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Probability Approaching Method (PAM) and Its Application on Fuel Management Optimization

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For multi-cycle reloading optimization problem, a new solving scheme is presented. The multi-cycle problem is de-coupled into a number of relatively independent mono-cycle issues, then this non-linear programming problem with complex constraints is solved by an advanced new algorithm—probability approaching method (PAM), which is based on probability theory. The result on simplified core model shows well effect of this new multi-cycle optimization scheme.

KEYWORDS: fuel management, optimization, multi-cycle, Probability Approaching Method

1. Introduction

Fuel assemblies in a reactor are discharged normally after several fuel cycles. Therefore, the refueling design of a nuclear power plant is in fact a multi-cycle problem. A multi-cycle reloading optimization problem involves the coupling of refueling processes of two consecutive fuel cycles. Compared with mono-cycle optimization problem, multi-cycle optimization problem is a more complicated combined optimization problem. The main difficulties consist in:

1. The evolution from mono-cycle to multi-cycle not only consists of a multiple increase of variables, but also changes the characteristics of the problem itself. A typical multi-cycle problem consists of two kinds of simultaneous combined relations. One is the combined relationship of the locations where the fuel assemblies are placed at; the other is the combined relationship of fuel loading patterns among the relevant consecutive fuel cycles. The characteristics of these two combined relationships are totally different from each other and the manners of the corresponding responses to the influences of the specified objective are also different. Therefore, the algorithms dealing with mono-cycle problems, such as BE, SA, GA, etc., can not be directly applied to solve multi-cycle optimization problems.

2. Referencing to the large computational task related to mono-cycle optimization problems, the computational task of a multi-cycle optimization problem by taking into account the interactive effects of several fuel cycles simultaneously will increase exponentially. The relevant computational task referring to a complicated multi-cycle optimization problem is unacceptable unless a very efficient algorithm or a much faster computer can be developed. Therefore, an algorithm which can be applied to solve a multi-cycle problem must be capable of efficiently saving computing capacity, i.e., with very good "intelligence".

The complexity of multi-cycle optimization problems as mentioned above has made these problems very difficult to solve, with so far no good way to cope with the multi-cycle optimization problems in reactor fuel management throughout the world. Few references relevant to the multi-cycle optimization problems in reactor fuel management can be found in open literature. The existing methods normally adopt a point reactor core model to carry out

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the multi-cycle optimization in advance and then use the optimized results from this simplified optimization method to guide the corresponding mono-cycle optimization process sequentially according to the order of each cycle, with the main steps stated below[1-3]:

Step 1: simplify the reactor core model, normally by partitioning the fuel assemblies in the reactor core to a number of batches (for instance 3 batches), and then use the average parameters (power, burn-up, etc.) of each batch as variables to carry out multi-cycle optimization calculation with a zero dimension reactor core model, so that the optimized parameters (batch power, batch burn-up, cycle length, etc.) of each fuel cycle under the umbrella of the multi-cycle optimization scheme are obtained;

Step 2: employ the results obtained from Step 1 as the guidance to carry out the mono-cycle optimization calculation sequentially for real reactor loading patterns with a commonly acceptable reactor core model in engineering design for each fuel cycle. Step 2 is required to make the core parameters of each fuel cycle as close as possible to the corresponding core parameters calculated from Step 1.

From the calculation results of the references mentioned above, one can conclude that this type of multi-cycle optimization method with a point reactor model can produce slightly better optimized results than the method of directly using mono-cycle optimization algorithm sequentially cycle by cycle, because it has considered the interactive effects of all relevant fuel cycles. However, this type of multi-cycle optimization method is still far from satisfactory. The following shortcomings have yet to be improved:

Because the assemblies in the reactor are only considered as few batches, and spatial influence of core can not be considered in the optimization, this zero dimensional reactor core model used in the optimization process is obviously too rough and differs too much from the situation of a real reactor. This shortcoming will no doubt rebate the optimization effect of this type of method.

In order to solve multi-cycle optimization problems more effectively, we have developed a new multi-cycle optimization scheme. This scheme can accurately take into account the details down to reactor fuel assemblies in processing the multi-cycle optimization calculation (i.e. the step 1 above). Of course, other problems will appear when a more accurate reactor core model is employed, but these newly encountered problems can be solved by applying a newly developed algorithm — probability approaching method (PAM), which is based on probability theory. We cautiously claim and predict that this new method with PAM can be effective in solving multi-cycle optimization problem in reactor fuel management.

2. New solving scheme for multi-cycle problem

2.1 De-coupling of the multi-cycle optimization problem

The basic idea embedded in the new solving scheme is to de-couple a multi-cycle optimization issue into a number of relatively independent mono-cycle issues. Assuming that 400 times of objective function evaluation are needed to solve a mono-cycle optimization problem, then 400^5 times of objective function evaluation are needed to directly solve a multi-cycle optimization problem with 5 cycles. However, if this multi-cycle optimization problem could be split into a number of relatively independent mono-cycle optimization problems, the order of the computing task would be reduced to $400\times5\times K$ times. Here K is the iteration time of the algorithm.

A typical multi-cycle optimization model is taken as instance. The assembly number in the core is M and the total cycle number considered is N. The cycle lengths of each cycle are appointed previously. The objective of the problem is to minimize the total cost of fresh assemblies. Then the multi-cycle problem can be expressed as follows.

Optimization objective:

$$\text{Maximize} \quad g = -\text{cost}(\varepsilon^k, n^k)$$

Variables:

$$X \equiv (\varepsilon^k, n^k, LP^k), \quad k = 1, 2...,N.$$

where ε^k and n^k are the enrichment and number of fresh assemblies used in the k-th cycle, LP^k is the BOC loading pattern of the k-th cycle.

Criteria constrained:

Peaking factor and fuel assembly burn-up limits

$$0 < F_i^k < F_{lim}, \quad 0 < Bu_i^k < Bu_{lim};$$
$$i = 1, 2...,M, \quad k = 1, 2...,N.$$

where F_i^k is the average assembly relative power in the k-th cycle . Bu_i^k is the burn-up values of each assembly in the core at the end of the k-th fuel cycle.

The cycle length of the k-th cycle must be equal to the value T^k , the appointed cycle length of the k-th cycle previously.

To de-couple the two different kinds of combined relations in multi-cycle problem, this problem is also solved in two steps, which is somewhat like the previous point-reactor model method. But in step1, the rough point-reactor model is replaced with a more accurate core model. And the problems caused by accurate core model are dealt with probability approaching method (PAM).

Step 1: replace the variables (ε^k , n^k , LP^k) of the problem with (ε^k , n^k , F_i^k), then carry out multi-cycle optimization. In this step, LP searching needs not to be considered.

Step 2: according to the F_i^k obtained in Step 1, carry out mono-cycle LP searching sequentially, making the relative power distribution in each cycle be seem as F_i obtained in step 1, and appointed cycle length be arrived.

Because accurate core model is utilized in step 1 here, a new constraint must be added in this step. This constraint is that the F_i^k given in Step 1 must exists corresponding loading pattern (LP) to realize, so that in step 2, the solving from F_i to LP can has solution.

The work in step 2 is only a number of mono-cycle LP searching problems, can be completed by existent common mono-cycle optimization methods. In the multi-cycle code we developed, this mono-cycle LP searching is carried out by statistic inductive algorithm, which is a high efficiency global optimization algorithm[4]. So the main difficulty is the work in step 1. And in next parts, we only describe how to solve the problem in step 1.

The multi-cycle problem in step 1 is as follows.

Optimization objective:

$$\text{Maximize} \quad g = -\text{cost}(\varepsilon^k, n^k)$$

Variables:

$$X \equiv (\varepsilon^k, n^k, F_i^k),$$
$$i = 1, 2...,M, \quad k = 1, 2...,N.$$

Criteria constrained:

peaking factor and fuel assembly burn-up limits

(a) $0 < F_i^k < F_{lim}$,

(b) $0 < Bu_i^k < Bu_{lim}$;

reload coupling conditions

(c) $Bu_i^k - F_i^k . T^k = Bu_j^{k-1}, \quad \varepsilon_i^k = \varepsilon_j^{k-1}$;

In each cycle, for the BOC (ε_i^k, Bu_i^k, F_i^k) corresponding to variable X (ε^k, n^k, F_i^k),

there must exist a set of (LP^k) making the core parameters (ε_i^k, Bu_i^k, F_i^k, T^k) can be realized. Thus, the spatial influence of core is considered in the calculation.

The problem defined above is a non-linear programming problem with constraints. It can be found that this non-linear programming problem has some characteristics. The objective and constraint (a) and (b) are very simple to evaluate from variable X, while the constraint (c) is complex and need core calculations. But in constraint (c), the relation between different cycles is independent. Thus, when the time-consuming core calculations are performed, the combined relations between consecutive cycles need not to be considered. Then the two kinds of different combined relations in multi-cycle problem are de-coupled. This makes it possible to solve this problem in acceptable computing time.

2.2 Solve the non-linear programming problem by PAM algorithm

The optimization problem above is a non-linear programming problem with constraints. Because the constraints are very complex, it is quite difficult to solve the problem by existent methods. To solve this problem effectively, a new algorithm is developed, named probability approaching method (PAM).

The non-linear programming problem above can be simply written as:

$$\begin{aligned} &\text{Maximize} \quad g(X) \\ &\text{s. t.} \quad f(X) = 0 \end{aligned}$$

The function g (X) is very simple, but the constraint f (X) = 0 is quite complex. For the non-linear programming problem with complex constraints above, because of the high complicacy of the constraint, it is very difficult to know feasible region distribution clearly in the whole searching zone. This is just what makes it difficult to solve the problem. Usually, what we can do involves only:

- to know feasible region distribution roughly in the whole searching zone.
- to know feasible region distribution clearly in a local searching zone.

According to these facts, a scheme for finding optimal solution based on probability theory is presented. The complex constraint is converted to a rough probability condition by using a statistic method, and the non-linear programming problem is converted into a probability problem. By solving this probability problem, the probability distribution of optimal solution is obtained, which tells us the suitable searching area of next step. Then the probability condition converted from the constraint is updated by using new statistic area, and the probability problem is solved again. Iteration is executed and the optimal solution can be obtained at last.

That is to say, the complex strict feasible regional distribution is approached by a probability distribution, which is converging more and more with iterations. So this algorithm is called probability approaching method.

In the following discussion, we call a solution feasible point if it satisfies the constraint f (X) = 0 (i.e. the point in feasible area determined by the constraint). Because the constraint is very complex, the distribution of feasible region is quite irregular. Common methods are difficult to describe it, while statistic method can describe it with a probability distribution [4]. This is just what we need in the new solving scheme mentioned above.

Feasible regional distribution determined by the constraint is irregular. Statistic method is used to give a rough probability description of it. First step is to select N1 number of feasible points X in the searching region randomly by core calculations; the following step is to make statistics to the distributions of L number of characteristic functions values with respect to these selected points and then to deduce L number of 1-D distribution curves. These L

numbers of distribution curves represent the probabilistic distributions of the feasible points X. The details will be explained in section 2.3.

Fig1 is the flowchart of the PAM algorithm.

2.3 Establishment of the probability distribution curve P_{valid} —X

This step is to give each solution X a probability of being a feasible point. To identify a solution (ε^k, n^k, F_i^k) is feasible or not, we need only identify the feasibility of its corresponding BOC (ε_i, Bu_i, F_i) in each cycle. If a approximation is adopted, we need only identify the feasibility of ($k_{\infty,i}$, F_i) in each cycle.

The distribution curve P_{valid}—X illustrates the probability that ($k_{\infty,i}$, F_i) is an feasible point. To obtain such a probabilistic distribution curve, two works are needed. One is to obtain N1 number of feasible points (statistic samples). This work can be completed by core calculations. The other work is to determine the statistic items, i.e., L number of characteristic functions. To give L number of statistic items, one needs to know what kind of factors are related to the event that ($k_{\infty,i}$, F_i) is a feasible point. In fact, when the parameters of the core structures and materials are specified, then the distribution of the feasible points ($k_{\infty,i}$, F_i) has been fixed. For a cylindrical reactor core with inner and outer zones, the actual relative power of the fuel assemblies always distributes in certain shape. For instance, the fuel assemblies with relative power level ranging 0.7~0.8 possess 5% of the entire core, the fuel assemblies with relative power value 0.8~0.9 occupy 7% and so on. Thus, the relevant statistical item is the distribution of the relative power F_i of the assemblies in the reactor core. Because F_i is also related to BOC k_∞ of the fuel assemblies, therefore we can write the following correlation:

$$F_i^* = F_i / k_{\infty,i}$$

Then, the distribution of F_i^* can be selected as the statistic item. That is to say, statistic items $\{ G_i \}$ are

$$G_i = \text{the percentage of } F_i^* \text{ in the range i (e.g. 0.4~0.5)}, \quad i = 1, 2, ..., L-1$$

Another statistic item is related with the cycle length T, because the feasible points ($k_{\infty,i}$, F_i) must satisfy the appointed cycle length T of this cycle. So another statistic item is

$$G_L = \sum_{i=1}^{M} \rho_i F_i \quad , \qquad \rho_i \text{ is assembly reactivity at EOC.}$$

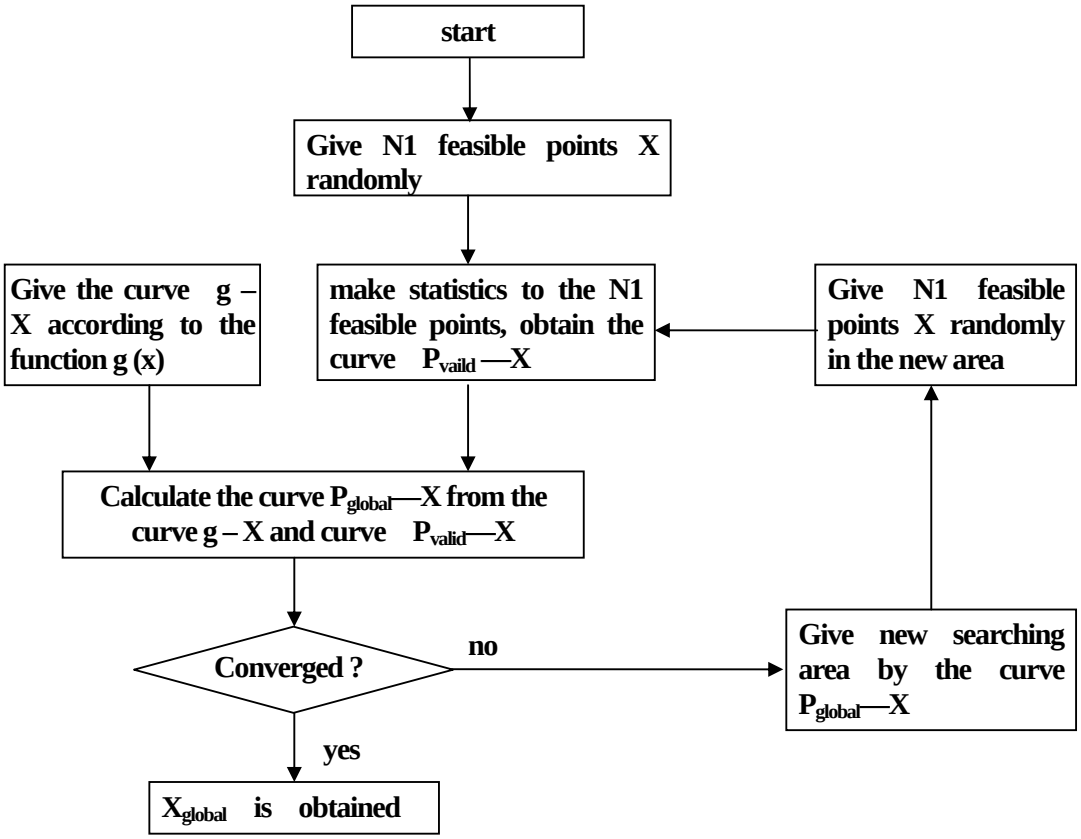

Fig.1 Flowchart of Probability Approaching Method

2.4. Establishing P_{global} —X curve from g —X and P_{valid} —X curves

The g —X curve in figure 2.a indicates the function curve of objective function g(X), and the P_{valid} —X curve in figure 2.b represents the probabilistic distribution curve of the feasible points, which is obtained from actual reactor core calculations and statistics in section *2.3*.

The ordinate g in figure 2.a is the value of the objective function and the ordinate P_{valid} in figure 2.b is the probability of the fact that X is a feasible point. The abscissa in figure 1 is the variable X to be optimized in the specified range and one point in X-coordinate stands for the values of a group of variables (ε^k , n^k , F_i^k), where i = 1, ..., M, k = 1, ..., N.

For a solution X1, if it is the global optimal point, two conditions must be satisfied:

(1) X1 is a feasible point;

(2) The solutions whose objective value is bigger than g (X1) are all unfeasible points.

Assume that these two events are independent, then the probability density function of the fact that X1 point is the global optimal point can be written as:

$$P_{global}(X1) = P1\times(1-P2).\ (1-P3).\ (1-P4).\ \ldots\ .\ (1-Pn).$$
$$= P1\times \lim_{n\to\infty} (1 - a / n)^n = P1\times e^{-a}$$

Here P1 is P_{valid} (X1), and

$$a\ (X1) = \int_{g(X)>g(X1)} P_{valid}(X)dX$$

.

where *a*(X1*)* is the value of the area of the shaded zone in fig 2.

Therefore, the probability distribution curve for the optimized points locating within the specified variable zone can be obtained, shown as figure 3.

The procedure to derive the value *a* is also easy to establish in coding the computer program by means of specifying sample points. For example, one can stochastically specify 1000 sample points X firstly in accordance with the distribution curve P_{valid}—X, then evaluate the values of g of their corresponding objective functions in curve g —X, in which the number n (X1) whose function value is greater than g (X1) is proportional to the shaded area a (X1). Therefore, a (X1) can be derived by multiplying n(X1) with a proportional coefficient being independent of X.

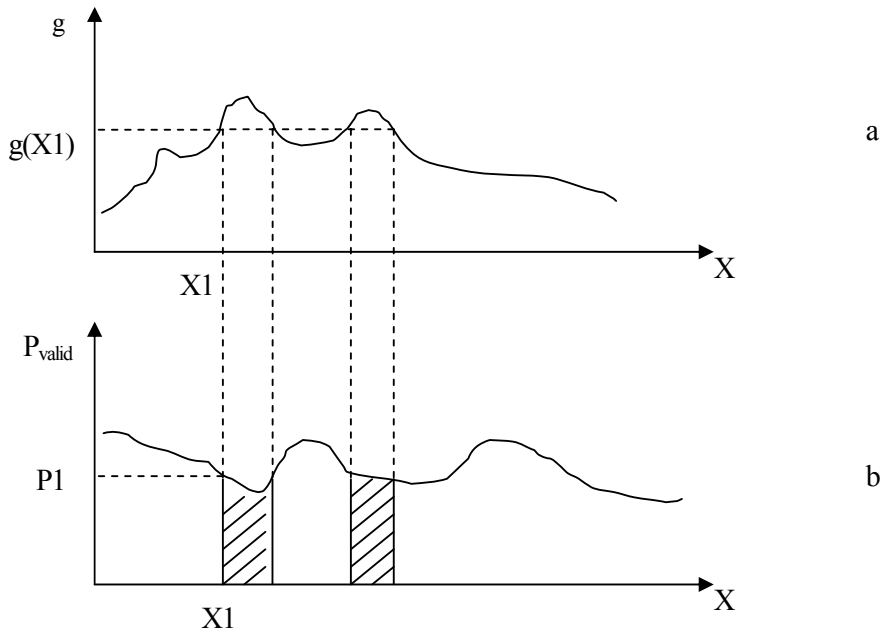

Fig 2 g —X curve and P_{valid} —X curve

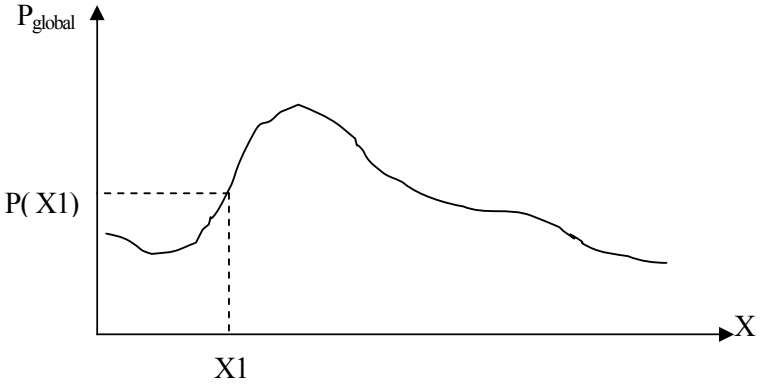

Fig 3 P_{global} —X curve

2.5 Realization of the PAM algorithm

When this scheme is realized on code, considering of the complexity of programming, some simplifications can be used. For example, when P_{global} —X curve is obtained from P_{valid} —X curve and g —X curve, the mathematical relation among them needn't to be satisfied strictly, it may be satisfied only on tendency.

In the multi-cycle code, the k_∞ information of individual assemblies is given previously. The enrichment is from 3.1% to 4.0%, and the burn-up is from 0 to 70000Mw.d/t. Then for each assembly, we can calculate its k_∞ from its ε and Bu directly.

In this algorithm, the new F_i^k is given randomly under the probability curve P_{valid}—X. It is related with fuel assemblies by the formula $F_i^* = F_i / k_{\infty, i}$.

A recommended parameter selection in the algorithm is N1=500 and L=20. Because the value of N1 must be large enough to represent the statistic property of problem, and the L number of characteristic functions can describe the feasible area distribution approximately.

3. Results on simplified models

The code utilizing the new solving scheme has been completed. A real multi-cycle model is used to test this code. This is the core of *CRUSAZ* Nuclear Plant in France. This core has totally 157 assemblies, and about 1/4 discharged in each EOC. The code used for core calculation is the code of Node Green's Function Method with Neumann boundary condition(NGFM-N) [5],which considering discontinue factor and pin power reconstruction. The multi-cycle optimization problem is as follows.

Parameters:

Cycle number: 4
Fresh fuel can be used: 3.2 % , 3.3 % , ... , 4.0 %
Cycle length T^k: 266 262 266 266 (fixed)
Stretchout: 20 20 20 20 EFPDs

Objective:

Minimize the total cost of fresh assemblies, that is to say,
Minimize $g = \text{cost}(\varepsilon^k, n^k)$
Because the prices of fresh assemblies are not given, the cost used here is simplified as the total mass of U^{235}.

Constraints:

1. $F_{xy} < F_{lim}$ (here F_{lim} =1.43) ;
2. Appointed cycle lengths T^k should be arrived.
3. Control rods position limitation.

Tab 1 is the result of traditional arrangement plus mono-cycle optimization. It uses 40 fresh assemblies in each cycle. The mono-cycle optimization objective in each cycle is to maximize the discharged burn-up and arrive the appointed cycle length.

Tab 2 is the result of multi-cycle optimization with probability approaching method. The total fresh fuel used in tab 2 is 2.67% less than that of tab 1, and the discharged burn-up of cycle 1, 2, and 3 (assemblies with 3.7% enrichment) is 2.74 % higher than that of tab 1.

The result in tab 2 has not considered the influence for cycle 5. In fact, at the EOC of cycle 4, we should leave enough fuel for the next cycle. If this influence is considered, another limitation is added. The limitation used here is that in cycle 5, 260 days cycle length must be

arrived with the 3.7 %×40 fresh assemblies.

Tab 3 shows the multi-cycle optimized result of problem with a limitation. The total fresh fuel used in tab 3 is 2.0 % less than that of tab 1, and the discharged burn-up of cycle 1, 2, and 3 (assemblies with 3.7% enrichment) is 2.86 % higher than that of tab 1.

Table 1 Traditional arrange (40 fresh assemblies) + mono-cycle optimized

Cycle	Fresh fuel used	Cycle length can obtained	Real operated length	Discharged Burn-up(GWd.t $^{-1}$)
Cycle 1	3.6 % × 40	266	266 + 20	44881
Cycle 2	3.8 % × 40	263	262 + 20	44168
Cycle 3	3.8 % × 40	266	266 + 20	43910
Cycle 4	3.8 % × 40	266	266 + 20	44752

Table 2 Multi-cycle optimized

Cycle	Fresh fuel used	Cycle length can obtained	Real operated length	Discharged Burn-up(GWd.t $^{-1}$)
Cycle 1	4.0 % × 36	266	266 + 20	45415
Cycle 2	3.9 % × 40	267	262 + 20	45710
Cycle 3	3.9 % × 40	273	266 + 20	45477
Cycle 4	3.2 % × 40	266	266 + 20	44975

Table 3 Multi-cycle optimized (limitation of cycle 5 is added)

Cycle	Fresh fuel used	Cycle length can obtained	Real operated length	Discharged Burn-up(GWd.t $^{-1}$)
Cycle 1	4.0 % × 36	266	266 + 20	45415
Cycle 2	3.9 % × 40	267	262 + 20	46166
Cycle 3	4.0 % × 36	270	266 + 20	45181
Cycle 4	3.6 % × 40	266	266 + 20	47058

4. Conclusions

Multi-cycle reloading optimization problem is very hard to solve. In this new solving scheme, the multi-cycle problem is de-coupled into a number of relatively independent mono-cycle issues, then the non-linear programming problem with complex constraints is

solved by a advanced new algorithm based on probability theory — probability approaching method. The result on real core model shows well effect of this new solving scheme. What's more, the PAM algorithm can also be applied on other non-linear programming problems which have very complex constraints.

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