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On Some Features of Quasi-Static Schemes in Reactor Kinetics

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The quasi-static method has shown to be an efficient tool for the time-dependent analysis of nuclear systems allowing a significant reduction of the computational effort needed for full space and energy transient calculations. The method is based on the factorization of the neutron density in the product of a shape and an amplitude function. The time integration of the balance equations is then carried out on a fast scale for the amplitude and on a much slower scale for the shape. Since the shape calculation is the most time consuming, this scheme allows a significant reduction of the computing time. In this work some specific features of the quasi-static technique are investigated, with special regard to the application of the method to source-driven systems.

KEYWORDS: Reactor Dynamics, Quasi-Static Method

1. Introduction

The quasi-static method has shown to be a very efficient tool for the time-dependent analysis of nuclear systems. Full calculations for multidimensional time-dependent problems need a relevant computational effort and quasi-statics allows a significant reduction of the computing time, while retaining the capability to give accurate representations of the energy and space evolution of the neutron population. The method is based on the factorization of the neutron density in the product of a shape and an amplitude function. [1, 2] The time integration of the model equations derived from the original balance equations is then carried out on a fast scale for the amplitude and on a much slower scale for the shape. The number of full space-energy calculations is thus highly reduced with a significant reduction of the computing time. The principle of quasi-statics has been also generalized successfully to stiff problems encountered in several different areas of applied science. [3]

In the standard application of the quasi-static method, the projection of the model equations is performed on the solution of the adjoint problem for the reference critical configuration. However, the method can be derived using any weighting function, although the choice of the importance function as a weight is privileged, since the amplitude turns out to be the component of the fundamental eigenfunction of the reference reactor in the full solution. [4] The method is consistent when the reference reactor is

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a steady-state critical system. However some problems unavoidably arise for source-driven systems. In this case it seems obvious to use as reference the source-injected initial steady-state system. However, no unique consistent adjoint can be defined. For instance, either the critical adjoint associated to an eigenvalue to make the homogeneous problem solvable or a source-driven adjoint can be used. In the latter case, an adjoint source must be defined. As a consequence, the question of the best choice of the weighting function naturally arises.

In this work some specific features of the quasi-static technique are investigated, with special regard to the application of the method to source-driven systems. The choice of different weighting functions is investigated, following recent works of the subject. [5]

2. Position of the problem

The starting problem is the solution of the neutron transport equation in presence of delayed emissions:

$$\begin{cases} \dfrac{\partial n(\mathbf{r},E,\mathbf{\Omega},t)}{\partial t} = \left[\hat{\mathrm{L}}(t) + \hat{\mathrm{M}}_p(t)\right] n(\mathbf{r},E,\mathbf{\Omega},t) + \displaystyle\sum_{i=1}^{R} \mathcal{E}_i(\mathbf{r},E,t) + S(\mathbf{r},E,\mathbf{\Omega},t), \\ \dfrac{1}{\lambda_i}\dfrac{\partial \mathcal{E}_i(\mathbf{r},E,t)}{\partial t} = \hat{\mathrm{M}}_i(t) n(\mathbf{r},E,\mathbf{\Omega},t) - \mathcal{E}_i(\mathbf{r},E,t), \qquad i = 1,2,...,R, \end{cases} \tag{1}$$

where the delayed neutron emissivity is defined as:

$$\mathcal{E}_i(\mathbf{r},E,t) = \lambda_i C_i(\mathbf{r},t)\frac{\chi_i(E)}{4\pi}. \tag{2}$$

The operators $\hat{\mathrm{L}}$, $\hat{\mathrm{M}}_p$ and $\hat{\mathrm{M}}_i$ are the leakage, prompt multiplication and delayed multiplication operators, respectively. All the other symbols have their usual meaning in standard reactor physics literature. The position of the problem requires to specify proper boundary and initial conditions.

The time-integration of system of equations (1) is searched in terms of a factorized solution as a product of the amplitude A and the shape φ:

$$n(\mathbf{r},E,\mathbf{\Omega},t) = A(t)\varphi(\mathbf{r},E,\mathbf{\Omega};t). \tag{3}$$

The factorization is then inserted into Eqs. (1), to obtain:

$$\begin{cases} A\dfrac{\partial \varphi}{\partial t} + \varphi\dfrac{dA}{dt} = A\hat{\mathrm{B}}\varphi + \displaystyle\sum_{i=1}^{6} \mathcal{E}_i + S, \\ \dfrac{1}{\lambda_i}\dfrac{\partial \mathcal{E}_i}{\partial t} = A\hat{\mathrm{M}}_i\varphi - \lambda_i\mathcal{E}_i, \qquad i = 1,2,...,R. \end{cases} \tag{4}$$

These equations can be regarded as a model for the shape φ, once the amplitude evolution is supposed to be known.

The equations for the amplitude are constructed by a projection operation on a weighting function w:

$$\begin{cases} \dfrac{dA}{dt}\left\langle w\right|\left.\varphi\right\rangle = A\left\langle w\right|\left.\hat{\mathrm{B}}\varphi\right\rangle + \displaystyle\sum_{i=1}^{6}\left\langle w\right|\left.\mathcal{E}_i\right\rangle + \left\langle w\right|\left.S\right\rangle, \\ \dfrac{1}{\lambda_i}\dfrac{d}{dt}\left\langle w\right|\left.\mathcal{E}_i\right\rangle = A\left\langle w\right|\left.\hat{\mathrm{M}}_i\varphi\right\rangle - \left\langle w\right|\left.\mathcal{E}_i\right\rangle, \qquad i = 1,2,...,R, \end{cases} \tag{5}$$

where the normalization condition is imposed:

$$\frac{d}{dt}\left\langle w\right|\left.\varphi\right\rangle = 0. \tag{6}$$

The procedure is applied using as shape initially the distribution of the reference reactor and later the last-calculated one. Suppose that after time t_0 the shape is updated at time T, system (4) is solved by a Euler implicit scheme:

$$\begin{aligned} &A(T)\frac{\varphi(T)-\varphi(t_0)}{T-t_0} + \varphi(T)\dot{A}(T) = A(T)\hat{\mathrm{B}}\varphi(T) + \\ &\sum_{i=1}^{6}\left[\mathcal{E}_i(t_0)e^{-\lambda_i(T-t_0)} + \int_{t_0}^{T} A(t')\hat{\mathrm{M}}_i\varphi(t_0)e^{-\lambda_i(T-t')}dt'\right] + S(T). \end{aligned} \tag{7}$$

The fulfillment of the normalization condition (6) is imposed iterating the solution of Eq. (7). Different strategies can be used for the procedure. [6] For instance, the amplitude is supposed to be continuous while its derivative is allowed to be discontinuous and obtained from Eqs. (5), using the latest available shape, or, alternatively and *more* physically, continuity on the total power is required with no restriction of continuity on both A and $\dot{A}$.

In a previous work [5] the problem of the weighting function was studied by simply looking at the results that can be attained by different choices. No clear conclusion could be reached and no general indications could be given, in the sense that the performance of a weighting function could be quite different and strongly dependent on the type of transient considered.

In this work paradigmatic system configurations are considered in diffusion theory, thus allowing to calculate analytically the full time behavior of the neutron flux $\Phi\left(x,t\right)$. Then the transient for the exact amplitude and shape functions is derived for any choice of the weight, having imposed a normalization condition. These functions are referred to as *exact*, since they are derived without introducing any approximation, since the amplitude function can be back-obtained from:

$$A(t) = \frac{\left\langle w\right|\left.\Phi(x,t)\right\rangle}{\left\langle w\right|\left.\Phi(x,t=0)\right\rangle}; \tag{8}$$

afterwards, the shape is derived as:

$$\varphi\left(x,t\right) = \frac{\Phi\left(x,t\right)}{A(t)}. \tag{9}$$

In the first part of the study, these functions are used to compute the kinetic parameters at the beginning of the transient and perform the corresponding point kinetic calcu-

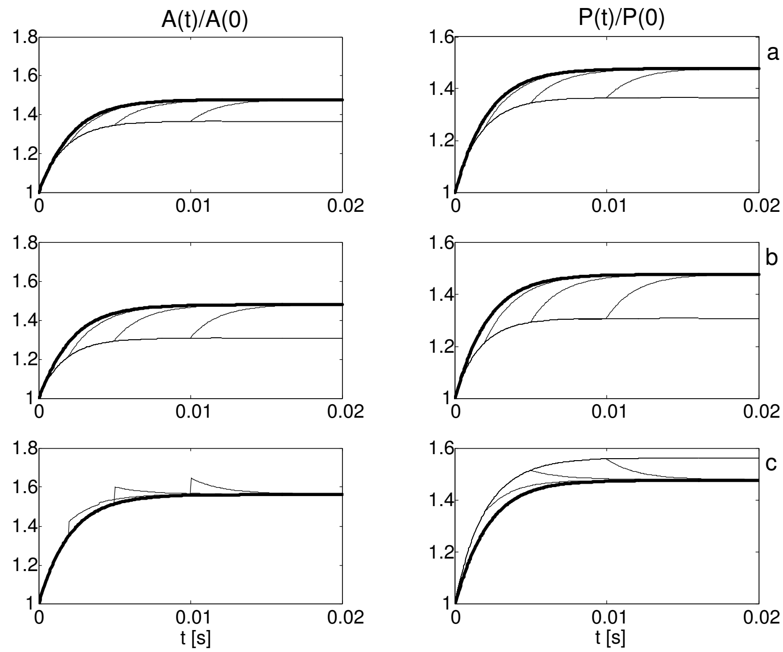

Fig. 1 Time evolution of amplitude function and power for different weigths (large system). a) constant; b) initial critical adjoint; c) final critical adjoint. The full solution is in bold and it is compared to quasi-statics with different number of shape recalculations. The transient involves a reactivity of 252 pcm.

lation. The scheme can be included into a quasi-static procedure, updating the kinetic parameters during the transient using the exact shape.

3. Discussion of results

The comparison of the results obtained can highlight the effect of the choice of the weighting function on the effectiveness of the quasi-static scheme. A transient for a system starting from criticality and evolving after the removal of a control device is considered. A one-dimensional problem is analytically solved. The control device with a small but finite thickness is symmetrically located (position identified by CD in figures). Quasi-statics is constructed to preserve the continuity of the system power. Therefore, the amplitude may be presenting discontinuities at the instants when the shape is updated. To characterize the evolution of the spatial distribution of the shape, the norm of the difference between the actual shape and the initial flux is used:

$$\xi(t) = \sqrt{\left\langle \left(\varphi\left(x,t\right) - \Phi\left(x,0\right)\right)^2 \right\rangle} \tag{10}$$

In Fig. 1 the time evolutions of the amplitude functions assuming different weights are compared (left column), together with the system power (right column) in a slab characterized by a half-thickness $H = 50L$. Due to the space distortion in the transient,

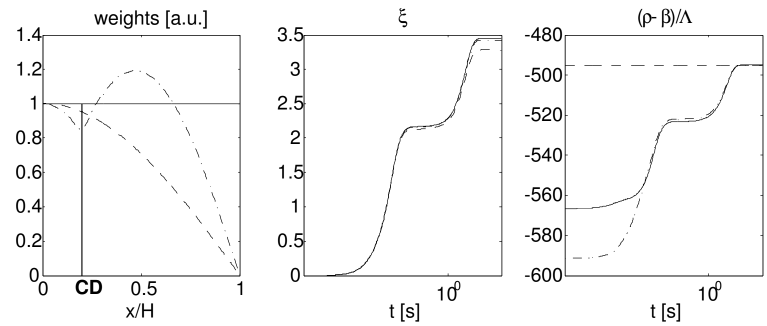

Fig. 2 Space shape of the adjoints adopted for the calculations presented in Fig. 1 (left). Evolutions of the spatiality parameter ξ (center) and of the kinetic parameter $(\rho - \beta)/\Lambda$ (right). Solid line: constant weight; dash-dotted line: initial critical adjoint; dashed line: final critical adjoint.

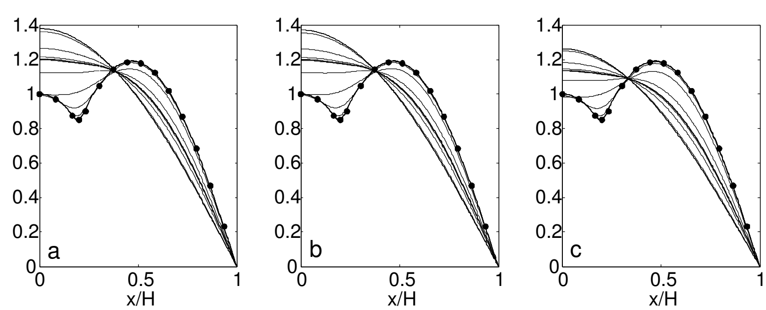

Fig. 3 Evolution during the transient of the shape for different choices of the weight; a: constant; b: initial critical; c: final critical. Dots indicate the initial flux distribution.

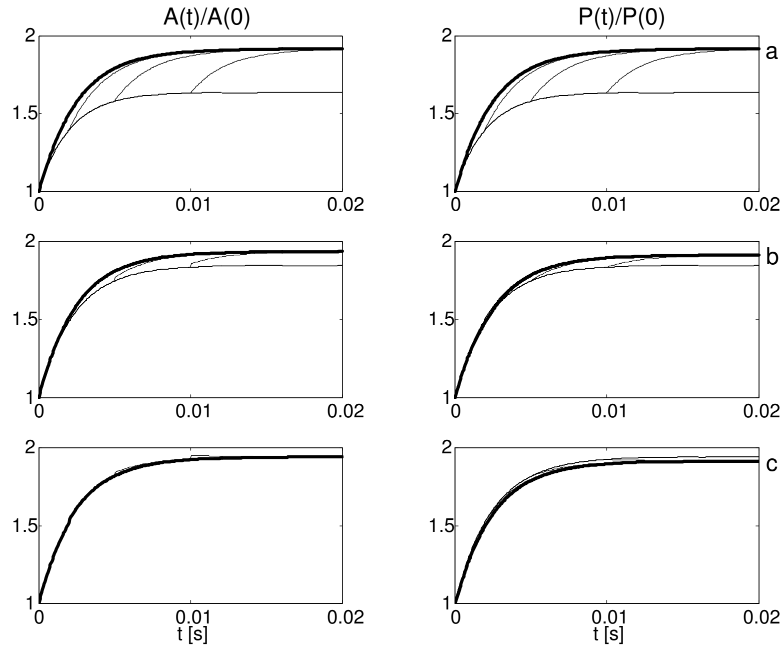

Fig. 4 Time evolution of amplitude function and power for different weigths (small system). a) constant; b) initial critical adjoint; c) final critical adjoint. The full solution is in bold and it is compared to quasi-statics with different number of shape recalculations. The transient involves a reactivity of 340 pcm.

it is possible to conclude that using as weight the initial (reference) critical adjoint is not the most appropriate choice. Reactivity is a crucial parameter to determine the evolution of the amplitude, and it can be seen that it is underestimated using the reference adjoint, while it is overestimated using the final system critical adjoint. This can be seen also in Fig. 2, where also the spatial behavior of the weights adopted and the evolution of ξ are reported. The observation of the graph for ξ shows a smaller distortion of the shape when the final adjoint is adopted. In this case it appears that the standard choice of the initial critical adjoint as weighting function is not the most effective, while the use of the final critical adjoint allows to obtain the best agreement between the exact power and the results obtained with point kinetic and quasi-static schemes. Unfortunately, in real cases the final adjoint is not always available, therefore the comparison is shown only to evidence that there is a best choice for the weighting function. Figure 3 gives a picture of the evolution of the shape in the three cases.

A smaller system ($H = 20L$) is now considered in Figs. 4 through 6. As expected, the spatial distortion due to the removal of the control device is much less important [5] even with a larger reactivity insertion. In this case the use of the initial adjoint performs better with respect to the constant adjoint. Also, the corresponding reactivity parameter undergoes a larger change in the transient.

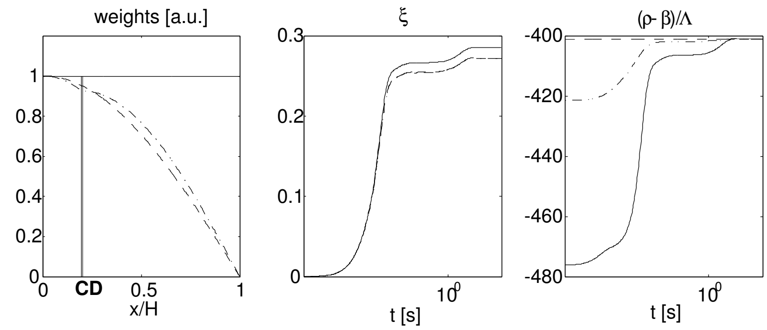

Fig. 5 Space shape of the adjoints adopted for the calculations presented in Fig. 4 (left). Evolutions of the spatiality parameter ξ (center) and of the kinetic parameter $(\rho-\beta)/\Lambda$ (right). Solid line: constant weight; dash-dotted line: initial critical adjoint; dashed line: final critical adjoint.

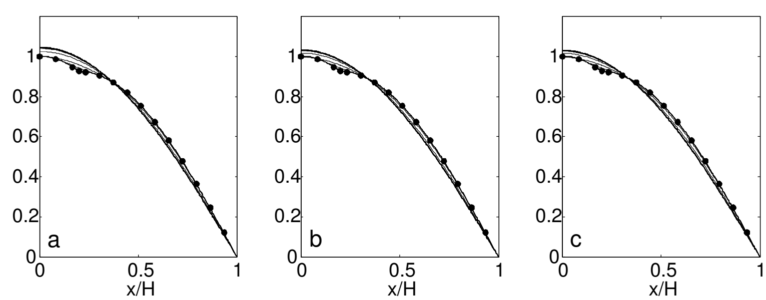

Fig. 6 Evolution during the transient of the shape for different choices of the weight; a: constant; b: initial critical; c: final critical. Dots indicate the initial flux distribution.

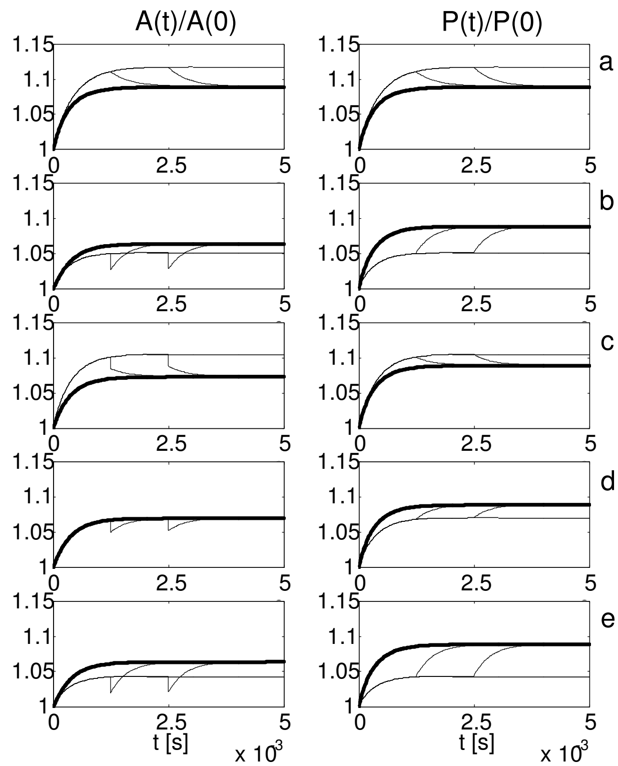

Fig. 7 Time evolution of amplitude function and power for different weigths in a sub-critical source-driven system. a) constant; b) initial critical eigenvalue adjoint; c) initial source (fission) adjoint d) final critical eigenvalue adjoint; e) final source (fission) adjoint. The full solution is in bold and it is compared to quasi-statics with different number of shape recalculations. The transient involves a reactivity of 275 pcm.

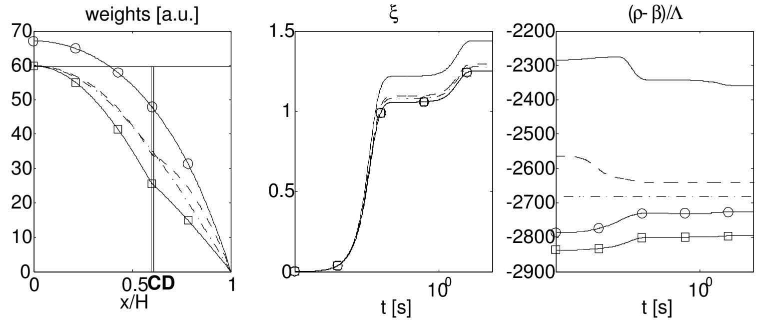

Fig. 8 Space shape of the adjoints adopted for the calculations presented in Fig. 7 (left). Evolutions of the spatiality parameter ξ (center) and of the kinetic parameter $(\rho - \beta)/\Lambda$ (right). Solid line: constant weight; squares: initial critical adjoint; dashed line: initial source-adjoint; dash-dotted line: final critical adjoint; circles: final source-adjoint.

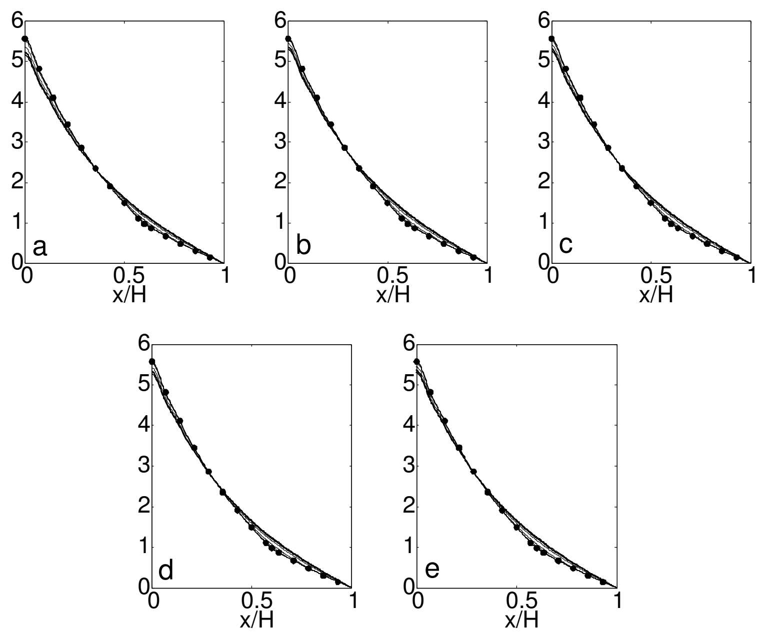

Fig. 9 Evolution during the transient of the shape for different choices of the weight; a: constant weight; b: initial critical adjoint; c: initial source-adjoint; d: final critical adjoint; e: final source-adjoint. Dots indicate the initial flux distribution.

Table 1 Relative computational effort using the full quasi-static method and the CEI modification. Computing times are normalized to the point kinetics value and refer to two reference transient calculations for a positive and a negative reactivity insertion into an accelerator-driven system.

	$\delta\rho = -1343\, pcm$		$\delta\rho = +1496\, pcm$	
number of shape recalculations	quasi-statics	CEI	quasi-statics	CEI
point kinetics	1.00	-	1.00	–
2	6.27	4.33	23.93	9.40
4	8.33	7.27	28.13	14.53
10	14.53	11.47	39.60	22.93
20	17.68	16.67	48.93	28.13

A subcritical configuration is now analyzed, Figs. 7 through 9; a localized source is assumed at the center of the slab having $20L$ half thickness. The initial k_{eff} is 0.98 and the removal of the control device leads to $k_{eff} = 0.98265$ with a corresponding reactivity insertion of 275 pcm. The first observation concerns the very little space distortion that can be induced in a strongly source-dominated system, as is clearly seen from Fig. 9. In this case, the best performance is obtained with the initial source-adjoint weight. However, this conclusion cannot be drawn for all situations, as other calculations may demonstrate.

Several changes in the scheme have been proposed to improve the performance of the quasi-static method. Some of them foresee an adjustment of the kinetic parameters between two shape updates. A possibility is to adjourn the shape a few times with a little computationally expensive explicit calculation. However, numerical instabilities are experienced in many situations. Alternatively, in this work a different philosophy has been tested. The shape is updated by an explicit step followed by an implicit calculation and a renormalization to fulfil condition (6) (Coupled Explicit-Implicit, CEI). Calculations for reference transients concerning positive and negative reactivity insertions in an accelerator-driven systems have proved the method performs satisfactorily. The following Table I reports the reduction of computational times that can be attained. It is seen that in certain cases the CEI method is little rewarding (in certain cases even penalizing), owing to the fact that for each shape recalculation the full method may reach the converged solution in just one iteration when the shape has been stabilized after the perturbation, while the CEI method always uses two full calculations and this may lead to an overall larger number of spatial calculations.

4. Conclusions

The discussion and the results presented show that, while for systems evolving from a nearly-critical situation the most obvious choice of the weighting function to generate the kinetic parameters to be used during a shape step in a quasi-static framework is the reference system importance function, this may be questionable when passing to source-driven systems. It is possible to evidence transients where the most effective weighting function turns out to be either a source-driven adjoint or even simply a constant function.

The study performed on schemes to enhance the effectiveness of quasi-statics does not reach conclusive results. However, it shows that there are wide possibilities to improve the performance of the procedure, still retaining the accuracy of the results.

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