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Simulation of Caliban Reactor Burst Wait Time and Initiation Probability Using a Point Reactor Model and PANDA Code

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Caliban assembly is a fast metal reactor operated by CEA/valduc. When used in the pulsed mode, without an external neutron source, there is a non reproducible time delay between the prompt critical state and the neutron burst. The experimental wait time distribution was constructed using 114 burst experiments.

The wait time is essentially the time before a persistent fission chain is initiated. When the delayed neutron precursors life time can be neglected the wait time can be calculated using the initiation probability which is the asymptotic value of the neutron persistence probability. The initiation probability can be computed using PANDA discrete ordinates neutron transport code.

In this paper the precursor lifetime has been taken into account using a time dependent point reactor model. The numerical results given by this model are in good agreement with the experiments.

KEYWORDS : Fission Chain Initiation, Fast Pulsed Reactor, Stochastic Neutronics

1. Introduction

Caliban fast metal reactor [1,3] is designed for pulsed neutron flux generation when operated above prompt critical. When these pulsed experiments are performed without an external neutron source there is a non reproducible time delay between the prompt critical state and the neutron burst. This wait time duration can be up to several seconds. A similar phenomena was observed with GODIVA II reactor [4] and theoretically explained by G. E. Hansen [5]. The wait time fluctuation is a stochastic neutron transport problem, it results mainly from the probability for one source neutron to initiate a persistent fission chain.

The experimental wait time probability distribution is obtained using time measurements performed during burst experiments with Caliban [3]. A stochastic neutronics time dependent with delayed neutrons point model developed by B. Lemaire [7] is used to calculate the wait time probability distribution.

In a first part we present the Caliban reactor and the operating procedures for burst generation. In a second part, stochastic neutronics methods used for initiation and persistence probability calculations are developed. In a third part numerical results concerning Caliban wait time and initiation probability are given.

2. Description of the experiments

2.1 Caliban reactor

Caliban facility is operated by CEA/Valduc since 1971. It is a fast metal reactor which can be used in steady state or pulsed mode. In this last case the reactivity is brought above prompt critical in order to produce a short intense neutron burst. A typical pulse is characterized by a

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width at half-maximum of 50 μs with a yield of about 5.8 10^{16} fissions.

The core weight is about 110 kg, it is made of 90% Uranium (with an enrichment of 93.5%) and 10% Molybdenum alloy. These values are given in weight. The reactor is a vertical cylinder with 195 mm diameter and 250 mm height. It is divided in a fixed top part and a bottom removable safety block.

The assembly reactivity is operated using three control rods and one excursion rod in UMo (10%). This rod is used for the fast injection of reactivity which permits in less than 200 ms the transition from a delayed super critical state to a prompt super critical state necessary to the neutron pulse emission. A central cavity is used for sample irradiation and perturbation experiments [2]. The delayed fission neutron fraction β is 660 10^{-5}.

The considered burst experiments were performed without an external source. The internal neutron source are mainly produced by the Uranium-235 spontaneous fissions. The internal source has been evaluated experimentally using the measured power with a given reactor sub critical state. A source level of 200 n/s was measured. [3]

2.2 Operating procedure

The operating procedure used for the burst experiments is the following one :

- The reactor is brought to a configuration corresponding to a delayed super critical state characterized by a total multiplication coefficient K=1.00120. This will be the state from which a prompt super critical configuration will be reached by insertion of the burst rod.
- The assembly is then set to a K=0.9 sub critical state during 15 minutes in order to obtain the extinction of the delayed neutron precursors.
- The reactor is brought again to the delayed sub critical state configuration of the first step. The state K=1.00120 is reached in about 60 seconds.
- The assembly is then maintained in this configuration during 20 seconds until the fast insertion of the excursion rod which brought the system to an above prompt critical state characterized by ΔK_p=8,3 cents (K=1.00720) in less than 200ms.
- After the prompt critical time a neutron burst has a non zero probability to occur. The burst time corresponds to a neutron chain multiplication until a given power is reached (P=1000 Watts).

2.3 Results

The wait times were measured in 114 burst experiments performed using the same operating procedure [2]. The results were collected in order to obtain the wait time probability distribution (cf. figure 1). The mean and the variance of the measured burst wait time are :

$$\bar{t} = 0.742s \quad \overline{t^2} - \bar{t}^2 = 0.487s^2$$

3. Burst wait time and initiation probabilities calculation

3.1 General

The wait time is characterized by the neutron burst event probability with time. Let P(t) be this probability, the wait time mean value and variance are given by :

$$\bar{t} = \int_0^{\infty} t \, P(t) \, dt \qquad \overline{t^2} = \int_0^{\infty} t^2 \, P(t) \, dt \qquad V(t) = \overline{t^2} - \bar{t}^2$$

P(t) is the probability that a neutron number greater than a given number N are in the assembly at time t.

When a persistent fission chain is initiated, the neutron multiplication is very fast. The time delay due to the multiplication can be neglected against the burst wait time. The probability that at least one neutron is in the system (persistence probability) will be used as the neutron burst probability.

In order to derive the persistence probability equation we use a methodology developed by B. Lemaire [7]. At first, the backward probability equation for the presence of n neutrons and p precursors is derived using a time dependent point reactor model. A specific case is the probability for zero neutron in the final state or extinction probability given that there was one neutron or one precursor in the initial state. The persistence probability (non extinction) is the probability for the complementary event. Persistence probability equation with a neutron source is derived using the single initial neutron persistence probability.

When the delayed neutron precursors lifetime is much larger than the prompt initiation time the persistence probability can be assimilated to its asymptotic (stationary) value, the initiation probability or persistence for an infinite final time. Regarding the initiation probability problem the stationary equation with neutron position and velocity dependencies can be solved using a discrete ordinates transport code.

3.2 Presence Probability for n Neutrons and p Precursors

The probability equation is derived within the framework of a time dependent point reactor model. On the other hand, in order to simplify the notation we consider the single precursor group case.

We use the following notations and definitions :

- θp is the mean lifetime for delayed neutron precursors.
- θ is the mean lifetime for neutrons.
- θ_c is the mean capture time.
- θ_F is the mean fission time. $\frac{1}{\theta} = \frac{1}{\theta_c} + \frac{1}{\theta_F}$
- f_i is the probability that i neutrons are emitted by one fission.
- g_j is the probability that j delayed neutron precursors are emitted by one fission.
- ν is the average number of neutrons (prompt and delayed) emitted by fission.
- k is the multiplication coefficient. $\frac{k}{\nu} = \frac{\theta}{\theta_F}$
- β is the fraction of delayed neutron precursors emitted by fission.
- $P(n,p,t/m,q,\tau)$ is the presence (transition) probability for n neutrons and p precursors at terminal time t given that there was m neutrons and q precursor in the system at initial time τ.

In order to derive the backward probability equation, the transition between the initial time τ and the final time t is decomposed by considering all the possible transitions through the intermediate states at time $\tau+d\tau$.

$$P(n,p,t/m,q,\tau)=\left(1-m\frac{d\tau}{\theta}-q\frac{d\tau}{\theta_p}\right)P(\bullet/m,q,\tau+d\tau)+$$
$$+m\frac{d\tau}{\theta_c}P(\bullet/m-1,q,\tau+d\tau)+$$
$$+q\frac{d\tau}{\theta_p}P(\bullet/m+1,q-1,\tau+d\tau)+ \tag{1}$$
$$+m\frac{d\tau}{\theta_F}\sum_{i,j=0}^{\infty}f_i g_j P(\bullet/m+i-1,q+j,\tau+d\tau)$$

Consider the right hand side of equation (1).

- The first term corresponds to the survival of the m neutrons and q precursors.
- The second term stands for the capture of one neutron.
- The third term describes the death of one precursor giving birth to one delayed neutron.
- The fourth term corresponds to a neutron induced fission giving birth to i prompt neutrons and j delayed neutron precursors.

Using the limit process the transition probability equation writes :

$$-\theta\frac{d}{d\tau}P(n,p,t/m,q,\tau)=-mP(\bullet/m,q,\tau)+q\frac{\theta}{\theta_p}\left(P(\bullet/m+1,q-1,\tau)-P(\bullet/m,q,\tau)\right)+$$
$$+m\frac{\theta}{\theta_c}P(\bullet/m+1,q-1,\tau)+m\frac{\theta}{\theta_F}\sum_{i,j=0}^{\infty}f_i g_j P(\bullet/m+i-1,q+j,\tau) \tag{2}$$

The backward equation is subject to terminal condition.

$$P(n,p,t/m,q,t)=\delta_{n,m}\,\delta_{p,q}$$

The single initial neutron probability equation is :

$$-\theta\frac{d}{d\tau}P(n,p,t/0,1,\tau)=-P(\bullet/1,0,\tau)+\left(1-\frac{k}{\nu}\right)P(\bullet/0,0,\tau)+$$
$$+\frac{k}{\nu}\sum_{i,j=0}^{\infty}f_i g_j P(\bullet/i,0,\tau)P(\bullet/0,j,\tau) \tag{3}$$

The Single initial precursor probability equation is :

$$-\theta_p\frac{d}{d\tau}P(n,p,t/0,1,\tau)=P(\bullet/1,0,\tau)-P(\bullet/0,1,\tau) \tag{4}$$

3.3 Persistence Probability

The persistence probability is the transition probability between an initial state and a final state with zero neutron and any number of precursors. The single initial neutron and single initial precursor persistence probabilities are :

$$a(\tau) = 1 - \sum_{p=0}^{\infty} P(0, p, t / 1,0,\tau) \qquad a_p(\tau) = 1 - \sum_{p=0}^{\infty} P(0, p, t / 0,1,\tau)$$

The single initial precursor persistence probability equation is derived using equation (4).

$$-\theta_p \frac{da_p}{d\tau} = a - a_p$$

The single initial neutron persistence probability equation is derived using equation (3).

$$-\theta \frac{da}{d\tau} = -a + \frac{k}{\nu}\left(1 - \sum_{i=0}^{\infty} f_i (1-a)^i \sum_{j=0}^{\infty} g_j (1-a_p)^j\right)$$

Using the χ and χ' coefficients defined by :

$$\chi_q = \sum_{i=q} \frac{i!}{q!(i-q)!} f_i \qquad \chi'_q = \sum_{j=q} \frac{j!}{q!(j-q)!} g_j \qquad C_q = \frac{\chi_q}{\nu(1-\beta)}$$

The persistence equation becomes :

$$-\theta \frac{da}{d\tau} = -a + \frac{k}{\nu}\left(1 - \sum_{q=0}^{\infty} \chi_q (-a)^q \sum_{q'=0}^{\infty} \chi'_{q'} (-a_p)^{q'}\right)$$

When the probabilities a and a_p are small the sum term in the left hand side of the equation can be truncated and we get the approximate equation :

$$-\theta \frac{da}{d\tau} = -a + \frac{k}{\nu}\left(\chi_1 a - \chi_2 a^2 + \chi'_1 a_p\right)$$

Using the χ and χ' coefficients definitions in term of ν, β and k the single initial persistence equation becomes :

$$-\theta \frac{da}{d\tau} = [(1-\beta)k - 1]a - (1-\beta)kC_2 a^2 + \beta k a_p$$

When six groups of delayed neutrons are defined the persistence probability equation system writes :

$$-\theta \frac{da}{d\tau} = [(1-\beta)k-1]a - (1-\beta)kC_2a^2 + k\sum_{j=1}^{6}\beta_j a_j$$

$$-\theta_j \frac{da_j}{d\tau} = a - a_j \qquad j = 1..6 \tag{5}$$

$$a(\tau = t) = 1 \qquad a_p(\tau = t) = 0$$

3.4 Persistence and Extinction Probability with a Neutron Source

Let $e_S(t/0,0,\tau)$ be the probability of zero neutron at the terminal time t given that there was no neutron and precursor at initial time τ with a neutron source of strength S. This is the extinction probability.

In order to derive the extinction probability equation the transition between the initial time τ and the final time t is decomposed by considering all the possible transitions through the intermediate states at time τ+dτ. Take the right hand side of equation (6). The first term corresponds to zero neutron source emission, the second term describes the one source neutron emission followed by its extinction at the terminal time.

$$e_s(t/0,0,\tau) = (1 - S\,d\tau)\,e_s(t/0,0,\tau + d\tau) + S\,d\tau\;e_s(t/1,0,\tau + d\tau) \tag{6}$$

Using the limit process, the extinction equation writes :

$$\frac{d}{d\tau}e_s(\tau) = Sa(\tau)e_s(\tau) \qquad e_s(\tau = t) = 1 \tag{7}$$

The persistence probability with source is :

$$a_S(t/\tau) = 1 - e_S(t/\tau)$$

3.5 Initiation Probability

The initiation probability is the probability to initiate a fission chain which remains persistent for any future time. For a constant reactivity level, it is solution of the stationary persistence equation. Within the point reactor model frame with the C_2 approximation the solution is :

$$a = a_j = \frac{K-1}{KC_2} \qquad a_S = 1 - e^{-Sa(t-\tau)}$$

More generally, without the approximation to a point model, the initiation probability is solution of an adjoint stationary transport equation with an extra non linear source term. [6]

$$\left(-\vec{\Omega}.\vec{\nabla}+\sigma_T\right)a(\vec{r},\vec{v},t)=\int\sigma_s(\vec{r},\vec{v},\vec{v}')\,a(\vec{r},\vec{v}',t)\,d^3v'+\int\bar{\nu}\sigma_F(\vec{r},\vec{v},\vec{v}')\,a(\vec{r},\vec{v}',t)\,d^3v'-$$
$$-\sum_{i=2}^{M}C_i(\vec{r},v)\left(-\int\frac{\chi(\vec{v})}{4\pi}a(\vec{r},\vec{v},t)\,d^3v\right)^i \quad (8)$$

The adjoint boundary conditions are :

$$a(\vec{r}_B,\vec{v},t)=0 \qquad \vec{n}_B.\vec{v}\geq 0$$

With a neutron source the initiation probability before t writes :

$$a_s(t)=1-\exp\left(-\int_{t_0}^{t}\left(\iint S(\vec{r},\vec{v},t)\,a(\vec{r},\vec{v},t)d^3r\,d^3v\right)d\tau\right) \quad (9)$$

4. Numerical Applications

4.1 Caliban Burst Wait Time Simulation

The Caliban burst wait time simulation is performed within the frame of the point reactor model. The burst probability is approximated by the presence probability of at least one neutron in the reactor (non extinction) $a_S(t)$.

Each point of the wait time cumulative distribution corresponds to a different final time t for the same initial time τ. For each final time the equation system (5,7) is solved backward in time starting from the final time until the initial time is reached. The initial time corresponds to the beginning of the delayed critical state (τ=-20.2s).

We used the following numerical data :

- $\theta_p=12.\ 10^{-9}$ s
- $c_2=0.98$
- S=200 n/s
- θ_j (second)=(80.645, 32.785, 9.009, 3.322, 0.885, 0.333)
- $\beta_j(10^{-5})$=(21.73, 143.79, 130.34, 260.69, 76.55, 26.90)

The total multiplication coefficient K is time dependent as shown on figure 1.

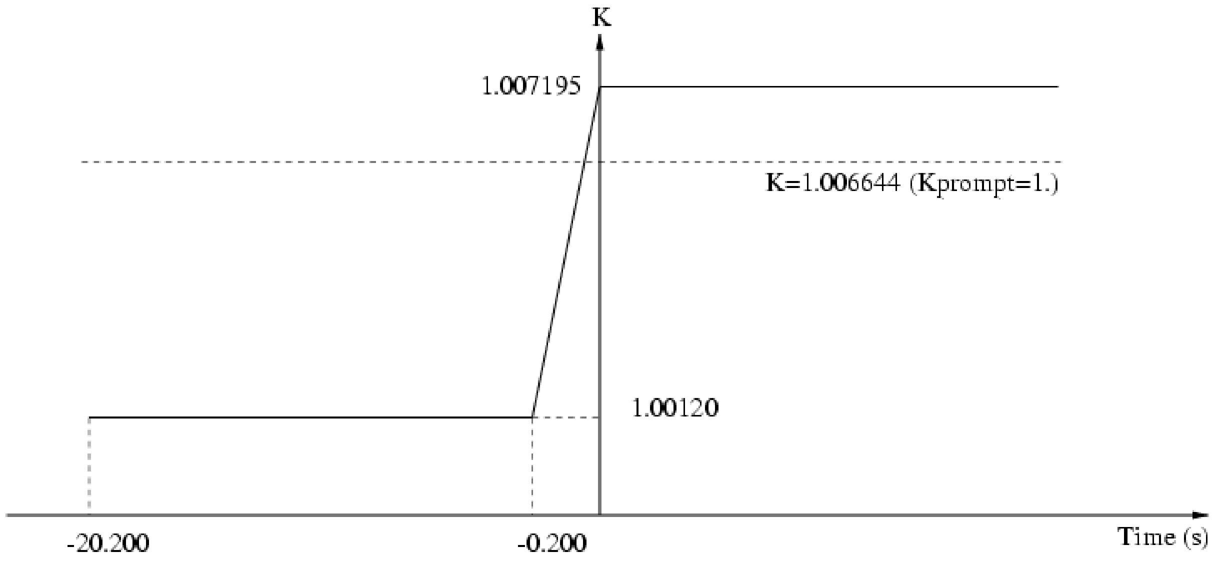

Fig.1 Time dependent total multiplication coefficient K value used in the numerical application.

On a numerical point of view, the equation system (6,7,8) is discretized using finite implicit differencing with an adaptive time step.

In the experimental cumulative distribution, the events which correspond to a burst happening before the end of insertion of the burst rod have been removed. These events have also been removed from the theoretical distribution which is then renormalized. The experimental and calculated cumulative wait time probability distributions are given in figure 2, both results are in good agreement.

Measured $\bar{t} = 0.742s \quad \overline{t^2} - \bar{t}^2 = 0.487s^2$

Calculated $\bar{t} = 0.769s \quad \overline{t^2} - \bar{t}^2 = 0.508s^2$

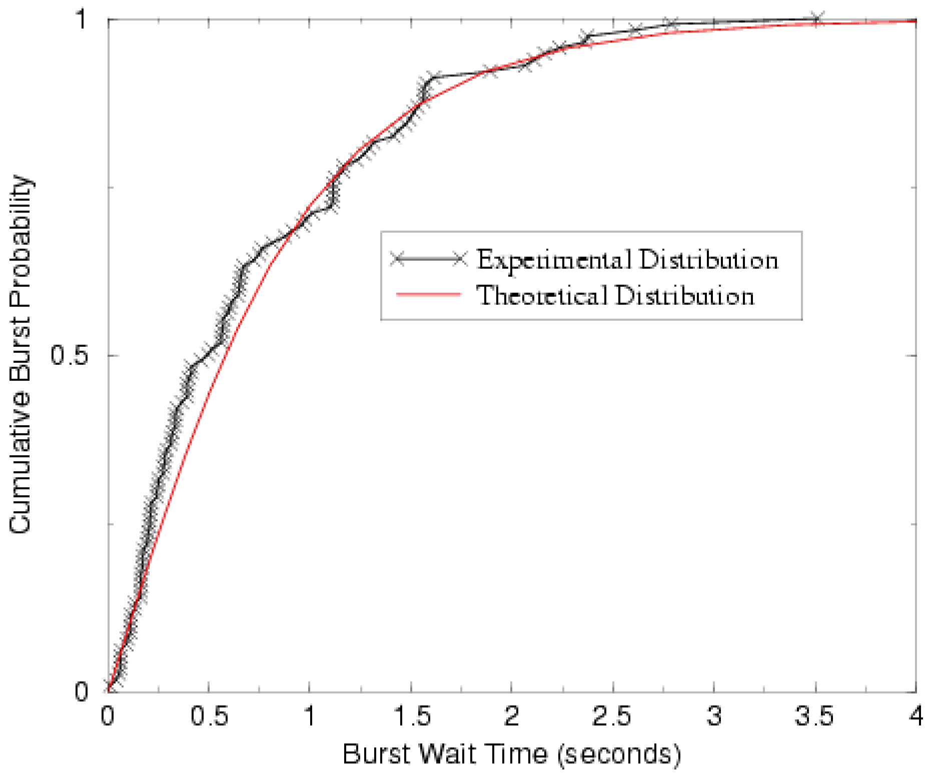

Fig.2 Experimental and calculated neutron burst cumulative distribution probability versus wait time.

4.2 Caliban Initiation Probability

The initiation probability with position and velocity dependencies is solution of the non linear stationary adjoint transport equation (4) which is solved using PANDA discrete ordinates (S_N) code. A simplified 2D modeling of Caliban reactor with an effective multiplication coefficient corresponding to the experimental prompt critical state (K=1.00720) is designed (cf. fig.3). The spatial mesh size is 24x34=816 cells.

The initiation probability map for a single fission neutron isotropically emitted in each cell is presented in figure 4. The PANDA calculation was performed using a 48 energy group neutron cross-section file originating from ENDF/B-VI continuous data files, P4 scattering moments Legendre expansion and S_{16} angular discretization.

The initiation probability for a fission source of strength S=200n/s uniformly distributed in the core is calculated using the single neutron probabilities with equation (10).

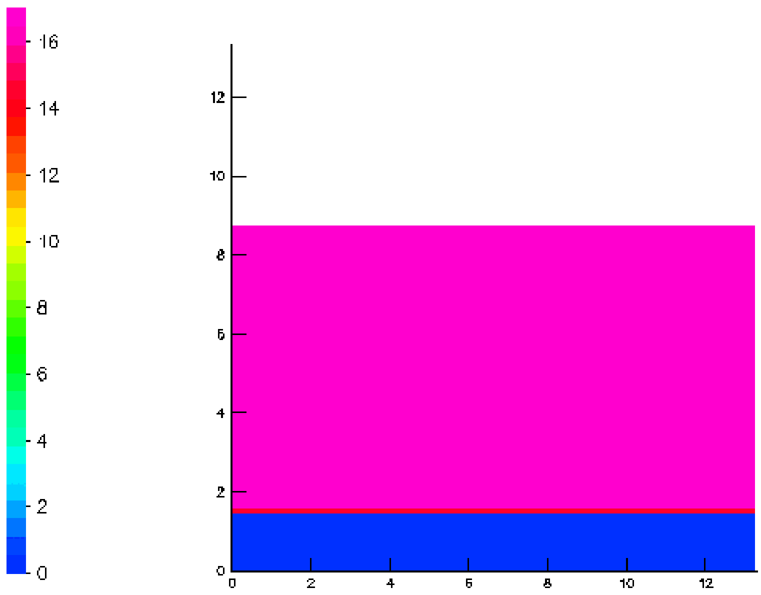

Fig.3 Caliban reactor modeling for PANDA initiation probability computation. The U5Mo Core and the central cavity are modeled in 2D-axisymetric with a Z=0 reflective boundary condition.

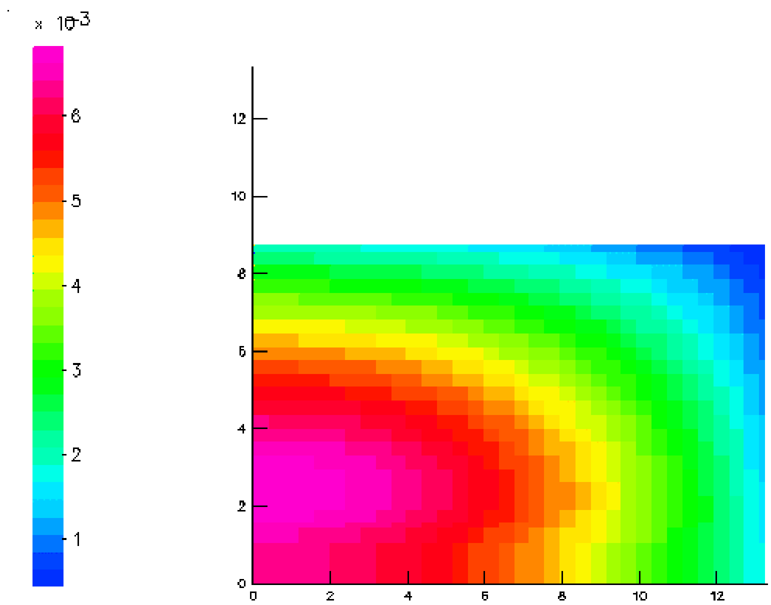

Fig.4 Caliban initiation probability map for a single initial fission neutron in each cell.

5. Conclusion

The probability distribution for the time delay between the prompt critical state and the neutron pulse was measured using burst experiments with Caliban fast pulsed reactor.

In order to understand this phenomenon a stochastic neutronics time dependent point reactor model taking into account the delayed neutron production was used. In this model, the multiplication time after initiation of a persistent fission chain is neglected and the persistence probability is given by the presence probability of at least one neutron (non-extinction probability). Numerical results obtained with this model are in good agreement with the experiments.

The neutron persistence probability asymptotic limit is the initiation probability which can be used to calculate the burst wait time when the delayed neutrons production dynamics can be neglected. The initiation probability with position and velocity dependencies is computed using the discrete ordinates code PANDA. In order to simulate burst experiments with PANDA the delayed neutrons treatment should be included in the time dependent version of the code.

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