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Efficient Hybrid NEM/BEM Transient Method

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This paper describes the implementation of the multi-group pin-by-pin SP_3 method in the Pennsylvania State University's (PSU) Nodal Expansion Method (NEM) core transient simulator code. The presented work is a part of the cooperative research between PSU and Politecnico di Torino (PT) and it was performed in a manner that the existing expertise at both institutions complemented each other.

Comparison was made to the VENUS II international benchmark for SP_3 and eight groups. The results demonstrated a deviation in the predicted k_{eff} as compared to experimental value of 1.00000 of 250 pcm and deviations in the prediction of pin powers as compared to reference values in the range of ±2.5%. The two-group diffusion calculations predicted well the eigenvalue but showed large errors in pin power predictions especially in the MOX assembly (about 9%). The two-group calculations are unable to model accurately neutron transport in the region where the flux is changing rapidly (at the core/reflection interface where MOX assemblies are located). The pin power error is reduced significantly with eight group calculations where in the diffusion case the maximum pin power error value is reduced to about 4.6% and in the SP_3 case to 2.5%.

KEYWORDS: Nodal Expansion Method (NEM), Boundary Element Method (BEM), SP_3 approximation, A_2 approximation, VENUS II International Benchmark

1. Introduction

In recent years the need for more accurate reactor simulator codes has been seen due to crucial changes in nuclear reactor design: MOX cores, higher burn-ups, advanced assemblies, control rods, and reactor concepts. All of these changes create a more heterogeneous neutronic environment in the reactor core, which in turn challenges the accuracy of diffusion theory.

The attractiveness of diffusion theory has its basis in well-developed calculation procedures and computation efficiency. A few assumptions are also used in conjunction with the diffusion theory to make this efficiency achievable; typically two energy groups are used and assemblies are represented by homogenized cross sections. These assumptions, together with the diffusion theory, give acceptable results for more traditional light-water reactor designs and typically calculate these results within a few minutes -for steady-state cases.

The challenge of using diffusion theory in any of the newer reactor designs is that material properties can change radically within a few neutron mean free paths. For example, a plutonium

fuel assembly is placed next to a standard uranium assembly in a MOX core. Higher burn-up cores have a similar problem in that the assemblies with the higher burn-up often have more plutonium when compared to a fresh uranium fuel assembly. Advanced reactor designs are pushing the envelope of 5 weight percent of U-235 and there is a possibility of replacing up to 68% of the core per reload cycle [1]. The use of part-length rods in Boiling Water Reactors also adds to the heterogeneity of advanced reactor designs. All of these new reactor designs can have material properties significantly changing from pin to pin within an assembly.

Due to the advancement of computer technology, higher order transport approximations can be implemented into core simulator codes, allowing for more accurate results for a wider array of modern core designs [2]. Correspondingly, it was decided that the Pennsylvania State University's Nodal Expansion Method Code (NEM) [3] should be updated. To complete this task in an efficient manner, it was decided that a higher order transport approximation would be used to calculate the partial currents for each assembly a few times during a standard NEM calculation, thus improving/updating the nodal partial currents. This non-linear iterative transport correction improved the accuracy of the NEM code without significantly increasing the computational time.

The decision of which higher order transport approximation to utilize was based on the ease of implementation or coupling the approximation with the preexisting NEM code, along with meeting the criteria for efficient computational time. The reasoning behind these decisions are given in detail for each new option in the following sections.

1.1 Reasoning for Simplified P_3

Among other transport methods, such as the method of characteristics and discrete ordinates, the multi-group pin-by-pin Simplified P_3 (SP_3) approximation was determined to be the most applicable choice for implementation into the preexisting core simulator code. This approach is supported by the obtained results of several researchers [4,5,6]. First, the SP_3 approximation can yield more accurate solutions of core transport problems than the diffusion approximation with considerable less computation expense than the discrete ordinates (S_N) or spherical harmonics (P_N) approximations [4]. Another advantage of SP_3 equations is that they can be solved by straightforward extensions of the common nodal diffusion methods with little computation resources overhead [6]. It has been shown elsewhere that if innovative acceleration techniques are utilized to improve the computation performance, the multi-group SP_3 pin-by-pin calculations can be used for practical core depletion and transient analyses [7,8].

1.2 Reasoning for A_N-BEM

Recent joint work at the University of Pisa, Italy, and the Politecnico di Torino, Italy, has demonstrated the equivalence between the SP_{2N-1} (for example SP_3) equations and the so-called A_N (for example A_2) equations [9]. The A_N equations have the simple structure of a system of multi-group diffusion equations, and are easily transformed into a set of boundary integral equations, which permits the application of a boundary element-response matrix (BEM) solution technique. The applicability of BEM to multi-group multi-dimensional neutron diffusion problems has been demonstrated previously [10]. The method rests upon a partial current formulation of BEM equations, that allows for a natural link with the response matrix technique of NEM model.

The A_2-BEM method was selected to be appropriate for local solution on a pin-by-pin basis in the framework of a homogeneous coarse mesh node. The coarse mesh node is typically in radial plane a homogenized fuel assembly with an axial height of six inches (15.24 cm). The fine mesh

cell is the radial plane of a homogenized fuel pin cell and the axial plane has a height of about 1.5 cm. The BEM method is applied to perform very detailed flux calculations in a transport A_2 approximation, using the above-described fine mesh spatial grid in the framework of the coarse mesh node.

2. Simplified P_3 Equations - Background

The Simplified P_3 equations are based on spherical harmonics (P_N), which is a method to approximate the transport equation by expanding the angular flux in a truncated series of Legendre Polynomials [11]. The spherical harmonics method dates back to 1926 [12]. In the 1960's, E.M. Gelbard first proposed the Simplified P_N equations. He substituted the three-dimensional Laplacian operators in the P_N equations for the second order derivative in the one-dimensional planar geometry [4]. This substitution was superficial and was not proven mathematically until recently. This lack of theoretical basis for the simplified P_N equations prevented their use for decades. In the early 1990's, Larsen, McGhee, and Morel presented the "first theoretical basis for the observed accuracy of the SP_N equations in multi-dimensional applications". They showed that these equations are higher-order asymptotic solutions of the transport equation "in a physical regime in which diffusion theory is the leading-order approximation" [4]. In the same time frame, "Pomraning showed that for odd N and an infinite homogeneous medium, the SP_N equations are a variational approximation to the one-group even-parity transport equation with isotropic scattering" [4]. To further strengthen the theoretical basis for the SP_N equations, Ciolini, Coppa, Montagnini, and Ravetto have shown that the A_N equations can be rigorously derived from the Boltzmann's transport equation and are equivalent to the SP_{2n-1} equations [9].

The SP_N method has been reviewed and tested by a number of companies and universities, such as: Studsvik, USA [6], NFI-Japan [8], and Purdue University, USA [13,14,15,16,2]. Studsvik has used SP_N (SP_L) in the QUANTM code to solve BWR assembly, PWR lattice, and shielding benchmarks problems. NFI-Japan has used SP_3 in the SCOPE2 code, and Purdue University has implemented SP_3 into the U.S. NRC neutronics code PARCS.

3. A_N Method and the Boundary Element Method (BEM) for the 3-Dimensional Cartesian Geometry

The A_N equations were first introduced by Coppa and Ravetto in 1980. Later in 1990, Spinard and Altac introduced the "Simplified Kernel" method (SK_N) which confirmed the resemblance of the A_N (SK_N) equations with diffusion equations [9]. Since then the A_N equations were shown to be equivalent to the SP_{2N-1} equations for the case of isotropic scattering [9]. This leads to a major advantage of these equations over the SP_N equations: the equations have a similar form to the diffusion equations. This makes implementing the equations into existing codes easier than the SP_N equations. This "diffusion like" form also permits these equations to be solved using a response matrix, which is referred to as the Boundary Element Method (BEM). The reason for combining the A_N method with the SP_N method is due to the equivalence of these two sets of equations.

3.1 A_2-BEM Equations

$$\nabla\cdot\left(\frac{\mu_\alpha^2}{\sigma(\vec{r})}\nabla\Phi_\alpha(\vec{r})\right)-\sigma(\vec{r})\Phi_\alpha(\vec{r})+q_0(\vec{r})=0 \qquad \begin{pmatrix}\alpha=1,2,...,N\\ q_0(\vec{r})=\sigma_s(\vec{r})\sum_{\beta=1}^{N}\omega_\beta\Phi_\beta(\vec{r})+S(\vec{r})\end{pmatrix} \qquad (1)$$

where

σ_s	= Scattering cross section	σ	= Total cross section
$\mu_\alpha^2=\lambda_\alpha$	= Eigenvalues	$\sum_{\beta=1}^{N}\omega_\beta\Phi_\beta(\vec{r})$	= Scalar flux
S	= External source	Φ_α	= Flux moments

The general formulation of the response matrix method used in BEM is as follows.

$$\hat{M}^{+}\mathrm{J}^{+}=\hat{M}^{-}\mathrm{J}^{-}+\Psi \qquad (2)$$

where

$$J^{\pm}=\left(J_1^{\pm},...,J_N^{\pm}\right)^t$$

$$\Psi^{\pm}=\left(\Psi_1^{\pm},...,\Psi_N^{\pm}\right)^t$$

$$\left[\hat{M}^{\pm}\right]_{\alpha\beta}=\hat{M}_{\alpha\beta}^{\pm}\pm\hat{C}\delta_{\alpha\beta}$$

C is a block diagonal matrix whose non-zero elements are c(j,k), that assume the values 0,1,1/2 depending on whether r is outside, inside, or is a smooth boundary point of the domain, respectively.

4. Nodal Pin-By-Pin SP_3

Before discussing the implementation of the SP_3 nodal into NEM or the coupling of NEM with A_2-BEM, it would be prudent to describe the Nodal Expansion Method (NEM) code. NEM is a few-group (up to 10 energy groups can be simulated) Three-Dimensional (3-D) transient nodal core model with three geometry-modeling options: Cartesian, Hexagonal-Z and Cylindrical (R-θ-Z) [19]. NEM is based on the transverse integration procedure and it was recently updated to utilize semi-analytical transverse-integrated flux representation and improved transverse leakage approximation [17]. The nodal coupling relationships are expressed in a partial current formulation. The time dependence of the neutron flux is approximated by a first order fully implicit finite-difference scheme (upgraded later with exponential transformation technique [17]), whereas the time dependence of the neutron precursor distributions is modeled by a linear time-integrated approximation. NEM is using the Response Matrix (RM) technique for inner iterations to calculate (update) ongoing partial currents for each spatial node in the framework of each energy group solution. The coarse-mesh rebalance (extended later to multi-grid technique [17]) and asymptotic extrapolation methods are used to accelerate convergence of the outer iterative solution process.

Both the transient and steady state SP_3 equations have been derived in the NEM terms. These equations need to have a similar form as the diffusion equation used in the NEM code, which typically solves the two group diffusion equations, but any number of energy groups can be specified.

4.1 Steady-State Simplified P_3 Method

The final version of the steady-state simplified P_3 equations used in NEM is as follows:

$$\begin{aligned} -D_g^1\nabla^2\Psi_g^1(\vec{r})+E_g^1\Psi_g^1(\vec{r})&=H_g^1(\vec{r})\\ -D_g^2\nabla^2\Psi_g^2(\vec{r})+E_g^2\Psi_g^2(\vec{r})&=H_g^2(\vec{r}) \end{aligned} \tag{3}$$

where

$$\begin{aligned} \Psi_g^1(\vec{r}) &= 2\psi_{g,2}(\vec{r})+\psi_{g,0}(\vec{r}) & \Psi_g^2 &= \psi_{g,2}(\vec{r})\\ D_g^1 &= D_{1,g}=\frac{1}{3\Sigma_{tr,1,g}} & D_g^2 &= \frac{27}{35}D_{2,g}=\frac{9}{35}\frac{1}{\Sigma_{t,3,g}}\\ E_g^1 &= \Sigma_{r,g}(\vec{r}) & E_g^2 &= \Sigma_{t,2,g}(\vec{r})\\ H_g^1 &= S_0+2\Sigma_{r,g}(\vec{r})\psi_{g,2}(\vec{r}) & H_g^2 &= \frac{2}{5}\left(\Sigma_{r,g}(\vec{r})\psi_{g,0}-S_0\right)\\ S_0 &= \frac{\chi_g}{k_{eff}}\sum_{g'}^{G}\nu_{g'}\Sigma_{f,g'}(\vec{r})\psi_{g',0}(\vec{r})+\sum_{g'\neq g}^{G}\Sigma_{s,g'\to g,0}(\vec{r})\psi_{g',0}(\vec{r}) \end{aligned}$$

4.2 Transient Simplified P_3 Method

The final version of the transient simplified P_3 equations used in NEM are as follows:

$$\begin{aligned} -D_g^1\nabla^2\Psi_g^1(\vec{r})+E_g^1\Psi_g^1(\vec{r})&=H_g^1(\vec{r})\\ -D_g^2\nabla^2\Psi_g^2(\vec{r})+E_g^2\Psi_g^2(\vec{r})&=H_g^2(\vec{r}) \end{aligned} \tag{4}$$

where

$$\begin{aligned} \Psi_g^1(\vec{r}) &= 2\psi_{g,2}(\vec{r})+\psi_{g,0}(\vec{r}) & \Psi_g^2 &= \psi_{g,2}(\vec{r})\\ D_g^1 &= D_{1,g}=\frac{1}{3\Sigma_{tr,1,g}} & D_g^2 &= \frac{4}{5}D_1\nabla^2+\frac{27}{35}D_3, D_3=\frac{9}{35}\frac{1}{\Sigma_{t,3,g}}\\ E_g^1 &= \Sigma_{r,g}(\vec{r})+\frac{1}{\upsilon_g\Delta t} & E_g^2 &= \Sigma_{t,2,g}(\vec{r})+\frac{1}{\upsilon_g\Delta t}\\ H_g^1 &= S_0^l+\frac{\psi_{g,0}^{l,old}(\vec{r})}{\upsilon_g\Delta t}-\frac{\nabla\psi_{g,1}^{l,old}(\vec{r})}{\upsilon_g\Delta t\Sigma_{tr,1,g}(\vec{r})+1}+2\psi_{g,2}^{l}(\vec{r})\left(\Sigma_{r,g}(\vec{r})+\frac{1}{\upsilon_g\Delta t}\right)\\ H_g^2 &= \frac{\psi_{g,2}^{l,old}(\vec{r})}{\upsilon_g t}-\frac{3\nabla\psi_{g,3}^{l,old}(\vec{r})+2\nabla\psi_{g,1}^{l,old}(\vec{r})}{5\upsilon_g t\Sigma_{tr,1,g}(\vec{r})+1}+\frac{2}{5}D_1\nabla^2\psi_{g,0}^{l}(\vec{r})\\ S_0 &= \sum_{g'}^{G}\nu_{g'}\Sigma_{f,g'}(\vec{r})\psi_{g',0}(\vec{r})+\sum_{g'\neq g}^{G}\Sigma_{s,g'\to g,0}(\vec{r})\psi_{g',0}(\vec{r}) \end{aligned}$$

5. Coupled NEM-RM / A_2-BEM

The way this option is designed, NEM calculates the nodal incoming partial currents for the response matrix and BEM updates the nodal out-going partial currents. It does this by solving the A_2 equations on a pin-by-pin basis using the most recent nodal incoming currents as boundary conditions. To prevent the computational time of a problem from rising to an impractical level, the NEM partial currents are only updated by A_2-BEM a few times (non-linear iteration)

throughout the entire NEM calculation. This is achieved starting with less stringent convergence criteria for the k_{eff}, point-wise nodal fission source, and the average wise nodal fission source. For standalone NEM, the convergence criteria are shown in Table 1.

Table 1. Convergence Criteria Governing the Update of Partial Currents

	k_{eff}	Point Fission Source	Average Fission Source
1^{st} update	1.0 E –03	1.0 E -01	1.0 E -01
2^{nd} update	1.0 E –04	1.0 E –02	1.0 E –02
3^{rd} update	1.0 E -05	1.0 E –03	1.0 E –03
4^{th} / final update	1.0 E -06	1.0 E -04	1.0 E -04

5.1 Coupling Scheme

An efficient scheme was introduced to couple the local A_2-BEM (pin-by-pin) solution with global core (assembly-by-assembly) calculation. In this two-level non-linear solution scheme the local refined calculations are not performed during each outer-iteration. Instead they are carried out only few times during the steady-state or given time step solution process, which results in significant improvement of the computation performance. To ensure convergence of the non-linear iteration process the following technique has been applied. Intermediate global (k_{eff}) and local convergence criteria (node-wise fission sources) were introduced to invoke performing A_2-BEM (pin-by-pin) solution. This is a part of an additional iteration level in which the higher order A_2 equations are solved, using the most recently computed nodal incoming partial currents in order to find more accurate values of the out-going partial currents. The figure below shows the general scheme of the coupling.

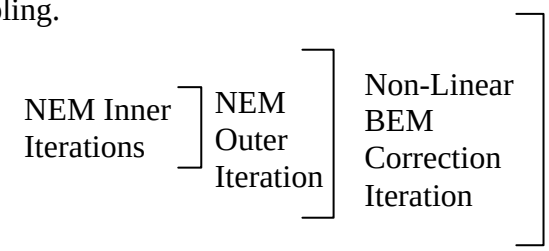

Figure 1. Coupling NEM-RM/A_2-BEM

5.2 Partial Current Mapping

The A_2-BEM method was selected as appropriate for local solution on pin-by-pin basis in the framework of a homogeneous coarse mesh node. The coarse mesh node is typically defined in radial plane as representing a homogenized fuel assembly with an axial height of 15.24 cm. The fine mesh cell is defined, such that, in radial plane it is a homogenized fuel pin cell and in axial plane it has a height of about 1.5 cm.

The strategy of deconstruction and reconstruction of the nodal partial currents is based in a first approximation on the geometric (area) weighting. For deconstruction of the nodal partial currents, for each face of each node, the nodal partial currents are divided by the number of cells along that face. Likewise, when the nodal partial currents are reconstructed, it is a sum over the cells along that face.

Investigation of more sophisticated approaches are left for future research. One option considered was to use a polynomial function instead of a linear function.

Two sets of boundary conditions are needed to solve the A_2 equations. The deconstructed incoming nodal currents are one set and the incoming third-moments are the other set. The third-moments are defined by the previous iteration step. Since the response matrix method is used, the localized partial moments are obtained from the outgoing moments from neighboring cells or those from the previous iteration step as:

$$J_{in}(x) = \bar{J}_{in} f(x) \tag{5}$$

where

$J_{in}(x)$ = Local distribution of partial incoming moments,

$\bar{J}_{in}(x)$ = Face-averaged partial incoming moment, and

$f(x)$ = Shape function normalized to the average incoming moment.

6. Verification and Validation

To verify and validate the results of the two transport theory models implemented into NEM, two different international benchmarks were utilized: OECD Benchmark L-336 and VENUS II. The OECD benchmark was used to verify the functionality and computational performance of the SP_3 Nodal and the coupled NEM/BEM models. Only two energy groups were used in testing due to the basic application of this benchmark, although in real applications more energy groups are required for SP_3 to achieve accurate results. The VENUS II benchmark was used to validate the accuracy of the NEM- RM/A_2-BEM.

6.1 International OECD Benchmark L-336

The international OECD Benchmark L-336 was first proposed in 1991 and was related to the assessment of pin-power distribution within heterogeneous core type configurations. For the original benchmark there were five quasi-core checkerboard configurations consisting of three different fuel assemblies, and a total of eight different homogeneous pin cells with different cross section sets. Only Case 3 was utilized to verify the implementation of NEM-RM/A_2-BEM and SP_3 nodal. Case 3 is four fuel assemblies – two are UO_2 and two are MOX in checkerboard pattern with zero flux boundary conditions on all sides of the problem.

The three-dimensional neutronics code PARCS [13] was used for single-assembly calculations to obtain the nodal homogenized cross sections. The homogeneous cross sections were generated by the method of flux-volume averaging.

To provide a reference solution for the purpose of this verification, the SP_3 nodal pin-by-pin calculations were performed. One can observe a steep gradient in power that occurs at the interface between uranium and plutonium assemblies, and that the center between four adjacent assemblies represents the most challenging part of the core.

The obtained NEM-RM/A_2-BEM results showed a maximum 4% over prediction of pin powers in uranium assemblies and 2% power under prediction in plutonium assemblies. These deviations are attributed to the simply geometry weighted spatial mapping approach used in NEM-RM/A_2-BEM calculations. More sophisticated techniques have to be investigated in the future to reduced the deviations. Table 2 gives the comparison of the calculated k_{eff} with the referenced SP_3 nodal eigenvalue for Case 3.

Table 2. k_{eff} for Case 3 with One Node Per Assembly

NEM-RM/A_2-BEM	1.01878
Reference SP_3 Nodal	1.01913
Error, pcm	-45

To test the transient implementations, the L-336, Case 3 benchmark again was utilized. The withdrawal of a control rod was modeled. To model the initial conditions with all rods inserted the uranium assemblies with guide tubes were replaced with the uranium assemblies with absorbers. The rod withdrawal transient was modeled by introducing the linearly changing cross sections for both pin-by-pin and nodal assembly models. The cross section rate change was calculated based on the axial height of 21.42 cm and withdrawal velocity of 4.76 cm/s. The correspondent withdrawal time of control rods equals 4.50 seconds. The NEM code has the capability to introduce the linear change of the cross sections such, that after 4.50 seconds of transient the values of cross sections are equal to those corresponded to unrodded uranium assembly. Both nodal SP_3 pin-by-pin and NEM-RM/A_2-BEM calculations were performed and the relative deviations in pin powers were analyzed for different time shapshots during the transient.

The maximum deviation in pin powers is somewhat higher than compared to initial steady state and around 5%.

6.2 VENUS II

The VENUS II international benchmark is based on actual measured data from the VENUS II reactor located at SCK•CEN in Belgium [18].

At this time, known data is only available for the 2-Dimensional VENUS II benchmark, while the 3-Dimensional benchmark is currently a blind benchmark. In either benchmark, the VENUS II reactor is a zero power critical reactor comprised of twelve 15x15 fuel assemblies with a pitch of a 17x17 PWR assembly. The four inner assemblies are comprised of a reflector, baffle, UO2 3.3 $^{w}/_{o}$ enriched fuel and ten Pyrex pins. The remaining eight fuel assemblies are comprised of 105 UO2 2.0 $^{w}/_{o}$ enriched pins and the remaining pins in each assembly are 2.7 $^{w}/_{o}$ high-grade plutonium [18].

This was modeled with eight energy groups. The transport, absorption, and scattering cross sections along with $\nu\Sigma_f$, $\kappa\Sigma_f$, chi-values for prompt and delayed neutrons, lambda, and beta were calculated by Purdue University using the HELIOS code. From this data, all needed information to run NEM-RM/A_2-BEM can be found.

Different calculations have been performed with NEM-RM/A_2-BEM code options for the 2-D VENUS II Benchmark. The following results were observed.

Table 3. 2-D NEM-RM/A_2-BEM VENUS II Results

Number of Energy Groups	Approximation	Core k_{eff}
2	Diffusion	0.99730
2	SP_3	1.0490
8	Diffusion	0.99400
8	SP_3	1.0250

The results obtained with SP_3 and eight groups demonstrated a deviation in the predicted k_{eff} as compared to experimental value of 1.00000 of 250 pcm and deviations in the prediction of pin powers as compared to reference values in the range of ± 2.5 %. The two-group diffusion calculations predicted well the eigenvalue but showed large errors in pin power predictions especially in the MOX assembly (about 9%). The two-group calculations are unable to model accurately neutron transport in the region where the flux is changing rapidly (at the core/reflection interface where MOX assemblies are located). The pin power error is reduced significantly with eight group calculations where in the diffusion case the maximum pin power error values is reduced to about 4.6% and in the SP_3 case to 2.5%.

7. Conclusions

The SP_3 transport theory model has been implemented within the framework of NEM for 3-D pin-by-in multi-group calculations. As compared to pin-by-pin multi-group NEM 3-D diffusion calculations, the increase of computation time is about 50% depending on the problem being analyzed. To further improve the computational efficiency and the performance of the transport theory model in order to be applied for practical transient analysis, an efficient scheme has been introduced to couple the local A_2-BEM (pin-by-pin) solution with the global core NEM diffusion (assembly–by-assembly) calculations. Since the local refined calculations are performed only a few times during a steady state or given time step calculation, the increase in computation time as compared to the NEM diffusion assembly-by-assembly calculations is about two times.

One case of the international OECD benchmark L-336 was used as a test problem for both steady-state and transient calculations performed with the developed SP_3 nodal pin-by-pin and NEM-RM/A_2-BEM models. Both models were demonstrated to be functional options of NEM and while the accuracy was found to be identical in terms of k_{eff}, and pin power predictions, the computation time of NEM-RM/A2-BEM was about eight times less than the SP_3 nodal pin-by-pin calculations.

The hybrid NEM-RM/A_2-BEM model with eight energy groups was also applied to the VENUS II international benchmark, where actual measured data is available [18]. The deviation in the predicted k_{eff} as compared to experimental value of 1.00000 was 250 pcm and the deviations in the prediction of pin powers as compared to reference values were in the range of ±2.5 %.

References

1. EPRI-1003133, “Optimum cycle Length and Discharge Burnup for Nuclear Fuel – A comprehensive Study for BWRs and PWRs Phase I: Results Achievable Within the 5% Enrichment Limit.”, December 2001.
2. C. Lee, T. Kozlowki, and T.. Downar. “Analysis of Control Rod Ejection Accident in MOX and MOX/UOX Cores with Time-Dependent Multigroup Pin-by-Pin SP3 Methods.” PHYSOR Meeting Seoul, Korea. October 2002.
3. B. Bandini, “A three-Dimensional Transient Neutronics Routine for the TRAC-PF1 Reactor Thermal Hydraulic Computer Code.”, Ph.D. Thesis. The Pennsylvania State University, May 1990.

4. P. Brantley, and E. Larsen, “The Simplified P3 Approximation,” Nuclear Science and Engineering: **134**, 1-21 (2000)
5. U. Shin, and W. Miller, “The Time-Dependent Simplified P2 Equations: Asymptotic Analyses and Numerical Experiments,” Nuclear Science and Engineering: **138**, 27-46 (1998)
6. K. Smith, “Multi-dimensional Nodal Transport Using Simplified P_L Method,” Proc. of Reactor Physics and Safety, Saratoga Springs, New York, p.223 (1986).
7. C.-Lee, T. Kozlowski; and T. Downar. “Analysis of Control Rod Ejection Accident in MOX and MOX/UOX Cores with Time-Dependent Multigroup Pin-by-Pin SP_3 Methods.” PHYSOR Meeting, Seoul, Korea, October 2002
8. M. Tatsumi, and A.Yamamoto, “Scope2: Object-Oriented Parallel code for Multi-group Diffusion/Transport Calculations in Three-Dimensional Fine-Mesh Reactor Core Geometry,” PHYSOR 2002, Seoul, Korea, October, 2002.
9. R. Ciolini, G.G.M. Coppa, B. Montagnini, P. Ravetto, "Simplified Pn and An Method in Neutron Transport", Progress in Nuclear Energy, 40, 245-272, (2002).
10. M. Maiani, B. Montagnini, "A Boundary Elements - Response Matrix Method for the Multi-group Neutron Diffusion Equations", Annals of Nuclear Energy, 26, 1341-1369, (1999).
11. E.E. Lewis, and W. Miller, “Computational Methods of Neutron Transport,” American Nuclear Society, La Grange Park, Illinois USA. (1993)
12. H. Greenspan, C. Kelber; and D. Okrent, “Computer Methods in Reactor Physics.” Argonne National Laboratory, Gordon and Breach Science Publishers. New York, London, Paris. 1968.
13. Purdue University, “User Manual for the PARCS-v2.2 Kinetic Core Simulator Module – Chapter 9,” August 2001
14. Purdue University, “Development Notes for PARCS Ver2.4beta”
15. C. Lee, T. Kozlowski, and T. Downar,. “Analysis of Rod Ejection Accident with Time-Dependent Multigroup Pin-by-Pin SP3 Methods.” School of Nuclear Engineering, Purdue University.
16. T. Kozlowski, C. Lee, and T. Downar. “Benchmarking of the Multigroup, Fine Mesh, SP3 Methods in PARCS with the VENUS-2 MOX Critial Experiments.” Purdue University, School of Nuclear Engineering, October 2002.
17. K. Ivanov, et al, “Nodal Kinetics Upgrade in the Penn State Coupled TRAC/NEM Codes,” Annals of Nuclear Energy 26 (1999), 1205-1219.
18 “Benchmark on the VENUS-2 MOX Core Measurements,” NEA/NSC/DOC (2000)7
19. “OECD/NEA AND U.S. NRC PWR MOX/UO2 Core Transient Benchmark,” Purdue University, West Lafayette, Indiana, U.S.A., 2003