathbf{W}$, we obtain the relation between of the spatial and angular moments of the neutron sources S_{gpkj}^z and of the neutron distribution function Ψ_{gpkj}^z:

$$S_{gpkj}^z = \sum_{g=1}^{G}\Sigma_{sg\leftarrow g'p0}^z\Psi_{g'pkj}^z - \Sigma_{sgp0}^z\Psi_{gpkj}^z + \frac{\delta_{p,0}\chi_g^z}{K_\infty}\sum_{g'=1}^{G}\nu\Sigma_{fg}^z\Psi_{g'00j}^z \quad \text{.....} \quad \text{.....} \quad \text{.....(12)}$$

where $S_{gpkj}^z = \int_{v_z}\int_{\vec{\Omega}} S_g^z(\vec{r},\vec{\Omega})j_{gpk}^z(\vec{r},\vec{\Omega})d\nu d\vec{\Omega} \Big/ \int_{v_z}\int_{\vec{\Omega}}\left[j_{jpk}^z(\vec{r},\vec{\Omega})\right]^2 d\nu d\vec{\Omega}$

To obtain the spatial dependence S_{gpkj}^z from Ψ_{gpkj}^z, we substitute expansion (10) in the first equation of the system (8) and obtain:

$$\Psi_{gpkj}^z(\vec{r},\vec{\Omega}) = \sum_{j'=1}^{J} S_{gpkj'}^z\int_{V_z} j_{j'pk}^z(r')G_{pk}^z(\vec{r},\vec{\Omega}/r')dv' + \int\int_{\Omega_s} g_s^z(\vec{r}_s',\vec{\Omega}')G^z(\vec{r},\vec{\Omega}/\vec{r}_s',\Omega')d\vec{r}_s'd\vec{\Omega}' \tag{13}$$

Then we multiply equation (13) by $j_{jpk}^z(\vec{r})Y_p^k(\vec{\Omega})$ and integrate over the volume of the z zone and over all the directions $\mathbf{W}$ and obtain the equation with respect to the spatial angular moments S_{gpkj}^z and Ψ_{gpkj}^z in the form:

$$\Psi_{gpkj}^z = \sum_{j'=1}^{J} S_{gpkj'}^z\int_{v'}\int_{v}\varphi_{jpk}^z(\vec{r})G_{p'k'}^{zpk'}(\vec{r}/r')\varphi_{jp'k'}^z(\vec{r}')dvdv' + \int_{v}\int_{s}\int_{\Omega'} g_{sg}^z(\vec{r}_s,\vec{\Omega}')*$$
$$G^{zpk}(\vec{r}/r_s',\vec{\Omega}')j_{jpk}^z(\vec{r})d\vec{\Omega}'d\vec{r}_s'dv \tag{14}$$

From equation (14) one can see, that the dependence Ψ_{gpkj}^z from S_{gpkj}^z contain the unknown surface pseudosources $g_{sg}^z(\vec{r}_s,\vec{\Omega}')$. To exclude the unknown surface pseudosources from equation (14), we will solve the onegroup problem in the whole multizone cell by the surface pseudosources method in G_N-approximation with the anisotropic source, expanded in series (10).

3. THE SURFACE PSEUDOSOURCES METHOD

We will discribed the essence of the surface pseudosources method[15,14] on an example of a cylindrical cell calculation. In this method the auxiliary problem is considered for every zone of the cell. Such problem is the one in an infinite homogeneous medium consisting of the material of the z zone considered. The volume source $S^z\left(\vec{r},\vec{\Omega}\right)$ is also extended to infinity. A surface pseudosources $g_s^z\left(\vec{r}_s,\vec{\Omega}\right)$ are introduced at an interior surfaces of a z zone. Then a solution in the surface pseudosources method for such z zone may be written as:

$$\Psi^z\left(\vec{r},\vec{\Omega}\right)=\int\limits_{\vec{\Omega}'}\int\limits_{V} S^z\left(\vec{r}',\vec{\Omega}'\right)G^i\left(\vec{r},\vec{\Omega}/\vec{r}',\vec{\Omega}'\right)d\vec{\Omega}'dV'+\int\limits_{\Omega_s}\int g_s^z\left(\vec{r}',\vec{\Omega}'\right)G^i\left(\vec{r},\vec{\Omega}/\vec{r}',\vec{\Omega}'\right)d\vec{r}_s'd\vec{\Omega}'$$

where V is the infinity space. (15)

To determine the unique solution of the transport equation (15) in the interior of a z zone, it is important to know the surface pseudosources $g_s^z\left(\vec{r}_s,\vec{\Omega}\right)$ for $\left(\vec{n},\vec{\Omega}\right)<0$, i.e. for neutrons coming into the zone interior. For $\left(\vec{n},\vec{\Omega}\right)>0$ the surface pseudosources $g_s^z\left(\vec{r}_s,\vec{\Omega}\right)$ may be extended with any way. In particular, for inplane geometries $g_s^z\left(\vec{r}_s,\vec{\Omega}\right)$ may be extended antisymmetrically, i. e.

$$g^z\left(\vec{r}_s,\vec{\Omega}\right)=-g^z\left(\vec{r}_s,-\vec{\Omega}\right) \tag{16}$$

Thus, using the Green function for the homogeneous infinite medium and the surface pseudosources, we pass from the old unknowns $\Psi^z\left(\vec{r},\vec{\Omega}\right)$ to the new ones $g_s^z\left(\vec{r}_s,\vec{\Omega}\right)$. And so we make for every zone of a cell.

Using the continuity conditions of the distribution functions $\Psi\left(\vec{r},\vec{\Omega}\right)$ at the zone boundaries, we obtain system of the integral equations for the surface pseudosources $g_s^z\left(\vec{r}_s,\vec{\Omega}\right)$ in the form:

$$\int\limits_{\vec{\Omega}'}\int\limits_{V} S^{z-1}\left(\vec{r}',\vec{\Omega}\right)G^{z-1}\left(\vec{r},\vec{\Omega}/\vec{r}',\vec{\Omega}'\right)d\vec{\Omega}'dV'+\int\limits_{\Omega'}\int\limits_{s} g_s^{z-1}\left(\vec{r}',\vec{\Omega}'\right)G^{z-1}\left(\vec{r},\vec{\Omega}/\vec{r}',\vec{\Omega}'\right)d\vec{r}_s'd\vec{\Omega}'=$$

$$\int\limits_{\vec{\Omega}'}\int\limits_{V} S^z\left(\vec{r}',\vec{\Omega}'\right)G^z\left(\vec{r},\vec{\Omega}/\vec{r}',\vec{\Omega}'\right)d\vec{\Omega}'dV'+\int\limits_{\Omega'}\int\limits_{s} g_s^z\left(\vec{r}',\vec{\Omega}'\right)G^z\left(\vec{r},\vec{\Omega}/\vec{r}',\vec{\Omega}'\right)d\vec{r}_s'd\vec{\Omega}'\ldots.(17)$$

We expand the Green functions and the surface pseudosources in the spherical harmonic functions $Y_n^m(\vec{\Omega})$:

$$\begin{aligned} G^z\left(\vec{r},\vec{\Omega}/\vec{r}',\vec{\Omega}'\right)&=\sum_{n'm'}^{\infty}\sum_{nm}^{\infty} G_{n'm',nm}(\vec{r}/\vec{r}')Y_{n'}^{m'}(\vec{\Omega}')Y_n^m(\vec{\Omega}) \\ g_s^z\left(\vec{r}',\vec{\Omega}'\right)&=\sum_{nm}^{\infty} g_{nm}^z(\vec{r}')Y_n^m(\vec{\Omega}') \end{aligned} \tag{18}$$

Limiting in the expansions of the surface pseudosources by the term with a number N, we obtain

G_N-approximation. Then we substitute expansions (18) in equation (17). Multiplying the obtained system of the integral equations (17) by $Y_n^m(\vec{\Omega})$ ($n \le N$) and then integrating over all $\vec{\Omega}$, we obtain system of the algebraic equations for the angular moments of the surface pseudosources. To solve this system of algebraic equations containing many of the zero elements, we use the matrix factorization method that is stable to the errors of approximations and roundings and the essence of that will be demonstrated below.

The distribution function $\Psi^z\left(\vec{r},\vec{\Omega}\right)$ expanded in the spherical harmonic functions $Y_n^m\left(\vec{\Omega}\right)$ inside of the z zone with the isotropic and anisotropic sources has the following form:

$$\Psi^z\left(\vec{r},\vec{\Omega}\right)=\sum_{pk}\sum_{nm} S_{pk}^{z}\, G_{pk}^{znm}\left(\vec{r}\right)Y_n^m\left(\vec{\Omega}\right)+\sum_{i=1}^{I}\sum_{nm}\sum_{n'm'} g_{n'm'}^{zj}\, G_{n'm'}^{znm}\left(\vec{r}\,/r_i\right)Y_n^m\left(\vec{\Omega}\right) \qquad (19)$$

where i is the number of the interior surface of the cell z zone ($i \le I$);

$S_{pk}^{z}=\int_v S_{pk}^{z}\left(\vec{r}'\right)G_{pk}^{znm}\left(\vec{r}\,/\vec{r}'\right)d\vec{r}'$.

Equating the angular moments of the distribution function (19) at the boundary between the first and second zones (the first zone is the cell central one) we obtain the equation in the matrix-vector form:

$$\mathbf{A}_{23}\,\vec{g}_{23}=\mathbf{A}_1\,\vec{g}_1+\mathbf{B}^1\left(S_{pk}^{1},S_{pk}^{2}\right), \qquad (20)$$

where $\mathbf{A}_{23}=\left[G_{n'm'}^{nm}(r_2/r_2'),\, G_{n'm'}^{nm}(r_2/r_3')\right]$;

$\mathbf{A}_1=\left[G_{n'm'}^{nm}(r_1/r_1')\right]$; $\mathbf{B}^1=\left[S_{pk}^{1nm},-S_{pk}^{2nm}\right]$; $\vec{g}_{kl}=\begin{bmatrix} g_{n'm'}^{k} \\ g_{n'm'}^{l} \end{bmatrix}$;

Further we transform equation (20) to the form:

$$\mathbf{A}_{12}\,\vec{g}_{12}=\mathbf{A}_3\,\vec{g}_3-\mathbf{B}^1\left(S_{pk}^{1},S_{pk}^{2}\right) \qquad (21)$$

where $\mathbf{A}_{12}=\left[G_{n'm'}^{nm}(r_1/r_1'),-G_{n'm'}^{nm}(r_2/r_2')\right]$; $\mathbf{A}_3=\left[G_{n'm'}^{nm}(r_2/r_3')\right]$

Solving equation (21) with respect to vector $\vec{g}_{12}$ we obtain

$$\vec{g}_{12}=\mathbf{A}_{12}^{-1}\mathbf{A}_3\,\vec{g}_3-\mathbf{A}_{12}^{-1}\mathbf{B}^1\left(S_{pk}^{1},S_{pk}^{2}\right) \qquad (22)$$

Further we equate the angular moments of the distribution function (19) at the boundary between the second and third zones and obtain

$$\mathbf{D}_{23}\,\vec{g}_{23}=\mathbf{A}_{45}\,\vec{g}_{45}+\mathbf{B}^2\left(S_{pk}^{2},S_{pk}^{3}\right) \qquad (23)$$

where $\mathbf{D}_{23}=\left[G_{n'm'}^{nm}(r_3/r_2'),\, G_{n'm'}^{nm}(r_3/r_3')\right]$; $\mathbf{B}^2=\left[0,-S_{pk}^{2nm},S_{pk}^{3nm}\right]$.

Substituting $\vec{g}_2$ from equation (22) into equation (23) we have the following equation

$$\mathbf{A}_{45}\,\vec{g}_{45}=\mathbf{F}_3\,\vec{g}_3+\mathbf{B}_1^2\left(S_{pk}^{1},S_{pk}^{2},S_{pk}^{3}\right) \qquad (24)$$

where $\mathbf{F}_3=\mathbf{D}_3\left[\mathbf{A}_{12}^{-1}\mathbf{A}_3\right]_2+\mathbf{D}_3$; $\left[\mathbf{A}_{12}^{-1}\mathbf{A}_3\right]_2$ is the lower half of the matrix $\mathbf{A}_{12}^{-1}\mathbf{A}_3$;

$$\mathbf{B}_1^2(S_{pk}^1, S_{pk}^2, S_{pk}^3) = -[\mathbf{D}_{23}]_2 [\mathbf{A}_{12}^{-1}\mathbf{B}^1(S_{pk}^1, S_{pk}^2)]_2 + \mathbf{B}_1^2(S_{pk}^2, S_{pk}^3)\,. \tag{25}$$

Thus, we obtain equation (24) coinciding with equation (20). And so on. At the last but one boundary of a cell we have the following equation:

$$\mathbf{F}_{2Z-3}\vec{g}_{2Z-3} = \mathbf{A}_{2Z-2,\sin k}\vec{g}_{2Z-2,\sin k} + \mathbf{B}_1^{Z-1}(S_{pk}^1, S_{pk}^2 \dots S_{pk}^Z) \tag{26}$$

where $\mathbf{F}_{2Z-3}$ is the matrix calculated by recurrent formula (24);

$\mathbf{B}_1^{Z-1}(S_{pk}^1, S_{pk}^2 \dots S_{pk}^Z)$ is the matrix calculated by recurrent formula (25).

To form the necessary boundary condition in the surface pseudosoures method at the cell external boundary, the additional neutron "sink" is placed at the cell boundary or outside the cell. Therefore, in the expression for the neutron distribution function (19) into the external zone the sink is appeared in the form:

$$\Psi_{\sin k}(\vec{r},\vec{\Omega}) = G_{10}(\vec{r},\vec{\Omega}/r_{\sin k}) * A \dots\dots\dots\dots\dots \tag{27}$$

where A is the sink capacity. The sink capacity is determined from the boundary condition (2). In the boundary condition "isotopic sink" the sink is placed at the cell external boundary. This boundary condition is close to the isotropic reflection [6]. In the boundary condition "combined sink" the several first fuel rows surrounding the fuel rod considered are taken into consideration exactly but the rest fuel rods are taken into consideration by "isotropic sink" placed at the external boundary of the supercell. This boundary condition is close to the mirror reflection in the actual lattice [6].

We transform equation (26) to the form:

$$\mathbf{A}_{2Z-3,2Z-2}\vec{g}_{2Z-3,2Z-2} = \mathbf{A}_{,\sin k}\vec{g}_{,\sin k} + \mathbf{B}_1^{Z-1}(S_{pk}^1, S_{pk}^2 \dots S_{pk}^Z) \tag{28}$$

Solving equation (28) we have

$$\vec{g}_{2Z-3,2Z-2} = \mathbf{A}_{2Z-3,2Z-2}^{-1}\left[\mathbf{A}_{,\sin k}\vec{g}_{,\sin k} + \mathbf{B}_1^{Z-1}(S_{pk}^1, S_{pk}^2 \dots S_{pk}^Z)\right] \tag{29}$$

Using the boundary condition (2) we obtain the following equation:

$$D_{2Z-2,\sin k}\vec{g}_{2Z-2,\sin k} = tok + \mathbf{B}^Z(S_{pk}^Z) \tag{30}$$

Substituting the $\vec{g}_{2Z-2}$ from equation (29) into equation (30) we write

$$\mathbf{F}_{\sin k}\vec{g}_{\sin k} = \mathbf{B}_1^Z(S_{pk}^1, S_{pk}^2 \dots S_{pk}^Z) + tok \tag{31}$$

Solving equation (31) we determine the vector $\vec{g}_{\sin k}$. Then we determine all rest angular moments of the surface pseudosources on the recurrent formula (29). In result we obtain the dependence of an angular moments of the surface pseudosources from the spatial angular moments of the volume sources $S_{gpj'}^z$ in the matrix-vector form:

$$\mathbf{g} = \mathbf{HS} \tag{32}$$

where $\mathbf{g}$ is the vector of all the angular moments of the surface pseudosources of the whole cell; $\mathbf{S}$ is the vector of the spatial angular moments of all volume sources of a whole cell; $\mathbf{H}$ is the matrix of combinations of the angular moments of the Green functions.

We substitute the dependence (32) in expression (19) and obtain

$$\mathbf{Y} = \mathbf{H}'\mathbf{S} \tag{33}$$

where $\mathbf{Y}$ is the vector of the angular moments of the neutron distribution function of the whole cell; $\mathbf{H}'$ is the matrix of combinations of the angular moments of the Green functions.

Besides (33) after solving onegroup equations by the surface pseudosources method we obtain the dependences of the first and second angular moments of the neutron distribution function at the inner and outer boundaries of a cell in respect of the spatial angular moments of the sources:

$$\mathbf{F} = \mathbf{H}''\mathbf{S} \tag{34}$$

where $\mathbf{F}$ is the vector of the first and second angular moments of the neutron distribution function at the inner and outer boundaries of a whole cell; $\mathbf{H}''$ is the matrix of combinations of the angular moments of Green functions.

It should be noted that the Green functions of the transport equation for the homogeneous infinite medium in the 1-dimension[18] and 2-dimension[19] cylindrical geometries have been obtained from the regular and singular elementary solutions (in sense K.M. Case[18]) of the transport equation for the cylindrical geometry. The regular and singular elementary solutions of the transport equation for an arbitrary geometry have been obtrained from the plane elementary solutions of the transport equation [18,19]. The 2- dimension cylindrical Green function, both parts which are written in the different cylindrical coordinats, have been obtained[20]. These Green functions have been used for a calculation of the cluster cells[20] of type RBMK, CANDU and to form the boundary condition " combined sink"[6].

4. THE SPATIAL-ENERGY PART OF THE PROBLEM

Having united the results of the solutions of the spatial-angular and energetic parts of a considered problem we obtain the spatial-energetic problem for the whole cell for all group in respect of the nodes, what for it is necessary to substitute the dependence S^{z}_{gpkj} from Ψ^{z}_{gpkj} (14) into equation (33). The K_∞ value being calculated in the problem (1)-(2), the method of the iteration on the source is used. In slowing-down region the equations are calculated one by one. In region of the neutron thermalization all the one-group equations connected are calculated by the method of the direct matrix solving. Having solved the equation with respect to the nodes, we determine the average group fluxes within the cell zones (equation (33)), the first and second angular moments of the neutron distribution function at the inner and outer boundaries of a cell (equation (34)) and a number of other functionals of a cell. The problems with an external neutron source (there are volume sources or incoming/outgoing current of neutron or both) may be solved also.

5. THE SPATIAL MOMENTS OF THE GREEN FUNCTION

Below we write down additions, which must be carried out, to take into account the linear anisotropy:

- To determine the obvious form of the function $\boldsymbol{j}^{z}_{j1}(\mathbf{r})$ for the current node.
- To obtain the obvious form of the current source i.e. to integrate over the infinite volume the first integral in equation (19).
- To integrate over the volume of the z zone the first term of equation (13) and the second one connected with the surface pseudosources.
- To solve all the onegroup equations by the surface pseudosources method in the approximation of the "plane" current sources and to obtain equations (33) and (34).

We determine an obvious form of the functions $\boldsymbol{j}^{z}_{j1}(\mathbf{r})$ for the current node in the asymptotic approximation in the form. :

$$\Psi_{10}(\boldsymbol{r}) \approx A_1 I_1\left(\frac{\boldsymbol{r}}{L}\right) + B_1 K_1\left(\frac{\boldsymbol{r}}{L}\right) \approx A_1^1 \boldsymbol{r} + B_1^1 / \boldsymbol{r} \tag{35}$$

Thus, a number of the simple functions: ρ, 1/ρ can be used for approximation. From these two functions we keep only the one function ρ how this was done for a flux node [3,6].

The angular moments of the Green function from the current source ρμ have the form:

$$\int_v \int_{4p} r'^2 G^z_{10}(\mathbf{r}, \mathbf{W}/r')dr'da' =$$

$$\sum_{nm} Y^m_n(\mathbf{W}) \left[\int_n \frac{A^0_n(\boldsymbol{n}) A^0_1(\boldsymbol{n}) F^n_{m,o}\left(\frac{\boldsymbol{r}}{\boldsymbol{n}}\right) \boldsymbol{n}\boldsymbol{r}^2 \left[-I_2\left(\frac{\boldsymbol{r}}{\boldsymbol{n}}\right)\right]}{\boldsymbol{n} N(\boldsymbol{n}, o)} d\boldsymbol{n} + \int_n \frac{A^0_n(\boldsymbol{n}) A^0_1(\boldsymbol{n}) \Phi^n_{m,0}\left(\frac{\boldsymbol{r}}{\boldsymbol{n}}\right) \boldsymbol{n}\boldsymbol{r}^2 K_2\left(\frac{\boldsymbol{r}}{\boldsymbol{n}}\right)}{\boldsymbol{n} N(\boldsymbol{n}, o)} d\boldsymbol{n} \right] 2p \tag{36}$$

here $A^0_n(\boldsymbol{n})$ are the n order polynomials: $A^0_0(\boldsymbol{n}) = 1$; $A^0_1(\boldsymbol{n}) = (1-c)$; $c = \Sigma_s / \Sigma_{tot}$; $\Phi^{n1}_{m,0}(x)$ and $F^n_{m,0}(x)$ are a combinations of the functions from the imaginary argument $I_l(x)$ and $K_l(x)$ accordingly; $N(\boldsymbol{n}, 0)$ is a multiplier; $\int_n f(\boldsymbol{n}) d\boldsymbol{n} = f(\boldsymbol{n}_0) + \int_0^1 f(\boldsymbol{n}) d\boldsymbol{n}$.

The spatial moment $(\boldsymbol{j}^z_{10}(\mathbf{r}) = \boldsymbol{r})$ of the first angular moment of the current source $(\boldsymbol{j}^z_{10}(\mathbf{r}') = \boldsymbol{r})$ has the form:

$$\int_{V'} \int_V j^z_{10}(\vec{r}) G^{z10}_{10}(\vec{r}/\vec{r}') j^z_{10}(r') dV dV' = -\frac{1}{3\Sigma(1-c\mu)} \tag{37}$$

where μ is the average cosine.

The spatial moments $(\boldsymbol{j}^z_{10}(\mathbf{r}) = \boldsymbol{r})$ of the first angular moment of the distribution function part connected with the surface pseudosources have the form:

$$\int_V \int_S \int_{\vec{\Omega}'} g_{sg}^{z}\left(\vec{r}_s, \vec{\Omega}'\right) G^{z10}\left(\vec{r} / \vec{r}_s', \right) j_{10}^{z}\left(\vec{r}\right) d\vec{\Omega}' d r_s' dV \Big/ \int_{v_z} \left[j_{jp}^{z}\left(\vec{r}\right)\right]^2 dV = \sum_{n'm'} Y_{n'}^{m'}\left(\vec{\Omega}\right) \frac{4}{r_2^4 - r_1^4} *$$

$$* \begin{cases} \int_n \dfrac{A_{n'}^{0}(\boldsymbol{n}) A_1^{0}(\boldsymbol{n}) \Phi_{m'0}^{n'}\left(\dfrac{\boldsymbol{r}'}{\boldsymbol{n}}\right)\left[\boldsymbol{r}_1^2 K_2\left(\dfrac{\boldsymbol{r}_1}{\boldsymbol{n}}\right) - \boldsymbol{r}_2^2 K_2\left(\dfrac{\boldsymbol{r}_2}{\boldsymbol{n}}\right)\right]}{N(\boldsymbol{n}, o)} d\boldsymbol{n}, \ldots . \text{ when } \ldots . \ \boldsymbol{r}' \le \boldsymbol{r}_1 < \boldsymbol{r}_2 \\ \int_n \dfrac{A_{n'}^{0}(\boldsymbol{n}) A_1^{0}(\boldsymbol{n}) F_{m'0}^{n'}\left(\dfrac{\boldsymbol{r}'}{\boldsymbol{n}}\right)\left[\boldsymbol{r}_2^2 I_2\left(\dfrac{\boldsymbol{r}_2}{\boldsymbol{n}}\right) - \boldsymbol{r}_1^2 I_2\left(\dfrac{\boldsymbol{r}_1}{\boldsymbol{n}}\right)\right]}{N(\boldsymbol{n}, o)} d\boldsymbol{n}, \ldots \text{ when } \ldots . \ \boldsymbol{r}' \ge \boldsymbol{r}_2 > \boldsymbol{r}_1 \end{cases} \quad (38)$$

6. RESULTS AND DISCUSSIONS

Above-written method has been realized in the option RATIA of the complex WIMS-SH-2.0 [21] for calculation of cylindrical cells. The typical cells of the BWR reactor has been calculated by this

Table I. The K_∞ values of a BWR cell with the MOX (7wt%) fuel at the room temperature.

Appro-ximation	Bound. cond.	Approximation	Water density g/cm^3 0,01	0,1	0,2	0,3	0,5	0,8	1,0
Trans-	"Isotro-	$\tilde{\tilde{G}}_1$ *)	0,898	0,9568 (-0,1)	0,9879 (-0,1)	1,0292 (0,1)	1,1136 (0,3)	1,2148 (0,4)	1,2642 (0,5)
		**)		[-0,1]	[-0,2]	[-0,2]	[<-0,1]	[<0,1]	[0,2]
		***)		{-0,1}	{-0,2}	{-0,3}	{-0,4}	{-0,3}	{-0,3}
port	pic	G_3*)	0,898	0,9621 (0,3)	0,9945 (0,4)	1,0344 (0,5)	1,1165 (0,5)	1,2159 (0,5)	1,2647 (0,5)
		**)		[0,2]	[0,2]	[0,1]	[0,1]	[0,15]	[0,2]
		***)		{-0,1}	{-0,2}	{-0,3}	{-0,4}	{-0,3}	{-0,3}
Linear	sink"	$\tilde{\tilde{G}}_1$	0,898	0,9575	0,9884	1,0280	1,1101	1,2095	1,2583
		***)		{-0,1}	{-0,2}	{-0,4}	{-0,4}	{-0,4}	{-0,4}
anisotr		G_3	0,898	0,9591	0,9901	1,0294	1,1109	1,2096	1,2581
		***)		{-0,1}	{-0,2}	{-0,4}	{-0,4}	{-0,4}	{-0,4}
Trans-	"Com-	$\tilde{\tilde{G}}_1$ *)	0,898	0,9571 (<0,1)	0,9894 (0,1)	1,0318 (<0,1)	1,1170 (0,2)	1,2182 (0,4)	1,2674 (0,4)
port	bined	G_3*)	0,898	0,9626 (0,3)	0,9965 (0,4)	1,0376 (0,5)	1,1204 (0,5)	1,2196 (0,5)	1,2681 (0,5)
Linear	sink"	$\tilde{\tilde{G}}_1$	0,898	0,9579	0,9905	1,0315	1,1148	1,2142	1,2626
anisotr		G_3	0,898	0,9599	0,9926	1,0331	1,1156	1,2143	1,2623

*) In round brackets the percentage difference of the given approximation from the same one for the linearly anisotropic case is shown for the same boundary condition.

**) In square brackets the percentage difference of the given approximation from the same one with the boundary condition "combined sink" for the linearly anisotropic case is shown.

***) In figured brackets the percentage difference of the given approximation from the same one with the boundary condition "combined sink" is shown for the same approximation of scattering cross section.

complex in the $\tilde{\tilde{G}}_1$ and G_3 approximations and in 28 groups. In the all calculations the temperature has been either room (300^0K) or hot (T_{fuel}=900K, $T_{moderator}$=597K, T_{clad}=594K). The water density has changed from 0,01 to 1.0 g/sm^3. It should be noted [20] that in lower $\tilde{\tilde{G}}_1$ -approximation of the surface pseudosources method at the zone boundaries of a cell it is better to equate the angular moments from the function $\vec{\Omega}\vec{n}\Psi_g(\vec{r},\vec{\Omega})$ but no from the one $\Psi_g(\vec{r},\vec{\Omega})$.

The typical three-zone cell [22] of the BWR reactor with the MOX fuel from UO_2+PuO_2 with 7wt% (Pu-239-61,3%; Pu-240-23,9%; Pu-241-9,6%; Pu-242-5,2%) was calculated in the transport and linearly anisotropic approximations (see Tables 1-2) with the boundary conditions "isotropic sink" and "combined sink". It should be noted that the first boundary condition is close to isotropic reflection [6], but the second one is close to the mirror reflection in the actual n-angular lattice [6]. In detail these boundary conditions have been described in paper [6]. By comparing the calculation results one can do the following conclusions:

- The difference in the K_∞ values between the transport and linearly anisotropic approximations in G_3-approximation of the surface pseudosources method is 0,5% and begin to decrease with the decrease of the water density less than the 0,3 g/cm^3.
- The difference of the same in the $\tilde{G}_1$-approximation is decreased with the decrease of the water density earlier because of rougher approximation than in G_3-one.
- The difference in the K_∞ values calculated with the boundary conditions: "isotropic sink" and "combined sink" in $\tilde{G}_1$ and G_3 approximations is -0,4-0,5%. The estimating calculations have shown that a 50% contribution of this difference has been given with the resonance of Pu240 at 1ev, the rest has been given with the resonances of the other nuclides. These results are in good agreement with the ones of paper [7], where the author have investigated the influence of the deficiency of the Wigner-Seitz approximation for Pu/MOX fuel pins on the K_∞ value.
- In modern codes the pin cell is usually calculated in the transport approximation with the white boundary condition in the cylindricalized cell. In present paper we suggest to calculate the square cell in the linearly anizotropic approximation. By comparing the results of the two calculations (see, Table I, II) one can see that the difference in the K_∞ value is 0,1-0,2%. Such result is less than in each effect considered above and it is connected with the mutual compensation of these two effects.

We fix one's attention on the difference in the K_∞ value between transport and linearly anisotropic approximations in the G_3-approximation. It is 0,5%. Such difference is connected with present in a fuel of the considerable quantity of Pu-240 (23,9%), having the "wide" resonance of absorption at 1 ev. In the WIMS-D4 code this resonance is taken into account of introduction of narrow energetic group in the energetic domain around 1 ev. The large absorption in the resonance domain of the Pu-240 results in the flux large gradient and hence in the considerable neutron current. Ultimately all this result in the perceptible influence of the anisotropic scattering on the cell characteristics in particular on the K_∞ value. Above described reasoning is confirmed with calculations. Removing the Pu-240 from a fuel, the difference in the K_∞ value between the transport and linearly anisotropic approximations in G_3- approximation for ρ_{Y_2O}=0,5 g/cm^3 is 0,17%. Thus, the contribution in the difference of the K_∞ value in 0,33% is connected with presence of the Pu-240 resonance at 1 ev.

It should be noted that in the surface pseodosources method the anisotropic scattering is taken into account both in one group calculations [12] (the first equation of the equation system (8)) and in multigroup calculation (the second equation of the equation system (8)). To estimate the contributions in the difference of the K_∞ value from accounting the anisotropic scattering in the one and multi group calculations, the following auxiliary calculation has been carried out for cell with the MOX fuel for ρ_{Y_2O}=0,5 g/cm^3 in the G_3-approximation with the boundary condition "isotropic sink". In this auxiliary calculation the anisotropic scattering is taken into account only in the onegroup calculations but in the multigroup calculation the isotropic approximation is taken. The K_∞ value calculated was equal to 1,1130. By comparing this result with the Table 1 results ($K_{\infty\ tr.\ ap.}$=1,1165, $K_{\infty\ lin.-anis.\ ap.}$=1,1109), one can see that accounting the anisotropic scattering in onegroup calculations gives 63% but in the multigroup calculation 37%.

Table II. The K_∞ values of a BWR cell with the MOX (7wt%) fuel at the hot temperature.

Approximation	Bound. condition	Approximation	Water density g/cm^3 0,01	0,1	0,2	0,3	0,5	0,8	1,0
Transport	"Isotopic sink"	$\tilde{\tilde{G}}_1$ *)	0,878	0,9288 (<-0,1)	0,9585 (<-0,1)	0,9976 (0,1)	1,0792 (0,4)	1,1794 (0,4)	1,2297 (0,5)
		**)		[-0,1]	[-0,2]	[-0,2]	[-0,1]	[<0,1]	[0,2]
		***)		{-0,1}	{-0,2}	{-0,2}	{-0,3}	{-0,3}	{-0,3}
		G_3*)	0,878	0,9335 (0,3)	0,9644 (0,4)	1,0024 (0,5)	1,0818 (0,5)	1,1804 (0,5)	1,2300 (0,5)
		**)		[0,2]	[0,2]	[0,1]	[0,1]	[0,1]	[0,2]
		***)		{-0,1}	{-0,2}	{-0,3}	{-0,4}	{-0,4}	{-0,3}
Linear anisotr		$\tilde{\tilde{G}}_1$ ***)	0,878	0,9294 {-0,1}	0,9588 {-0,2}	0,9964 {-0,3}	1,0757 {-0,5}	1,1742 {-0,5}	1,2237 {-0,5}
		G_3 ***)	0,878	0,9308 {-0,1}	0,9604 {-0,2}	0,9977 {-0,4}	1,0764 {-0,5}	1,1742 {-0,5}	1,2234 {-0,5}
Transport	"Combined sink"	$\tilde{\tilde{G}}_1$ *)	0,878	0,9291 (-0,1)	0,9599 (-0,1)	1,0000 (<0,1)	1,0825 (0,2)	1,1829 (0,4)	1,2330 (0,5)
		G_3*)	0,878	0,9339 (0,2)	0,9664 (0,4)	1,0055 (0,5)	1,0857 (0,5)	1,1842 (0,5)	1,2336 (0,5)
Linear anisotr		$\tilde{\tilde{G}}_1$	0,878	0,9299	0,9609	0,9998	1,0803	1,1789	1,2281
		G_3	0,878	0,9316	0,9628	1,0013	1,0810	1,1789	1,2278

*) In round brackets the percentage difference of the given approximation from the same one for the linearly anisotropic case is shown for the same boundary condition.

**) In square brackets the percentage difference of the given approximation from the same one with the boundary condition "combined sink" for the linearly anisotropic case is shown.

***) In figured brackets the percentage difference of the given approximation from the same one with the boundary condition "combined sink" is shown for the same approximation of scattering cross section.

CONCLUTIONS

As the pin cell calculations shown, the influence of the boundary conditions and of the anisotropic scattering in the considered calculations was conditioned on the whole by the large absorption resonance of the Pu 240 at about 1 ev. Such behavior was connected with the perturbation of the neutron distribution function round of this resonance. Fortunately, although the influence of each of these two effects is perceptible, they have the different signs, therefore the joint influence appears to be no large. It would be interested to investigate the influence of these effects in the time of the continuous energy slowing-down of neutrons in the resonance energy region both on the K_∞ value and on the temperature coefficients of reactivity.

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