PHYSOR 2004 -The Physics of Fuel Cycles and Advanced Nuclear Systems: Global Developments
Chicago, Illinois, April 25-29, 2004, on CD-ROM, American Nuclear Society, Lagrange Park, IL. (2004)

A NEW DE-HOMOGENISATION METHOD FOR LOCAL POWER RECONSTRUCTION

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The transport calculation of a MTR core takes too much computation time for design studies and approximations have to be introduced : diffusion equation is used and the assemblies are homogenized. Nevertheless, safety studies require the knowledge of the local power of the fuel plates. The standard reconstruction method assumes the detailed flux shape in an assembly as a superposition of a macroscopic flux in the core and a microscopic one at the level of the assembly. Because of the strong flux gradients, the MTR core-reflector interface is obviously the most difficult point for the reconstruction method. A constant flux hypothesis in the assembly leads there to discrepancies which may rise by up to 22% on the Horowitz reactor core.

In this paper, we develop a method to improve the accuracy of the local power reconstruction, required for the design studies of the Horowitz reactor.

Let f be an unknown function, which represents the fine pin power distribution on the core. We suppose a parameterized function p to be the best approximation of f. For that, we introduce a semi-norm and the minimization is made classically with the least square method on each assembly. The solution is a piecewise polynomial and positive function. In order to close the system for cubic polynomials, needed at core-reflector interfaces, constraints based on physical arguments are used.

The discrepancy against Monte-Carlo simulation reaches so only 6.5% on the fuel plate power peak.

KEYWORDS: local, power, reconstruction, MTR, JHR

1 Introduction

Special fuel assembly geometry, small dimensions and a heterogeneous composition characterize the future European Material Testing Reactor, called Jules Horowitz Reactor. These specifications result in strong flux gradients in the core-reflector interface assemblies, and necessitate a simulation, which can handle these increased gradients. To ensure safety, the determination of the detailed local power distribution in the core and the maximum local power peaking factor must be as accurate as possible with a reasonable execution time. That's why a design route called HORUS3D is being developed [3]. The neutronics route is based on a homogeneous diffusion model [4]. So, a de-homogenization procedure must be available in order to recover a fine heterogeneous power distribution. We will see that the standard de-homogenization procedure, based on a classical factorization principle, is unable to recover an accurate local power distribution, due to the strong flux gradients near the reflectors. So a new method is developed, based on a macroscopic flux reconstruction.

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2 General background

2.1 JHR description

The Julius Horowitz Reactor is the future European Material Testing Reactor [1,2]. The design studies are split in different phases. Preliminary design studies allowed to define of the main options of the core and its elements, and to characterize a reference configuration. The Detailed Design Studies will optimize the JHR performances by testing modifications from the reference configuration.

Its assembly would be composed of 18 cylindrical fuel plates maintained by 3 stiffeners (Fig. 1). The central cavity can host either aluminium filler, a hafnium control rod or an irradiation test device.

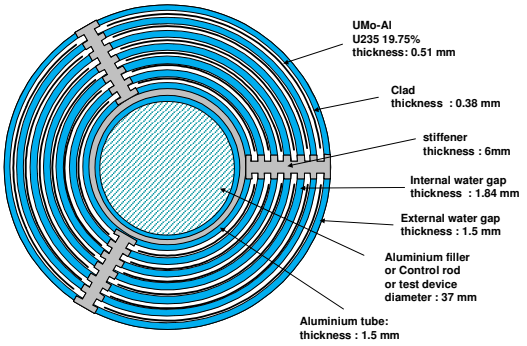


Figure 1 : **Cross-section of the JHR assembly**

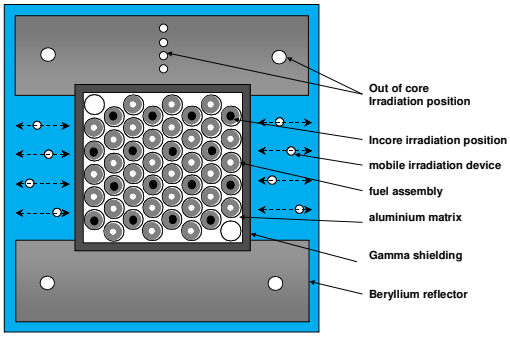


Figure 2 : Cross-section of the JHR core

The JHR will be cooled and moderated by light water. The preliminary core consists of 46 assemblies (Fig. 2), arranged in a triangular lattice inside a rectangular aluminum matrix. It is boarded on two sides by a beryllium (Be) reflector. The other two sides are left free (H_2O) in order to introduce mobile irradiation devices.

An adapted and consistent neutronics/thermal-hydraulics code package, named HORUS3D (HOrowitz Reactor simulation Unified System) is developed in order to fulfill the needs of the Detailed Design Studies [3].

2.2 Computations

The JHR assembly neutronics route is performed with the APOLLO2 [8] code and its 172 groups library. The complex geometry (Figure 1) necessitates the use of the exact 2D collision probability method. The optimized slowing-down model with a Doppler cross-section broadening for the first resonance [4,8], developed for PWR UOX fuels, can be used. Hence, the fuel plates power distribution on the assembly (in infinite media). The cross sections are homogenized on the assembly and collapsed on an adapted 6-energy group mesh.

The 3D core neutronics route uses the new finite elements developments in the CRONOS2 code [5], allowing splitting the hexagonal cells in 6 triangles. This leads to 145 000 prisms and the tedious task of constructing the core geometry is assumed by the procedural possibilities of the CRONOS2 code.

The triangular spatial mesh allows the correct representation of the fuel part. The reflector parts would necessitate a rectangular mesh. Therefore, an adapted flux homogenization is made in APOLLO2 using the exact 2D Pij solver TDT on a quarter section of the core. 22 homogenized media are so handled in the radial CRONOS2 geometry.

Here is the radial power shape (axially integrated). One can observe the strong gradient at the core interface due to the beryllium reflector. Nevertheless, the assembly power results are in very good agreement against Monte-Carlo simulation even at the reflector interface (discrepancies don’t exceed 1.5 %).

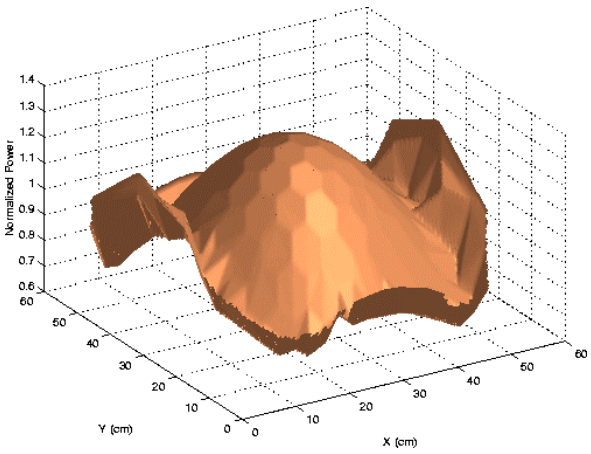


Figure 3 : Integrated radial power shape

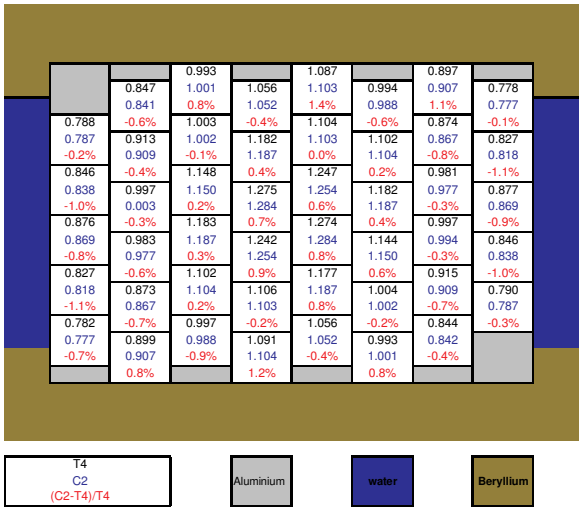


Figure 4 : Assembly power map discrepancies with TRIPOLI4 results

This Monte-Carlo calculation has been run by TRIPOLI4 code at BOL for enough histories to produce stable, accurate results even in terms of local power (0.14% uncertainties). Therefore, it can constitute a reference calculation to assess the developed deterministic schemes at BOL.

At this point, this scheme allows determining the power in each prism. A methodology has to be developed to reach the power of the fuel plates.

The Standard Method (SM) consists in multiplying directly the assembly power density, obtained previously in an infinite media calculation, by the core power density on each prism. A special numbering is used to identify each element of the core as in the figure below.

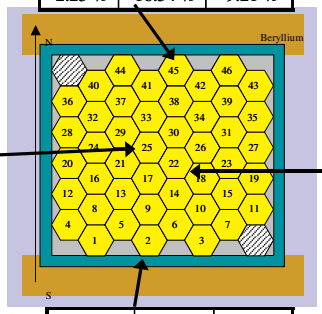
$$\frac{(\text{SM} - \text{TRIPOLI4})}{\text{TRIPOLI4}}$$

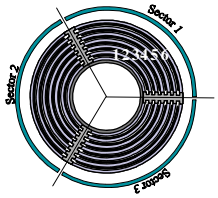
sect1	sect2	sect3
0.53 %	**22.01 %**	11.29 %
1.69 %	19.54 %	11.56 %
0.98 %	14.17 %	8.18 %
-0.23 %	6.18 %	3.96 %
-1.71 %	-4.29 %	-1.39 %
-2.25 %	-16.54 %	-9.21 %

sect1	sect2	sect3
0.36 %	0.01 %	-1.25 %
1.04 %	0.34 %	0.47 %
0.30 %	1.01 %	0.69 %
0.18 %	0.79 %	0.81 %
0.18 %	0.88 %	1.27 %
0.97 %	1.63 %	**2.20 %**

sect1	sect2	sect3
-0.70 %	-0.10 %	-0.40 %
0.20 %	2.43 %	1.47 %
0.87 %	1.89 %	0.99 %
0.66 %	2.11 %	0.80 %
1.00 %	2.39 %	0.63 %
2.30 %	**2.96 %**	0.80 %

sect1	sect2	sect3
21.98 %	1.01 %	9.40 %
19.48 %	1.65 %	9.30 %
14.64 %	0.79 %	7.41 %
6.03 %	-0.22 %	4.05 %
-4.32 %	-1.49 %	-1.74 %
-16.96 %	-3.19 %	-8.46 %





One can observe that the Standard Method is enough for the assemblies in the center of the core, where the flux is quite flat. At the reflector interface, discrepancies exceed 20% due to the strong gradient.

3 Description of the method

One has to develop a method using the core calculation without extra development in the code and allowing to determine the local power with an error about 10%.

We note :

C_i^j the *j*-th pin of the *i*-th triangle of an assembly *n* (*i = 1..6, j = 1..6)* ,

and Ω_i^j the *j*-th pin of the *i*-th sector of an assembly *n* (*i = 1..3, j = 1..6)* .

Thus, $\Omega_i^j = C_{2i-1}^j + C_{2i}^j$, (*i = 1..3, j = 1..6*).

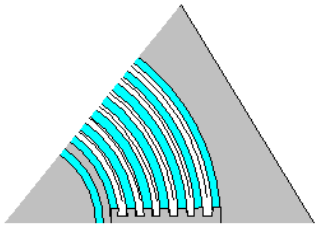


Figure 5 : 6×1/2 **pins** in a triangle

We wish to estimate an unknown function *f*, which is defined on a domain Ω (an assembly), and represents the fine power. Therefore, *f* would be a positive and continuous function of two variables *x* and *y*. We consider that power mainly comes from U235 fissions. Thus, the most interesting domains in an assembly are the fuel plates (Fig. 3), where fissions occur. Let's assume that the integration of *f* over a Ω_i^j gives the plate power. Following the factorization principle of the equivalence procedure which assess the "microscopic" power (at the pin level) to be a product of a global power (at the assembly level) by a correctly normalized local power (obtained by APOLLO2 calculations),

$$\iint_{\Omega_i^j} f \, dS \quad \approx \quad Ass(j) * \iint_{\Omega_i^j} \tilde{f} \, dS \qquad (1)$$

where the symbol *Ass(j), j=1..6* denotes local power obtained by the assembly computation and $\tilde{f}$ an undetermined function defined on the assembly which represents "macroscopic" power over the assembly. $\tilde{f}$ must be positive and continuous. The only unknown is the shape of $\tilde{f}$ on each assembly of the core. Therefore, an approximation of $\tilde{f}$ would be the solution of the problem.

Let a parameterized function P_k (with *k* unknown parameters) be the best approximation of $\tilde{f}$. The core computation provides constant homogeneous power on each of the six triangles domains of an assembly. The constant power on each triangle must be equal to the integration of P_k parameterized function over each fuel pin of the considered triangle (Fig. 3)

$$\sum_{j=1}^{6} \iint_{C_i^j} P_k \, dS \approx \quad Core(i) \qquad (2)$$

The symbol *Core(i), i=1..6* denotes the "global" power in each of the six triangles of the assembly (obtained by CRONOS2 computation). Thus, system (1) is a set of 6 equations. If *P* is the solution of (1) for an assembly n, the *j-th* plate *i-th* sector power value of this assembly is given by :

$$\left(Ass(j) \right) * \left(\iint_{\Omega_i^j} P \, dS \right) * \left(\frac{\sum_{k=1}^{6} \iint_{\Omega_i^k} dS}{2 * \iint_{\Omega_i^j} dS} \right) \qquad (3)$$

In expression (2), the third factor is a plate volume normalization. The two other factors refer to the superposition of global and local power.

The approximation of $\tilde{f}$ by P_k must be the best in the least square sense ($min \left\| P_k - \tilde{f} \right\|$).

The semi-norm invoked is defined as follows : $\left\| \bullet \right\| = \left(\sum_{i=1}^{6} \left(\sum_{j=1}^{6} \iint_{C_i^j} \bullet \, dS \right)^2 \right)^{\frac{1}{2}}$.

4 POLYNOMIAL APPROXIMATION

Several parameterized function can be used. A polynomial ratio approximation for example (as $Q=\frac{ax+by+c}{ex+fy+g}$) will lead to a non-linear system. As the elaborated method is to be implemented in the CRONOS2 code, an approximation that leads to linear system is preferred. Thus, a set of simple functions such as polynomials can be used for the approximation. This leads to the linear system : $A^tA\xi=A^t\beta$ where *:*

$$A=\begin{pmatrix} \sum_{j=1}^{6}\iint_{\Omega_1^j}x^n dS & \sum_{j=1}^{6}\iint_{\Omega_1^j}y^n dS & \cdots & \sum_{j=1}^{6}\iint_{\Omega_1^j}dS \\ \sum_{j=1}^{6}\iint_{\Omega_2^j}x^n dS & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \sum_{j=1}^{6}\iint_{\Omega_6^j}x^n dS & \cdots & \cdots & \sum_{j=1}^{6}\iint_{\Omega_6^j}dS \end{pmatrix}$$ with 6 lines and $\frac{(k+1)(k+2)}{2}$ columns, and $\beta=\begin{pmatrix} Core(1) \\ \vdots \\ Core(6) \end{pmatrix}$

Example:

$P_1=ax+by+c$ leads to $A\xi=\beta$ (via Eq (1)) with :

$$A_{i1}=\sum_{k=1}^{6}\iint_{C_i^k}xdS, A_{i2}=\sum_{k=1}^{6}\iint_{C_i^k}ydS, A_{i3}=\sum_{k=1}^{6}\iint_{C_i^k}dS, i=1..6$$

$$\xi=\begin{pmatrix} a \\ b \\ c \end{pmatrix}, \beta=\begin{pmatrix} Core(1) \\ \vdots \\ Core(6) \end{pmatrix}$$

Matrix terms are integrands of monomials over assembly triangles pins. The analytical results of the integrands are written in the cylindrical coordinates (because of the pin's cylindrical geometry) with respect to local assembly Cartesian coordinates. As the core is periodic, the pin geometry does not vary with assembly position, and we can then invoke a translation invariance to note that the dependence of the solutions on the position variables will occur through the combination :

$$\begin{cases} X=x_0+x \\ Y=y_0+y \end{cases}$$

(x_0,y_0) denotes the center of the considered assembly ; X,Y and x, y are respectively global and local position variables in the core in the considered assembly (Fig. 2).

As long as the trial function P_k is not changed, the solutions from many different assemblies (different core power distribution) can be obtained using the same matrix A in Eq (1).

5 Optimization problem

A computational inversion of matrix A in Eq (1) is required to determine polynomial coefficients. The question of existence and uniqueness of the solution must be addressed. On the one hand, the system leads to 6 equations per assembly, corresponding to the power on each triangle. On the other hand, the number of the unknown coefficients depends of the trial function P_k.

In the case of $P_1=ax+by+c$, there are 6 equations for 3 unknowns. In general, there is no guarantee that these equations will have any solution at all. Thus, the estimated solution to this problem is formulated in terms of least squares. For a parabolic polynomial, we are in the case of we have 6 equations and 6 unknowns. For k≥3, one has an infinite of solutions and we have to add different constraints to resolve the system.

6 Basis function analysis

Since the neutron flux is flat in core center assemblies and has great gradients near the reflectors, a natural way to overcome these difficulties is to subdivide the core into sub-regions and to introduce a piecewise approximation from one assembly to another. Therefore, the power density function is allowed to be discontinuous at the assembly boundaries. A choice of trial functions must be made, and then evaluate in the different sub-regions, so that each domain problem can be solved independently. In this section, we compare specific choices of basis functions and discuss the properties of these choices that make them either succeed or fail.

Since core computation provides six values per assembly, there are only six equations to be satisfied. It forces us to use low order polynomials as often as possible, so as to be sure that the equations are closed and that there is no need to introduce new constraints.

In the center of the core, the relatively flat power is approximated by first order polynomials with a ±2 % target accuracy (in comparison with TRIPOLI4 reference calculations), which is satisfactory. At the core-reflector interface, first order polynomial approximation can not take into account the high flux gradients. These assemblies require high order polynomial approximations, which means that additional information must be supplied to achieve closure. We wish to estimate the accuracy and robustness of a high order trial function approximation, without introducing new constraints. We will refer to a known solution obtained by the TRIPOLI4 reference calculations, which provides 18 plate power values per assembly, what causes the system to be over determined. We solve this ideal system in the least square sense. Let P_k be a high order polynomial, the ideal system is constructed as follows :

$$Ass(j)*\left(\iint_{\Omega_i^j} P_k dS\right)*\left(\frac{\sum_{k=1}^{6}\iint_{\Omega_i^k} dS}{2*\iint_{\Omega_i^j} dS}\right)=Tripoli(i,j), j=1..3, i=1..6$$

Parabolic solution : $P_2 = ax^2 + by^2 + cxy + dx + ey + f$

This polynomial give good results on the fuel plate, but we can observe variations inside the assembly leading to negative values, physically not acceptable. Moreover, A is ill-conditioned with the core triangular element resolution, although there are 6 equations for 6 unknown coefficients in this case. This is due to symmetric properties in the hexagonal assembly. For this reason, some additional equations are required.

Cubic solution : $P_3 = ax^3 + by^3 + cx^2 y + dxy^2 + ex^2 + fy^2 + gxy + hx + iy + j$

Even if the pin power accuracy is correct, the scheme displays violent oscillations with large negative values for the P_3 approximation, which is physically not acceptable.

Truncated cubic solution : $P_3t = ax^3 + by^3 + cx^2 + dy^2 + ex + fy + g$

P_3t is a cubic polynomial with $x^n y^m$, $(n>0, m>0)$ terms truncated. It seems to be the most reasonable function since it reveals good pin power accuracy and a physically acceptable shape.

We have relied on the known TRIPOLI4 reference solution to evaluate the accuracy achieved by each choice of polynomial approximation in the different core domains. This analysis leads us to apply an adaptive P_1 or P_3t approximations.

7 P1+P3t Approximation

7.1 Self adjusting algorithm for P1/P3t scheme

In the case of strong flux gradients, the assembly is calculated by the P3t scheme; a relatively flat flux (in the middle of the core) is calculated with the P1 scheme. We have to consider whether an assembly is "worth" being calculated by P1 or P3t scheme. The algorithm developed exploits the differences between the six CRONOS2 macroscopic power values inside an assembly to detect the case of a strong flux gradient and its direction. In the case of a relatively flat flux over an assembly, the six triangles values are roughly the same, which means that the assembly is worth being calculated by P1 scheme. This defines a heuristic criterion to apply (self-adjust) P1 or P3t approximations in the whole core.

7.2 Additional equations for the P3t scheme

In the case of a strong flux gradient, the basis function analysis has shown that a P_3t polynomial would provide physically reasonable solutions, insofar as the system is perfectly closed. The P_3t approximation involves 6 equations for 7 unknowns. It is clearly of interest to include a few more equations in the system to achieve closure. Next, we give four constraints equations based on physical arguments :

- *"central" constraint* :

Local power involves (thermal) neutron flux, multiplied by the fission cross section (multiplied by energy provided by fission). Since the fuel is localized, we do not expect the neutron flux to display great variations in the middle of a standard assembly. We note that the P_3t polynomial solution displays an unexpected decrease in the center of the assembly. To overcome this undesired phenomenon and to prevent large possible errors, we assume that the integral of the central flux can be represented by an average of macroscopic power values in the assembly, as follows :

$$\int_{\theta=0}^{2\pi}\int_{r=0}^{R} P_3 t \; rdrd\theta \quad = \quad \frac{\sum_{i=1}^{6} Core(i)}{6} * \pi R^2 \quad , with\ R = 1$$

- *"great derivate" constraint* :

At the core-reflector interface, there is a strong flux growth, which is generally underestimated. This constraint aims at increasing gradients of the polynomial solution in the reflector direction.

$$\left[\frac{d}{dx} + \frac{d}{dy}\right](P_3 t)\Big|_{(x,y)=(\alpha,\beta)} \quad = \quad \pm\omega$$

If the gradient is in the ± x-axis direction :
$\beta = 0$, and $\alpha = \pm$ constant (> assembly dimensions)
If the gradient is in the ± y-axis direction :
$\alpha = 0$, and $\beta = \pm$ constant (> assembly dimensions)
ω is a great value which depends on the estimation of the gradient.

- *"P_3t ' =0 in the center of an assembly" constraint* :

This constraint is introduced so as to prevent the solution from oscillating in the center of the assembly.

$$\left[\frac{d}{dx} + \frac{d}{dy}\right](P_3 t)\Big|_{(x,y)=(0,0)} \quad = \quad 0$$

- *"conservative" constraint* :

The initial system plus new constraints equations are solved in the least square sense. We expect the scheme to be as accurate as possible, not only at a microscopic level, but also at a macroscopic one : the summation of the reconstructed pin power over an assembly is not supposed to differ from the exact macroscopic power values over the assembly (provided by core code computations). That's why we introduce this constraint :

$$\sum_{i=1}^{6}\sum_{j=1}^{6}\iint_{C_i^j} P_3 t \, dS \quad = \quad \sum_{i=1}^{6} Core(i)$$

Numerical results have illustrated the validity of this model in the case of high flux gradients near the reflector. The question naturally arises as to whether the accuracy and robustness of the developed method extends to more complex problems such as the presence of an absorber rod, which causes an important local decrease of the flux.

8 Results

Reference Monte Carlo calculations were obtained with the TRIPOLI4 code without any simplifications; all geometry details were represented and punctual cross sections were used. 50 Millions of particles were generated.

The fuel plate power distribution is particularly well determined [Figure 6]. Discrepancies do not exceed 16% and 98% of them are within an interval of ±10%. The most important errors are located in the core's corner assemblies which are bordered by the beryllium and the pool water; but the assembly normalized power is here lower than 1. The plate power peak, located at the beryllium reflector interface, is well estimated. The discrepancy does not exceed 6.5%. Moreover, discrepancies are negligible (~1%) on the core center, where the assembly power reaches the maximum.

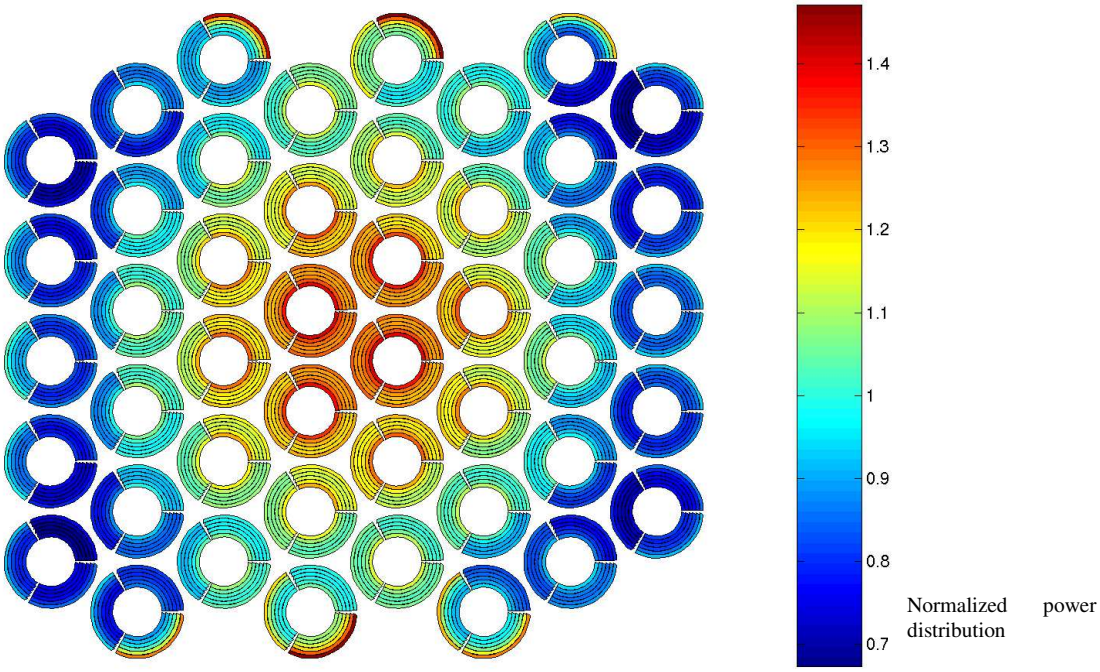


Figure 6 : HORUS3D local power distribution – STD configuration

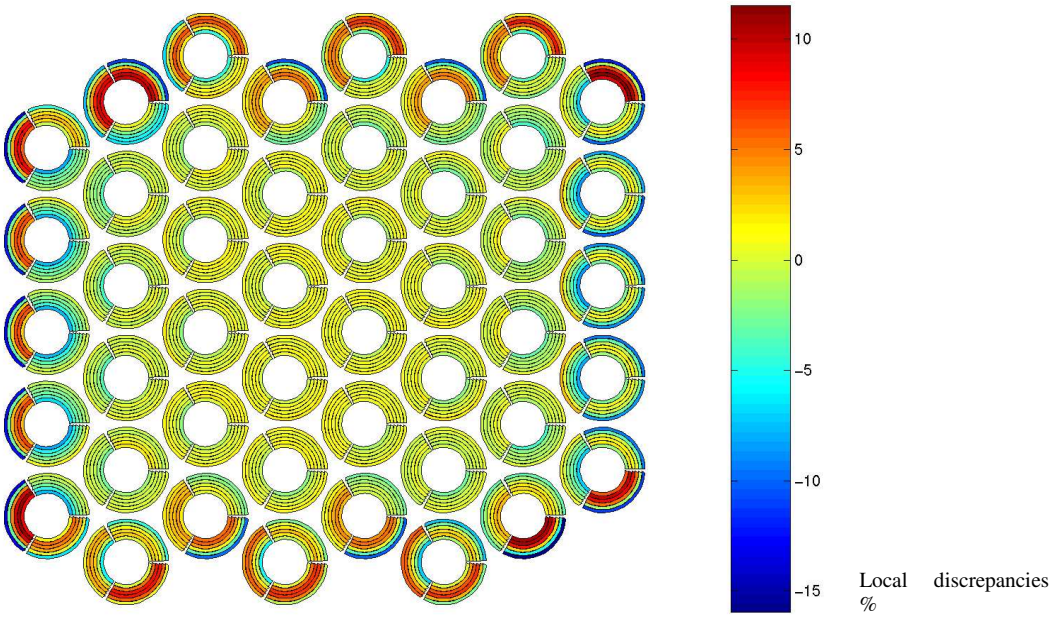


Figure 7 : Local power discrepancies with TRIPOLI4 results – STD configuration

The method has been recently improved. The power distribution can now be obtained on azimuthal sub-sectors of the assembly. This functionality is particularly important due to the strong gradients in this MTR core.

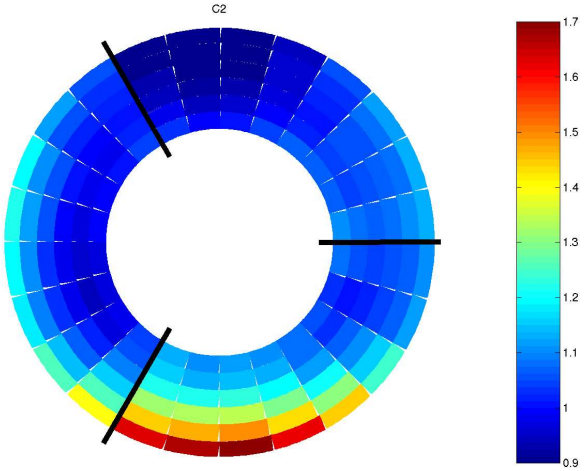


Figure 8 : Azimuthal power distribution in the hottest assembly

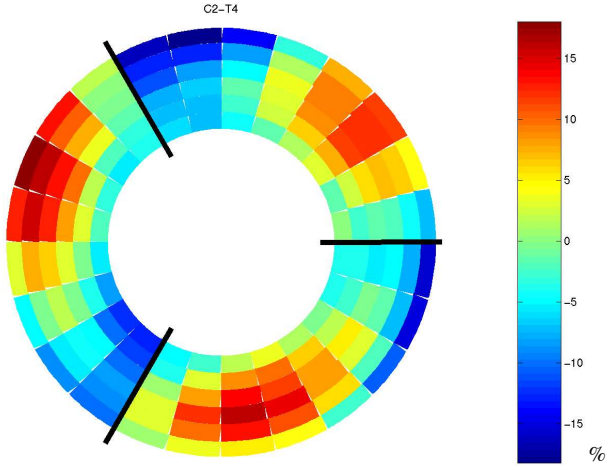


Figure 9 : Azimuthal power discrepancies with TRIPOLI4 results – hottest assembly

9 Conclusions

We have developed a self-adjusting piecewise P_1/P_3t polynomial approximation of the "macroscopic" flux over each assembly of the JHR core. Following the conventional equivalence procedure based on the factorization principle, power is reconstructed pin-by-pin as a product of this "macroscopic" flux by a correctly normalized local flux

Whereas conventional de-homogenization methods will show a large rise in relative errors near the beryllium reflector, the discrepancy of our method against Monte-Carlo simulation reaches only 6.5% on the fuel plate power peak. Moreover, the model is simple enough to be suited for design studies, since it does require only little computation time.

We will continue on the assessment of our method considering time evolution and the various experimental devices which may be introduced.

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