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Improvement of the SPH Method for Multi-assembly Calculations

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Abstract: In this paper, improvement of the SPH method (the improved SPH method) is proposed. The SPH method is commonly used in pin-by-pin mesh core calculations to reduce cell-homogenization error. The investigation revealed that the normalization condition of the SPH factor in the conventional SPH method is not appropriate for multi-assembly calculations in which different assembly types are adjacent. Since the conventional normalization condition does not incorporate flux discontinuity between assemblies, cell homogenization error in assembly peripheral region becomes larger.

In the improved SPH method, the SPH factor is divided by an averaged "cell-level" discontinuity factor obtained in each fuel assembly. Though the SPH factor is somewhat changed from the conventional value, no additional homogenization parameters (e.g. discontinuity factor) is necessary in core calculations.

Test calculations were carried out in a simplified one-dimensional slab and two-dimensional PWR colorset geometries. The calculation results showed that the improved SPH method effectively reduce the cell-level homogenization error especially in assembly peripheral region.

Since we can easily implement the improved SPH method by slight code modifications, it can be a promising candidate of the cell-homogenization method for pin-by-pin core calculations.

KEYWORDS: SPH method, SPH factor, multi-assembly calculation, generalized equivalence theory, pin-by-pin core calculation, SCOPE2

1. Introduction

Improvement of the SPH method to reduce cell-homogenization error in multi-assembly geometry is presented in this paper.

The SPH method is a homogenization technique that reduces spatial homogenization errors.[1],[2] No additional homogenization parameter is necessary in the SPH method since the homogenization parameter (the SPH factor) is incorporated in cross sections. This feature of the SPH method promotes its application to large-scale fine-mesh calculations, e.g. three-dimensional pin-by-pin whole core calculations, since extra memory storage for homogenization parameters is not necessary.[3],[4]

Another popular homogenization method is the generalized equivalence theory (GET) that utilizes discontinuity factors at node interfaces.[5] Though GET is usually used in advanced nodal codes with assembly-level discontinuity factors, it can be also applied to more detail calculations, e.g. a cell-homogenized pin-by-pin core calculation, with cell-level discontinuity

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factors. However, since a discontinuity factor is assigned to each node surface, considerable storage for discontinuity factors will be necessary for the pin-by-pin fine mesh core calculations using GET. This feature could be shortcoming of GET in large scale calculations.

Some of the authors compared cell-homogenization capability of the SPH and GET methods in the simplified pin-by-pin geometry that simulates PWR colorset configurations.[6] The calculation results revealed that homogenization error (error in pin-by-pin absorption rate) of the SPH method were larger than that of GET, especially at interface region between different assembly types.

The investigation of homogenization error suggested a source of error in the current SPH method; the normalization condition used in the SPH method may not be appropriate. In order to resolve this issue, the improved SPH method is newly proposed in this paper. The proposed method is simple and easily retrofitted present codes that utilize the SPH method.

As described above, the SPH method has advantage in the treatment of homogenization parameters since the parameters are incorporated in cross sections. Therefore, performance improvement of the SPH method will be useful for large scale calculations, e.g. three-dimensional pin-by-pin core calculations.

2. Comparison of SPH and GET

2.1 Brief description of the methods

Brief descriptions of the SPH method and GET will be provided in this section to make this paper thorough. Detail descriptions of these methods are found in Refs. [2] and [5],

In the SPH method, cell-average reaction rate obtained in heterogeneous calculation is preserved by multiplying a correction factor (SPH factor) to cell-average cross sections as shown in Eq.(1).

$$\tilde{\Sigma}_k^{ave} = \mu_k \Sigma_k^{ave} \tag{1}$$

where,

$\tilde{\Sigma}_k^{ave}$: SPH corrected homogeneous cross section of region k,

μ_k : the SPH correction factor of region k,

Σ_k^{ave} :the flux-volume weighted cross section of region k.

The correction factor, i.e. the SPH factor, is calculated by Eq.(2).

$$\mu_k = \frac{\bar{\phi}_k^{het}}{\phi_k^{hom}} = \frac{\dfrac{\sum_{i \in k} V_i \phi_i^{het}}{\sum_{i \in k} V_i}}{\phi_k^{hom}} \tag{2}$$

where,

ϕ_i^{het} :neutron flux of fine (heterogeneous) region *i* obtained by cell-heterogeneous calculation,

$\bar{\phi}_k^{het}$:average neutron flux of coarse (homogeneous) region k obtained by cell-heterogeneous calculation,

ϕ_k^{hom} :average neutron flux of coarse region k obtained by cell-homogeneous calculation using the SPH corrected cross section given in Eq.(1).

Neutron flux level in the homogeneous calculation is normalized as follows:

$$\frac{\sum_i V_i \phi_i^{het}}{\sum_i V_i} = \frac{\sum_k \bar{V}_k \phi_k^{hom}}{\sum_k \bar{V}_k} \tag{3}$$

where,

$\overline{V}_k = \sum_{i \in k} V_i$: volume of homogeneous region k.

By inserting Eq.(2) to Eq.(1), we obtain the following:

$$\tilde{\Sigma}_k^{ave} \phi_k^{\hom} = \Sigma_k^{ave} \overline{\phi}_k^{het} \quad (4)$$

Equation (4) indicates that the reaction rate of region k in the heterogeneous calculation is preserved in the homogeneous calculation by applying the SPH factor.

The SPH factor depends on neutron flux in homogeneous system, and the homogeneous flux depends on the SPH factor through the SPH corrected cross sections. Therefore, the SPH factor must be obtained by iterative calculations in the homogeneous system.

In GET, discontinuity of neutron flux at surface area of homogenized region is defined as follows in order to preserve reference net neutron current of the heterogeneous system.

$$DF = \frac{\phi^{het,sur}}{\phi^{\hom,sur}}, \quad (5)$$

where,

DF : discontinuity factor,

$\phi^{het,sur}$: surface flux of a region obtained by cell-heterogeneous calculation.

$\phi^{hom,sur}$: surface flux of a region obtained by cell-homogeneous calculation.

The previous studies revealed that the homogeneous calculation cannot simultaneously preserve the surface flux and the neutron net current of the heterogeneous calculation, i.e. only the surface flux or the neutron net current can be preserved in the homogeneous calculation. Therefore, discontinuity for surface flux defined by Eq.(5) is introduced in GET in order to preserve the net current of the heterogeneous calculation. By preserving the net current, reaction rates of homogeneous regions can be identical to those of the heterogeneous calculation. As discussed in the previous chapter, the discontinuity factor is defined for each surface and must be explicitly taken into account during homogeneous calculations. Therefore, memory storage required for homogeneous calculation with GET is larger than that with the SPH method. Especially, in the case of pin-by-pin core calculations, memory storage for the discontinuity factors is considerably large.

It should be noted that the discontinuity factor and the SPH factor relate each other. When one discontinuity factor is assigned to a homogeneous region, it can be taken into account by dividing cross sections by the discontinuity factor[7],[8]. Compared with Eq.(1), a discontinuity factor for a homogeneous region is equivalent to inverse of the SPH factor. In other words, the iteration calculation to obtain the SPH factor is equivalent to that estimating a discontinuity factor for a homogeneous region.

2.2 Comparison of the SPH and GET results

Performances of the two different homogenization methods, i.e. SPH and GET, were compared in simplified pin-by-pin geometries that represent typical situations of PWR colorset assembly configuration. Though the actual PWR colorset configuration is two-dimensional geometry, it is simulated by one-dimensional slab geometry. In order to compare homogenization parameters, i.e. the SPH and the surface discontinuity factors, one-dimensional geometry is more suitable than two-dimensional geometry from the viewpoint of clarity and simplicity. Calculated configurations are shown in Fig.1. Calculations were carried out by the following procedures:

(1)Heterogeneous structure in each cell was homogenized by the cell-heterogeneous single assembly calculations and homogenization parameters (i.e. the SPH factor and discontinuity factors for each cell) were also calculated in single assembly geometry. In the case of the SPH factor calculations, cell-homogeneous calculations in single assembly geometry were repeatedly executed to obtain the converged SPH factors.

(2)Then the cell-homogenized cross sections and the homogenization parameters obtained in the single

assembly calculations were supplied to the colorset configuration with cell-homogenized geometry and cell-homogenized calculation in colorset geometry was carried out.

(3)The reference solution was obtained by the independent cell-heterogeneous calculation in the colorset configuration shown in Fig.1. The homogenization parameters were also estimated in colorset geometry for comparison.

(4)By comparing cell-averaged absorption rates of the cell-homogeneous calculation to the reference value, i.e. those of the cell-heterogeneous calculation, we can evaluate the performance of the homogenization methods. When the cell-homogenized calculation well reproduces the cell-heterogeneous calculation in colorset geometry, performance of the homogenization method to generate the cell-homogenized cross sections is considered as good.

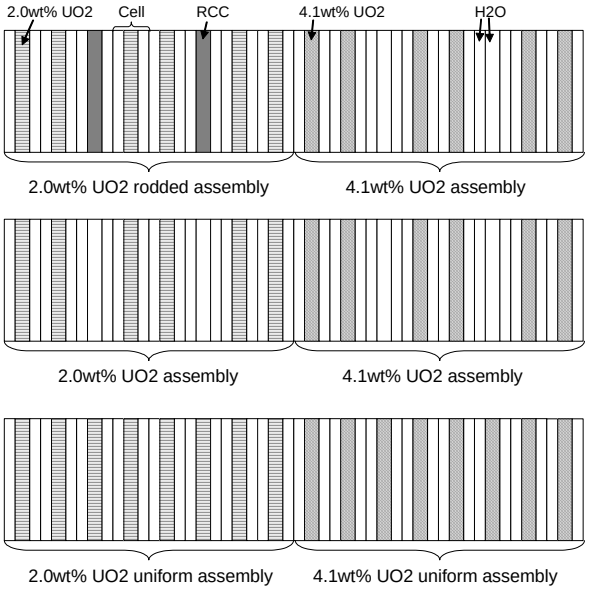

Fig.1 Simplified PWR colorset configurations (one-dimensional slab geometry)

Other calculation conditions are listed below:

- Reflective boundary condition is used both for the single and colorset assembly geometries.
- One-group, uniform fixed source problem is assumed to represent typical situation of thermal group of LWR lattice calculations.
- Intensity of the neutron source is 1.0 cm^{-3}.
- Cross sections are shown in Table I, which are typical values in thermal group of each material.
- Dimensions for fuel and moderator are 0.6cm and 0.45cm, respectively. Consequently, cell pitch is 1.5cm (=0.6+0.45*2).
- Analytic nodal method based on the diffusion theory is used both for the cell-homogeneous and the cell-heterogeneous calculations. No spatial truncation error (i.e. mesh discretization error) exists since calculations were carried out in one-dimensional geometry with known neutron source distribution. Though transport calculations are usually used for cell calculations, diffusion theory is used in this calculation. Therefore, errors caused by spatial truncation and diffusion approximation can be ignored in this calculation and we can focus to the spatial homogenization error.
- Cell-homogenized cross sections are derived using the flux-volume weighting method.

Table 1 Cross sections

Material	Cross secion(1/cm)	
	absorption	total
4.1wt% UO2	0.32	0.69
2.0wt%UO2	0.18	0.55
RCC	7.6	7.6
H2O	0.019	1.3

Calculation results are shown in Tables 2~4. These Tables indicates that results of the SPH method show larger discrepancy than those of GET. In one-dimensional geometry, a discontinuity factor for a homogeneous region is sufficient to reproduce the reference results. Since the SPH factor is equivalent to the inverse of the discontinuity factor as discussed in the previous section, performance of SPH and GET can be equivalent. However, Tables 2~4 clearly show that discrepancy of the SPH method is larger than that of GET.

Table 2 Comparison of absorption rate (2.0wt%UO2+RCC-4.1wt%UO2, simplified slab geometry)

Cell	Absorp. rate	Error in absorp. rate	
	REF	DF(GET)	SPH
U200	0.725	0.00	0.00
U200	0.566	0.00	0.00
RCC	2.260	0.00	0.00
U200	0.449	-0.02	0.01
U200	0.449	-0.05	0.02
RCC	2.249	-0.07	0.65
U200	0.558	-0.13	1.90
U200	0.706	-0.13	3.52
U410	1.163	0.15	-2.74
U410	1.293	0.09	-1.03
GT	0.222	0.04	-0.44
U410	1.359	0.02	-0.19
U410	1.358	0.01	-0.08
GT	0.222	0.00	-0.04
U410	1.283	0.00	-0.02
U410	1.139	0.00	-0.01

Table 3 Comparison of absorption rate (2.0wt%UO2 -4.1wt%UO2, simplified slab geometry)

Cell	Absorp. rate	Error in absorp. rate	
	REF	DF(GET)	SPH
U200	1.146	-0.07	-0.08
U200	1.231	-0.08	-0.09
GT	0.321	-0.13	-0.15
U200	1.282	-0.23	-0.25
U200	1.270	-0.30	-0.34
GT	0.311	-0.53	-0.61
U200	1.146	-0.98	-1.13
U200	0.976	-0.77	-1.11
U410	1.341	1.04	1.33
U410	1.366	0.77	0.89
GT	0.228	0.33	0.38
U410	1.374	0.17	0.20
U410	1.364	0.10	0.11
GT	0.222	0.04	0.05
U410	1.284	0.02	0.02
U410	1.139	0.02	0.02

Table 4 Comparison of absorption rate (2.0wt%UO2 -4.1wt%UO2 uniform, simplified slab geometry)

Cell	Absorp. rate	Error in absorp. rate	
	REF	DF(GET)	SPH
U200	0.999	-0.07	-0.07
U200	0.998	-0.09	-0.10
U200	0.996	-0.14	-0.16
U200	0.993	-0.23	-0.26
U200	0.984	-0.37	-0.43
U200	0.967	-0.57	-0.69
U200	0.931	-0.76	-1.00
U200	0.855	-0.69	-1.22
U410	1.173	1.09	1.57
U410	1.065	0.69	0.90
U410	1.024	0.37	0.45
U410	1.009	0.19	0.22
U410	1.003	0.09	0.10
U410	1.001	0.04	0.04
U410	1.001	0.02	0.02
U410	1.000	0.02	0.02

2.3 Comparison of the SPH and GET homogenization parameters

In order to resolve the cause of error in the SPH method, homogenization parameters were compared. The homogenization parameters, i.e. the surface discontinuity and the SPH factors, are shown in Tables 5~12. Since inverse of a SPH factor is equivalent to a discontinuity factor, it is also shown in Table5~12. Note that homogenization parameters were estimated not only in the single assembly geometries, but also in the colorset assembly geometries to investigate discrepancy. The homogenization parameters obtained in the colorset assembly geometries exactly reproduce reference results obtained in cell-heterogeneous colorset assembly geometries.

From Tables 5~12, we can find the following observations:

- The surface discontinuity factors obtained in the single assembly geometry (Tables 5~9) are similar to those obtained in the colorset geometry (Tables 10~12).
- In contrast to GET, the SPH factors obtained in the single assembly geometry are different from those obtained in the colorset assembly geometry. For example, Tables 8, 9 (single assembly results) and Table 12 (colorset assembly result) clearly indicates discrepancy between single assembly and colorset assembly results. Tables 8 and 9 show that the SPH factors in uniform single assembly is 1.0 but those in colorset assembly (Table 12) are varied between 0.955 and 1.035.
- Small discrepancy can be found in Table 3 for both GET and SPH. From Tables 5 and 7, flux discontinuity between 2.0wt%UO2 and 4.1wt% UO2 fuel assemblies is 1.016/1.031~0.985. In case of the SPH method, flux discontinuity between the assemblies is 1.018/1.023~0.995, which is similar to that obtained by GET, i.e. 0.985. Since flux discontinuity in GET and SPH is similar in this case, discrepancy of GET and SPH is almost equivalent in this case.
- In contrast to Table 3, we can find considerable difference between GET and SPH in Table2. From Tables 6 and 7, flux discontinuity between 2.0wt%UO2+RCC and 4.1wt% UO2 fuel assemblies is 1.019/1.031~0.988. On the other hand, that in the SPH method is 0.920/1.023~0.899, which is different from that of the GET. This difference is a root cause of the considerable discrepancy found in Table 2.

Table 5 Homogenization parameters (2.0wt%UO2, simplified slab geometry)

Cell	Homogenization parameter			
	DF(left)	DF(right)	SPH	1/SPH
U200	1.016	1.022	0.983	1.018
U200	1.010	1.029	0.994	1.006
GT	1.000	1.000	1.023	0.978
U200	1.026	1.016	0.997	1.003
U200	1.016	1.026	0.997	1.003
GT	1.000	1.000	1.023	0.978
U200	1.029	1.010	0.994	1.006
U200	1.022	1.016	0.983	1.018

Table 6 Homogenization parameters (2.0wt%UO2+RCC, simplified slab geometry)

Cell	Homogenization parameter			
	DF(left)	DF(right)	SPH	1/SPH
U200	1.019	1.000	1.087	0.920
U200	1.040	0.945	1.045	0.957
RCC	1.383	1.328	0.714	1.400
U200	0.952	1.023	0.997	1.003
U200	1.023	0.952	0.997	1.003
RCC	1.328	1.383	0.714	1.400
U200	0.945	1.040	1.045	0.957
U200	1.000	1.019	1.087	0.920

Table 7 Homogenization parameters (4.1wt%UO2, simplified slab geometry)

Cell	Homogenization parameter			
	DF(left)	DF(right)	SPH	1/SPH
U410	1.031	1.037	0.977	1.023
U410	1.025	1.046	0.989	1.011
GT	1.000	1.000	1.034	0.967
U410	1.043	1.031	0.992	1.008
U410	1.031	1.043	0.992	1.008
GT	1.000	1.000	1.034	0.967
U410	1.046	1.025	0.989	1.011
U410	1.037	1.031	0.977	1.023

Table 8 Homogenization parameters (2.0wt%UO2 uniform, simplified slab geometry)

Cell	Homogenization parameter			
	DF(left)	DF(right)	SPH	1/SPH
U200	1.017	1.017	1.000	1.000
U200	1.017	1.017	1.000	1.000
U200	1.017	1.017	1.000	1.000
U200	1.017	1.017	1.000	1.000
U200	1.017	1.017	1.000	1.000
U200	1.017	1.017	1.000	1.000
U200	1.017	1.017	1.000	1.000
U200	1.017	1.017	1.000	1.000

Table 9 Homogenization parameters (4.1wt%UO2 uniform, simplified slab geometry)

Cell	Homogenization parameter			
	DF(left)	DF(right)	SPH	1/SPH
U410	1.032	1.032	1.000	1.000
U410	1.032	1.032	1.000	1.000
U410	1.032	1.032	1.000	1.000
U410	1.032	1.032	1.000	1.000
U410	1.032	1.032	1.000	1.000
U410	1.032	1.032	1.000	1.000
U410	1.032	1.032	1.000	1.000
U410	1.032	1.032	1.000	1.000

Table 10 Homogenization parameters (2.0wt%UO2+RCC-4.1wt%UO2, simplified slab geometry)

Cell	Homogenization parameter			
	DF(left)	DF(right)	SPH	1/SPH
U200	1.019	1.000	1.032	0.969
U200	1.040	0.945	0.993	1.007
RCC	1.383	1.328	0.678	1.475
U200	0.952	1.023	0.946	1.057
U200	1.023	0.952	0.946	1.057
RCC	1.330	1.382	0.677	1.477
U200	0.945	1.039	0.990	1.010
U200	1.001	1.017	1.027	0.974
U410	1.033	1.037	1.011	0.989
U410	1.026	1.046	1.022	0.979
GT	1.000	1.000	1.068	0.936
U410	1.043	1.031	1.024	0.976
U410	1.031	1.043	1.024	0.976
GT	1.000	1.000	1.068	0.936
U410	1.046	1.025	1.021	0.979
U410	1.037	1.031	1.009	0.991

Table 11 Homogenization parameters (2.0wt%UO2-4.1wt%UO2, simplified slab geometry)

Cell	Homogenization parameter			
	DF(left)	DF(right)	SPH	1/SPH
U200	1.016	1.022	1.012	0.988
U200	1.010	1.029	1.023	0.977
GT	1.000	1.000	1.053	0.949
U200	1.026	1.015	1.026	0.974
U200	1.017	1.025	1.025	0.976
GT	1.000	1.000	1.050	0.952
U200	1.032	1.004	1.017	0.983
U200	1.030	1.001	0.992	1.008
U410	1.042	1.032	0.953	1.049
U410	1.030	1.044	0.955	1.047
GT	1.000	1.000	0.997	1.003
U410	1.044	1.030	0.955	1.047
U410	1.031	1.043	0.954	1.048
GT	1.000	1.000	0.995	1.005
U410	1.046	1.025	0.951	1.051
U410	1.037	1.031	0.940	1.064

Table 12 Homogenization parameters (2.0wt%UO2-4.1wt%UO2 uniform, simplified slab geometry)

Cell	Homogenization parameter			
	DF(left)	DF(right)	SPH	1/SPH
U200	1.017	1.017	1.035	0.966
U200	1.017	1.017	1.035	0.967
U200	1.017	1.017	1.034	0.967
U200	1.017	1.016	1.034	0.967
U200	1.018	1.016	1.032	0.969
U200	1.018	1.014	1.029	0.971
U200	1.020	1.011	1.023	0.977
U200	1.024	1.002	1.009	0.991
U410	1.042	1.026	0.970	1.031
U410	1.037	1.030	0.961	1.041
U410	1.034	1.031	0.957	1.045
U410	1.033	1.032	0.956	1.047
U410	1.032	1.032	0.955	1.047
U410	1.032	1.032	0.955	1.047
U410	1.032	1.032	0.955	1.047
U410	1.032	1.032	0.955	1.047

The above discussion indicates that root cause of the discrepancy for the SPH method is inappropriate treatment of flux discontinuity between assemblies. This is more clearly indicated in case of Table 4. In this case, the SPH factor obtained in single assembly calculation is 1.0 (Table 8, 9). The SPH factors obtained by single assembly suggest no flux discontinuity exist between the assemblies. However, actual surface discontinuity factors for 2.0wt% and 4.1wt% uniform assemblies are 1.017 and 1.032, respectively, which are different from unity and indicate that there is flux discontinuity between assemblies.

In the next chapter, the improved SPH method, which offers solution for this issue, will be discussed.

3. Improvement of the SPH method

3.1 Description of the improved SPH method

Discussion in the previous section clarified that the source of homogenization error in the SPH method is incorrect representation of flux discontinuity between different types of assemblies. Then, why the flux discontinuity between assemblies is not accurately captured in the SPH method?

The SPH factors are estimated by ratio of neutron flux from a heterogeneous calculation and that from a homogeneous calculation. In the traditional SPH factor derivation, average values of heterogeneous and homogeneous neutron fluxes throughout the system are normalized to the same value. Consequently, the obtained SPH factors are normalized so that their average value is approximately unity. This normalization is necessary to obtain unique set of SPH factors.[1],[2] However, the

normalization condition in the current SPH method drops out the flux discontinuity between assemblies and results some discrepancy.

Fortunately, the normalization condition of the SPH method can be altered.[9] Therefore, normalization condition can be modified to adequately incorporate flux discontinuity between assemblies, which is "the forgotten factor" in the current SPH method. This is the basic concept of the improved SPH method.

The flux discontinuity between assemblies is calculated from the following equation:

$$DF_{SPH} = \frac{\Phi^{het,sur}}{\Phi^{hom,SPH,sur}} \tag{6}$$

where,

DF_{SPH} :flux discontinuity factor for the improved SPH method,

$\Phi^{het,sur}$:surface flux of assembly obtained by cell-heterogeneous calculation,

$\Phi^{hom,SPH,sur}$:surface flux of assembly obtained by cell-homogeneous calculation including the SPH factor.

By considering the discontinuity factor for the improved SPH method, neutron flux discontinuity between assemblies can be adequately taken into account. Detail calculation procedure will be shown later.

One of the merits of the SPH method is no additional homogenization parameter is necessary in homogeneous calculations as described in introduction. However, direct application of Eq.(6) requires additional homogenization parameter in homogeneous calculation and will decrease the merit of the SPH method. In order to improve this point, the following simplification is taken into account in the improved SPH method.

The discontinuity factor of Eq.(6) is obtained in each cell of assembly peripheral (outer most) region. Since direct treatment of these discontinuity factors is somewhat complicated especially in multi-dimensional geometry, we assume that the discontinuity factor is constant throughout the assembly surface. By using this assumption, just one discontinuity factor is obtained for each assembly. When a discontinuity factor is obtained for an assembly, this discontinuity factor can be incorporated by dividing cross sections in the assembly.

Actual calculation procedure of the improved SPH method is summarized below:

(1) A cell-heterogeneous calculation is carried out in single assembly geometry
(2) The SPH factors are derived using the results obtained in (1). This is the same procedure to estimate conventional SPH factors.
(3) Evaluates average of assembly surface flux from the cell-homogenized calculation with SPH corrected cross sections, i.e. from the result of (2).
(4) Estimates average of assembly surface flux from the cell-heterogeneous calculation, i.e. from result of (1).
(5) Using Eq.(6), estimates "assembly averaged cell-level discontinuity factor" from the results of (4) and (3).
(6) Divide the SPH factors obtained in (2) by DF_{SPH} of Eq.(6) to get improved SPH factors. The improved SPH factor implicitly contains flux discontinuity between different types of assemblies.

3.2 Applications

The improved SPH method was applied to the test problems. The first one is that discussed in the previous section, i.e. simple one-group, one-dimensional slab problem, aiming to clarify influence of the improved SPH factor. The second one is more general, i.e., two-dimensional, multigroup, multi-assembly (colorset) geometry of PWR. The second problem will reveal performance of the improved SPH method in actual situation.

For the first problem, the 2.0wt%UO2+RCC-4.1wt%UO2 colorset configuration, which was discussed in the previous section, was re-analyzed using the improved SPH method. The calculation results are shown in Table 13. Tables 13 and 2 indicate that error in absorption rate is significantly reduced using the improved SPH method and is identical to that using GET. Table 14 shows the improved SPH and conventional SPH factors with inverses of these values. Note that the assembly averaged cell-level discontinuity factors obtained by Eq.(6) are 1.108 and 1.008 for 2.0wt%UO2+RCC and 4.10wt%UO2 assemblies, respectively. From Table 14, flux discontinuity between these two assembles using the improved SPH factors is 1.019/1.031~0.988, which is equivalent to GET as shown in Tables 6 and 7.

Table 13 Comparison of absorption rate using the improved SPH method (2.0wt%UO2+RCC-4.1wt%UO2, simplified slab geometry)

Cell	Absorp. rate	Error in absorp. rate	
	REF	DF(GET)	Improv. SPH
U200	0.725	0.00	0.00
U200	0.566	0.00	0.00
RCC	2.260	0.00	0.00
U200	0.449	-0.02	-0.02
U200	0.449	-0.05	-0.05
RCC	2.249	-0.07	-0.07
U200	0.558	-0.13	-0.13
U200	0.706	-0.13	-0.13
U410	1.163	0.15	0.15
U410	1.293	0.09	0.09
GT	0.222	0.04	0.04
U410	1.359	0.02	0.02
U410	1.358	0.01	0.01
GT	0.222	0.00	0.00
U410	1.283	0.00	0.00
U410	1.139	0.00	0.00

Table 14 Comparison of the SPH factors and inverse of the SPH factors (2.0wt%UO2+RCC, simplified slab geometry)

Cell	Homogenization parameter			
	SPH	Improv. SPH	1/SPH	1/(I.SPH)
U200	1.087	0.982	0.920	1.019
U200	1.045	0.944	0.957	1.059
RCC	0.714	0.645	1.400	1.551
U200	0.997	0.900	1.003	1.111
U200	0.997	0.900	1.003	1.111
RCC	0.714	0.645	1.400	1.551
U200	1.045	0.944	0.957	1.059
U200	1.087	0.982	0.920	1.019

Table 15 Comparison of the SPH factors and inverse of the SPH factors (4.1wt%UO2, simplified slab geometry)

Cell	Homogenization parameter			
	SPH	Improv. SPH	1/SPH	1/(I. SPH)
U410	0.977	0.970	1.023	1.031
U410	0.989	0.981	1.011	1.019
GT	1.034	1.026	0.967	0.974
U410	0.992	0.984	1.008	1.016
U410	0.992	0.984	1.008	1.016
GT	1.034	1.026	0.967	0.974
U410	0.989	0.981	1.011	1.019
U410	0.977	0.970	1.023	1.031

The second problem treats a more general case, i.e. two-dimensional PWR colorset calculation. The colorset arrangement of 17x17 PWR fuel assemblies, which is shown in Fig.2, was used. The colorset geometry consists of low (2.0wt%) and high (4.1wt%) enrichment assemblies. The reference calculation in cell-heterogeneous geometry was carried out by the CASMO4 code[10] in nine groups. The CASMO4 code performs transport calculation in the colorset geometry based on the method of characteristics.[11] Since the

CASMO4 code directly treat the cell-heterogeneous geometry, i.e., pellet, clad and moderator, it does not suffer from the cell-homogenization error.

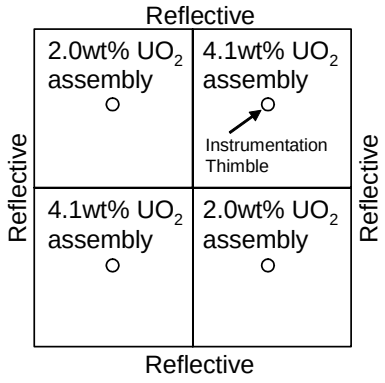

Fig.2 PWR colorset assembly configuration (four 17x17 fuel assemblies)

The cell-homogeneous calculation was carried out by the SCOPE2 code.[4] It performs pin-by-pin core calculation in three-dimensional geometry based on multi-group advanced nodal transport method. The advanced nodal method is used to reduce the discretization error of pin-by-pin mesh, whose radial dimension is about 1cm. Cross sections of SCOPE2 code is generated from CASMO4 calculations in single assembly geometry. No colorset calculation was used for cross section generation. The SPH method is utilized to capture homogenization effect of pin cells. The improved SPH method described in the previous section was implemented in the cross section tabulation code for SCOPE2, which edits cell-homogenized cross section obtained from the CASMO4 calculations and prepares tabulated cross section for SCOPE2. Note that no modification was made in SCOPE2 for the improved SPH method. Nine groups calculation was also carried out by the SCOPE2 code.

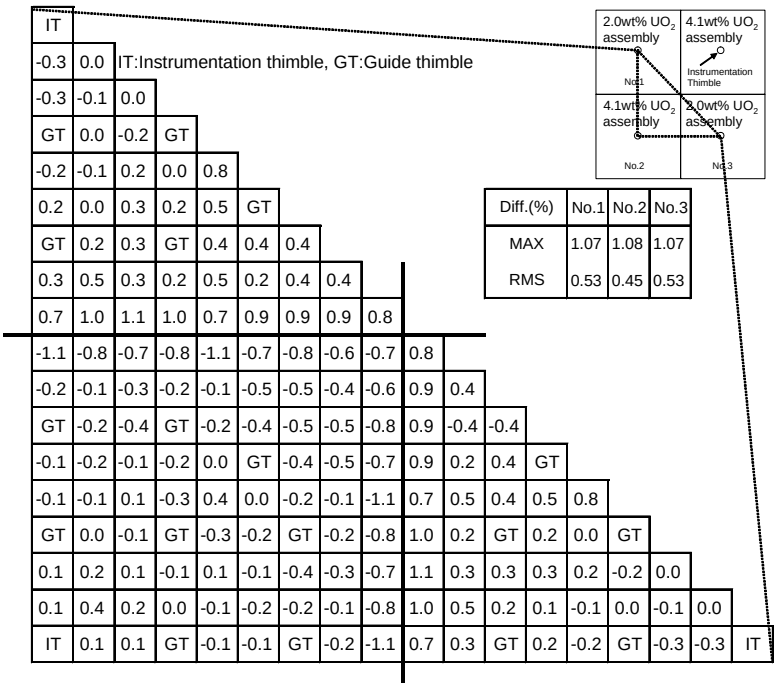

Fig.3 Comparison of pin power distribution predicted by SCOPE2 (conventional SPH method) and CASMO4 in central zone of 2x2 assembly geometry of 2.0wt% and 4.1wt% UO2 fuel assemblies.

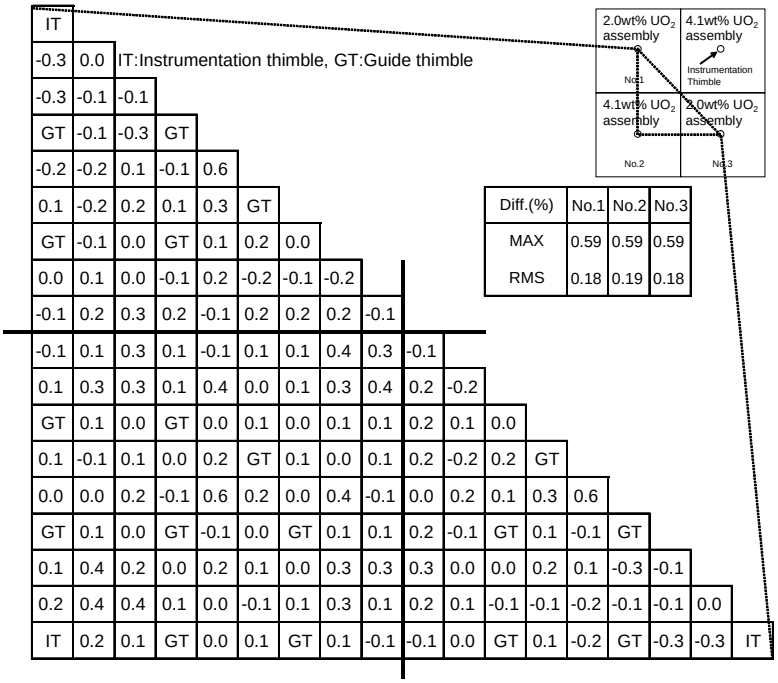

Fig.4 Comparison of pin power distribution predicted by SCOPE2 (improved SPH method)and CASMO4 in central zone of 2x2 assembly geometry of 2.0wt% and 4.1wt% UO2 fuel assemblies.

Calculation results of the conventional and the improved SPH methods are shown in Fig. 3 and 4. These figures show pin-wise power distribution in the center part of the colorset geometry and the SCOPE2 results are compared with the CASMO4 result. Figures 3 and 4 clearly indicate that the improved SPH method effectively reduces errors of the pin-powers. Since neutron flux discontinuity between assemblies is adequately incorporated in the improved SPH method, error of pin-powers especially in assembly peripheral region is reduced.

The above results show that the improved SPH method can effectively reduce cell-homogenization error especially in assembly peripheral region. We can easily retrofit the improved SPH method to pin-by-pin core calculations since only calculation procedure of the SPH factors is slightly modified. This indicates that the improved SPH method is an effective and practical homogenization technique for pin-by-pin core calculations.

4. Conclusion

The improved SPH method is proposed in this paper. By using the improved SPH method, cell-level flux discontinuity between different types of assemblies, which is dropped out in the conventional SPH method, is accurately taken into account. Consequently, the improved SPH method gives more accurate results than the conventional SPH method especially at the assembly periphery region in multi-assembly geometry. Test calculations in simple one-dimensional slab and two-dimensional PWR colorset geometries showed that errors of pin-powers can be effectively reduced by the improved SPH method. Since the implementation of the improved SPH method is easy, it will be practical and useful homogenization method for pin-by-pin fine-mesh core calculations.

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