

**Activities of working party on
“Subcritical Core of Accelerator-Driven System” under
the Research Committee on Reactor Physics of AESJ and JAERI**

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The Research Committee on Reactor Physics under the Atomic Energy Society of Japan and the Japan Atomic Energy Research Institute organized the working party (ADS-WP) on “Subcritical Core of Accelerator-Driven System”. The ADS-WP investigated reactor physics of subcriticality from the viewpoint of the accelerator driven system (ADS) since subcriticality has been almost studied from the viewpoint of critical safety. The working party was set in July 2001 and it worked for two years. The activities of the ADS-WP are (Work-I) theory of subcriticality, (Work-II) benchmark of subcritical core, (Work-III) setting of subcriticality level of ADS and (Work-IV) monitoring of subcriticality. These activities clarified about the important issues related to the subcriticality or the subcritical core from the wide ranges of theory, analysis, calculation, design and monitoring for ADS. The activities were already summarized and the report will be published in March 2004.

KEYWORDS: *Working party, Accelerator Driven System, Subcritical core, Theory, Critical Experiment, Benchmark calculation, Subcriticality level, Monitoring*

1. Introduction

The Research Committee on Reactor Physics under the Atomic Energy Society of Japan (AESJ) and the Japan Atomic Energy Research Institute (JAERI) was set Working Parties (WPs) to review and research special issues related to reactor physics of Accelerator Driven System (ADS).

The first ADS-WP was set in July 1999 and researched for two years on the following issues;

- Neutron transport calculations in high energy range,
- Static and kinetic (safety-related) characteristics of subcritical system,
- System design including ADS concepts and technology developments required.

These activities of the first ADS-WP have been summarized and reported in JAERI-Review-2001-047¹⁾.

After the first ADS-WP, the second ADS-WP was organized in July 2001. The second ADS-WP investigated about “subcriticality” from the viewpoint of ADS though subcriticality has been almost studied from the viewpoint of critical safety. The issues worked by the second ADS-WP were as follows;

Work-I Theory of subcriticality

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Subcriticality is not fully studied in reactor physics at this stage since the present reactor physics is theoretically constructed assuming "criticality". In the ADS-WP, subcriticality was investigated in theory as the Work-1.

Work-II Benchmark of subcritical core

Although the accuracy of present calculation is high enough for a critical core, the accuracy is not sure for a subcritical core. The ADS-WP worked on the problem about the calculation for a subcritical core as the Work-II. As the Work-II, first, the ADS-WP examined subcritical core experiments by four critical assemblies in Japan. From those examinations, three benchmark calculation problems were made based on the experiments of the three critical assemblies among four. The members of the ADS-WP calculated the benchmark problems using their original calculation tools and libraries. The calculated results were compared and summarized.

Work-III Setting of subcriticality level of ADS

Subcriticality level of ADS is very important in the design of ADS. However, a certain concept does not exist for setting subcritical level of ADS and some typical levels such as 0.97 or 0.95 have been employed. The ADS-WP investigated other studies concerning the subcriticality level of ADS, and organized the concept for setting subcritical level of ADS.

Work-IV Monitoring of subcriticality

To operate ADS safely, subcriticality must be monitored during the operation of ADS. There are many methods to measure criticality or subcriticality. It is significant to examine the possibility of those methods as the monitoring method of subcriticality during operation. The ADS-WP worked about the monitoring of subcriticality.

2. Theory of subcriticality (Work-I)

2.1 Theory of subcriticality²⁾

For a critical system, even if the system is under critical, effective multiplication factor, k_{eff} , is appropriate for expressing subcriticality since the critical system is not far from critical state and the k_{eff} is close to unity. The k_{eff} is obtained by solving the following equation without external sources.

$$L\phi^e(r, E) = \frac{1}{k_{eff}} P\phi^e(r, E) \quad (1)$$

L , P and $\phi^e(r, E)$ are the removal operator, the production operator and the neutron flux for a critical state, respectively.

Total of fission neutrons, S , in a subcritical state is important because it closely relates to the total power of the ADS. S is expressed as Eq. (2) if using k_{eff} .

$$S = \frac{k_{eff}}{1 - k_{eff}} Q_{eff} \quad (2)$$

$$\text{where } S = \iint P\phi(r, E) drdE, \quad Q_{eff} = \frac{\iint \phi^\dagger q(r, E) drdE}{\iint \phi^\dagger P\phi(r, E) drdE} S$$

The external neutron source is $q(r, E)$. $\phi(r, E)$ and $\phi^\dagger(r, E)$ are the solution of

Eq.(3).

$$L\phi(r, E) = P\phi(r, E) + q(r, E), \quad L^\dagger \phi^\dagger(r, E) = \frac{1}{k_{eff}} P^\dagger \phi^\dagger(r, E) \quad (3)$$

Dagger on the right shoulder of operators means the adjoint operators. As shown in Eq. (2), instead of total of external source ($Q = \iint q \, drdE$), effective neutron source must be introduced to rigorously express the total source.

For the subcritical system with the external source, multiplication factors by Kobayashi and Nishihara can exactly and simply express the situation of neutron multiplication. The multiplications are as follows;

$$\text{Fission neutron multiplication rate: } k_f = \iint GP\phi \, drdE / \iint P\phi \, drdE \quad (4)$$

$$\text{Source neutron multiplication rate: } k_q = \iint Gq \, drdE / \iint q \, drdE \quad (5)$$

$$\text{Subcritical multiplication rate: } k_{sub} = (k_f S + k_q Q) / (S + Q) \quad (6)$$

In the three definitions, the importance function, $G(r, E)$, is defined by the following equation;

$$L^\dagger G(r, E) = \nu \Sigma_f(r, E) \quad (7)$$

k_f is the quantity which expresses the multiplication rate of the neutrons at the next generation by the fission neutrons, k_q is that which expresses the multiplication rate of the neutrons at the next generation by the source neutrons. k_{sub} , which is the average of k_f and k_q , corresponds to the multiplication factor of a subcritical system.

By using those, S can express as Eq. (8).

$$S = k_f S + k_q Q, \quad S = \frac{k_q}{1 - k_f} Q, \quad Q = \frac{k_{sub}}{1 - k_{sub}} Q \quad (8)$$

As the last of Eq. (8), it is shown that S is simply expressed by Q and k_{sub} .

Using the importance function $G(r, E)$, dynamic one-point equation, Eqs. (9) and (10), is also derived.

$$l \frac{\partial}{\partial t} S = (-1 + k_p(1 - \bar{\beta}))S + \sum_i \lambda_i k_{d,i} C_i + k_q Q \quad (9)$$

$$\frac{\partial}{\partial t} C_i = \bar{\beta}_i S - \lambda_i C_i \quad (10)$$

In Eq. (9), the prompt and delayed fission neutron multiplication rates are newly introduced instead of the fission neutron multiplication rate in the static state.

2.2 New terms related to subcriticality

As mentioned above, several new terms are required to express subcriticality exactly. Some of them are already known and are widely employed to discuss about subcriticality. However, the definite names are not established for those, for example, ϕ^* . The ADS-WP discussed about the names for them and the following names are recommended.

$$Q_{eff} = \left(\frac{1}{k_{eff}} - 1 \right) S: \quad \text{Effective neutron source,}$$

$$\phi^* = Q_{eff} / Q: \quad \text{Neutron source effectiveness,}$$

$$k_{sub} = \frac{S}{S + Q} : \quad \text{Subcritical multiplication rate,}$$

k_f : Fission neutron multiplication rate,

k_q : Source neutron multiplication rate

These names were proposed to the colleague in the research field of reactor physics in Japan.

3. Benchmark of subcritical core (Work-II)

3.1 Setting of calculation benchmark problems

The ADS-WP examined the subcritical core experiments by the four critical assemblies (CAs) in Japan. From those examinations, three benchmark calculation problems were set based on the experiments of the three CAs among four. The three CAs are as follows;

- Fast Critical Assembly (FCA), JAERI,
- Thermal Critical Assembly (TCA), JAERI
- Kyoto University Critical Assembly (KUCA), Kyoto University

In this paper, the experiments and the calculation benchmark of FCA are described hereinafter.

3.1.1 Experiment³⁾

In order to grasp the characteristics of ADS, some experiments were executed at FCA. In the experiments, a subcritical core simulated ADS was mocked up using a ²⁵²Cf neutron source and tungsten samples. The following cores with different test regions were made;

Case-A: a reference critical core without a test region,

Case-B: a subcritical core with a tungsten target region and a tungsten beam region,

Case-C: a subcritical core with a tungsten target region and a void beam region,

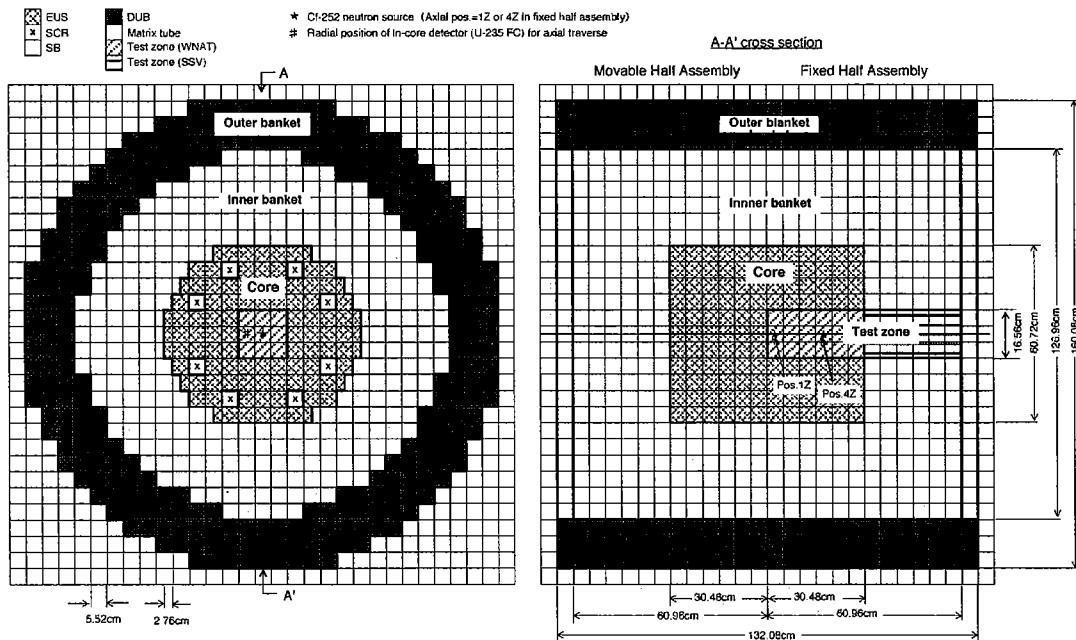


Fig.1 Subcritical core configuration of FCA experiment (Case-C)

Case-D: a subcritical core with a void target region and a void beam region of ADS.
 Case-C, which is shown in Fig.1, is a simulated core of a typical ADS. Two positions (1Z and 4Z) of the neutron source were set in the subcritical cores to clarify the dependence on neutron source position. For those cores, several parameters such as the multiplication factor and the reaction rate distribution of ^{235}U were measured.

3.1.2 Benchmark problem

Based on the experiments, benchmark problems are made. Two core models of two-dimensional R-Z and three-dimensional XYZ were adopted. Each material region of those models was homogenized. For those problems, the followings are required to calculate; (a) effective multiplication factor, (b) subcritical multiplication rate, (c) reactivity effect of test region, (c) axial distribution of ^{235}U reaction rate, (d) effective delayed neutron fraction, (e) neutron generation time.

3.2 Calculation result

More than ten members of the ADS-WP joined to analyze the problems by using their familiar codes and nuclear data libraries. The code calculated were MCNP4c, MVP, THREEDANT (3D-XYZ), CITATION (2D-RZ), DORT for the FCA problems. In this paper, the ^{235}U reaction rate and the subcritical multiplication rate are typically described.

3.2.1 Reaction rate of ^{235}U

For the reference critical core (Case-A), every calculation results were agreed well. As a typical result of the subcritical core, Fig.2 shows the axial distribution of the ^{235}U reaction rate. The disagreement among the calculations is observed at the neighbor of the neutron source. This disagreement is mostly due to the difference of the multiplication factor, which is shown in the figure, since the neutron multiplication of the subcritical core also results in the neutron distribution. This is a unique point of the physics of subcritical core.

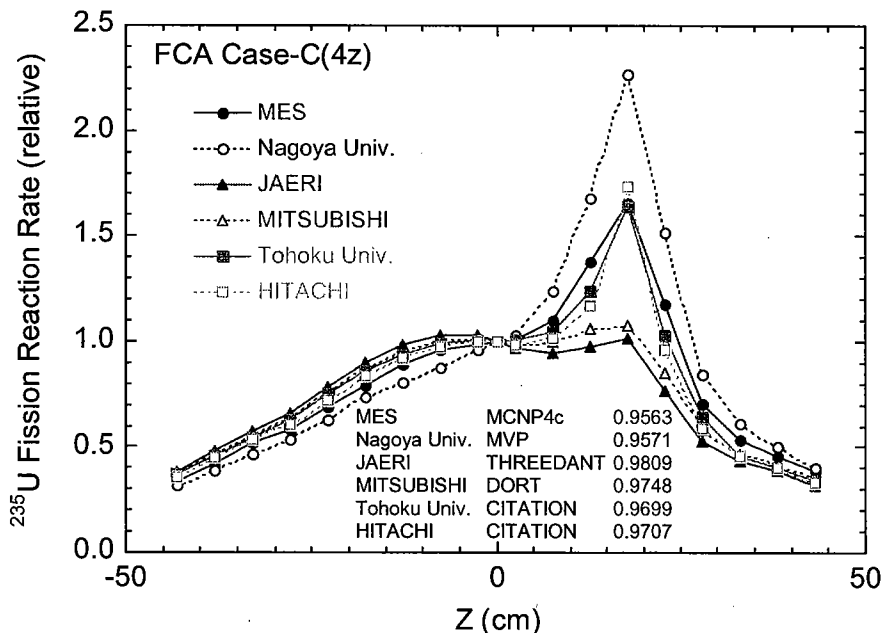


Fig.2 Analyzed reaction rate of ^{235}U for FCA experiment (Case-C)

Table 1 Subcritical multiplication rate k_{sub} for two neutron source positions (1z and 4z)

Neutron source position; 1Z (core center)					
	Code	Library	Case-B	Case-C	Case-D
MES	MCNP4c	ENDF/B-Vi	0.9612	0.9615	0.9722
Nagoya Univ.	MVP	JENDL-3.3	0.922	0.936	0.9394
JAERI	THREEDANT	JENDL-3.2	0.9860	0.9840	0.9948
MITSUBISHI	DORT	JENDL-3.2	0.9814	0.9810	0.9904
Tohoku Univ.	CITATION	JENDL-3.3	0.9889	0.9886	0.9933

Neutron source position; 4Z (near core boundary)					
	Code	Library	Case-B	Case-C	Case-D
MES	MCNP4c	ENDF/B-Vi	0.9405	0.9399	0.9646
Nagoya Univ.	MVP	JENDL-3.3	0.938	0.932	0.937
JAERI	THREEDANT	JENDL-3.2	0.9741	0.9779	0.9934
MITSUBISHI	DORT	JENDL-3.2	0.9709	0.9700	0.9874
Tohoku Univ.	CITATION	JENDL-3.3	0.9807	0.9800	0.9913

3.2.2 Subcritical multiplication rate

The subcritical multiplication rate k_{sub} is summarized in Table 1. Comparing the subcritical multiplication rates between 1Z and 4Z, the subcritical multiplication rate is different for the different neutron source position. This is also a unique point of the physics of subcritical core.

Comparing the values by THREEDANT, DORT and CITATION are rather agreed, however, those are largely different from the values of MCNP4c and MVP. These differences are not well understood yet. But, it is confirmed that the large disagreement among the various calculations exist and the accuracy for a subcritical core is not enough at this stage. The disagreements among the calculations are shown in other cases and for other benchmark problems such as TCA benchmark. It is clearly confirmed that more efforts are required for improving accuracy for a subcritical core.

4. Setting of subcriticality level of ADS (Work-III)

4.1 Investigation of the past study

For setting the subcriticality level of ADS, some studies are recently reported by OECD/NEA⁴⁾, Kim⁵⁾ and Iwasaki⁶⁾. The similar studies have been performed at the design of the fast reactor "Monjyu". In the ADS-WP, those studies are reviewed. From the comparison of those, it was found that those studies adopted similar ideas for setting the subcriticality level of ADS.

4.2 Concept of subcriticality level setting

The ADS-WP summarized those and the following concept is organized for setting the subcritical level of ADS at the beginning of cycle on the operation condition.

Concept of subcriticality level setting:

$$k = 1.0 - \Delta k_{Temp} - \Delta k_{Acc} - \Delta k_{Burn} - \Delta k_{Margin} - \Delta k_{Error} \quad (11)$$

where k : Subcriticality level at the beginning of cycle on the operation condition

Δk_{Temp} : Reactivity change between the hot (operation) and cold operations

Δk_{Acc} : Reactivity inserted by accidents

Δk_{Burn} : Reactivity of burnup during the cycle

Δk_{Margin} : Reactivity of margin for operation

Δk_{Error} : Reactivity of error by measuring and calculating neutron multiplication.

4.3 Application to ADS

The ADS-WP examined the concept to some typical ADS cores. As one of the results, the following remarks are concluded if the concept is applied to ADS.

- Δk_{Burn} is most important. The accuracy of Δk_{Burn} should be studied first.
- Δk_{Burn} is also important. It must be determined whether the accident with 100% void is considered since the void reactivity coefficient is positive in typical ADS cores.
- Δk_{Error} may be large for ADS core with MA.

5. Monitoring of subcriticality (Work-IV)

5.1 Measuring method of subcriticality

To operate ADS safely, subcriticality must be monitored during the operation of ADS. The monitoring method of subcriticality is indispensable for developing ADS. There are many methods to measure criticality or subcriticality. It is significant to examine the availability of those methods as the monitoring method of subcriticality. The method examined in the ADS-WP are the neutron multiplication method, the period method, the rod drop method, the compensate method, the pulsed neutron source method and the noise analysis method.

The neutron multiplication method directly measure the neutron counting rate of a detector. However, the method is not available for measuring the subcriticality since the neutron counting rate depends on not only subcriticality but also the number of source neutrons which largely varies with accelerator condition or operation.

There are many other methods that does not directly depend on the counting rate. The typical of those are the period method, the rod drop method, the compensate method, the pulsed neutron source method and the noise analysis method. The first three of them are not adequate to measure the subcriticality under operation since those requires the precise measurement at critical condition. The remains, the noise analysis method and the pulsed neutron source method, are examined in detail in the ADS-WP since those method does not require the measurement of the number of source neutrons.

5.2 Feasibility as monitoring of subcriticality

For examining the feasibility as the monitoring of subcriticality for ADS, it is important that there are two accelerator operation modes of direct beam and pulse beam. The ADS-WP examined considering the two modes.

The pulsed neutron source method measures the time variation of neutrons after the

induction of neutron burst. It is clear that the method is basically available to monitor the subcriticality for the pulse beam mode. Furthermore, it is concluded that the pulsed neutron source method is also effective if the area ratio method or the Simmons-King method are taken. The pulsed neutron source method can be employed to measure the subcriticality for both the two beam modes. However, for applying this method to a deep subcritical core, the effect of the higher-order space distribution must be corrected. For this point, Garelis and Russell⁷⁾ and Gozani⁸⁾ have proposed a new method to reduce the effect of the higher-order distribution by ignoring the time region just after the beam induction. Kosály and Fischer⁹⁾ has also proposed another method which employs the space integration of the measured data by several neutron detectors.

On the other hand, the noise analysis method, which includes the Feynman-a method, the Rossi-a method and the frequency analysis method, is also available to monitor the subcriticality of ADS Especially the noise method can be employed for the direct beam mode since the noise analysis method has originally developed for a subcritical core with a external source neutron. However, it should be noted that the accuracy of the method is a little bit poor since the method requires measuring the variance and the conditional probability of neutron detection. For the feasibility to the pulse beam, the noise method has been also studied. Kitamura^{10),11)} has developed a new technique by expanding the noise method into the pulse beam mode and demonstrated by using the KUCA.

6. Summary

The Research Committee on Reactor Physics under the Atomic Energy Society of Japan and the Japan Atomic Energy Research Institute organized the working party (ADS-WP) on "Subcritical Core of Accelerator-Driven System". The ADS-WP investigated reactor physics of subcriticality from the viewpoint of the accelerator driven system (ADS) since subcriticality has been almost studied from the viewpoint of critical safety. The working party was set in July 2001 and it worked for two years. The activities of the ADS-WP are

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