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Coarse-Mesh Discretized Low-Order Quasidiffusion Equations for Subregion Averaged Scalar Fluxes

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In this paper we develop homogenization procedure and discretization for the low-order quasidiffusion equations on coarse grids for core-level reactor calculations. The system of discretized equations of the proposed method is formulated in terms of the subregion averaged group scalar fluxes. The coarse-mesh solution is consistent with a given fine-mesh discretization of the transport equation in the sense that it preserves a set of average values of the fine-mesh transport scalar flux over subregions of coarse-mesh cells as well as the surface currents, and eigenvalue. The developed method generates numerical solution that mimics the large-scale behavior of the transport solution within assemblies.

KEYWORDS: neutron transport, coarse-mesh discretization

1. Introduction

The present computational methodologies for reactor analysis are based on full-core diffusion calculations and assembly-level transport calculations in which the many-group transport equation is solved in isolated assemblies with reflective boundary conditions on fine spatial grids. An alternative approach was recently developed [1–7]. To account for the complicated transport effects in full-core calculations, a new methodology is based on the low-order quasidiffusion (LOQD) equations [8, 9]. Assembly group data is generated by means of single-assembly transport calculations that use special albedo boundary conditions which reproduce effects of an unlike neighboring assemblies [2, 6].

In the recent papers [4, 7], we developed methods for approximating of the LOQD equations on coarse grids that preserves exactly a set of spatial Legendre moments of the fine-mesh transport solution over coarse-mesh cells (e.g., assembly or quarter assembly). The results showed significant improvement in accuracy of the coarse-mesh solution. Another way to improve the coarse-mesh solution is to consider methods that preserve average values of the scalar flux over subregions inside each coarse cell. In such case it is necessary to formulate the weighted balance equations of different forms for another set of unknown functions. This will also require a new type of assembly group data.

In this paper we present a finite element discretization scheme of the LOQD equations on coarse grids that preserves exactly average values of the fine-mesh transport scalar flux over sets of pin cells within coarse-mesh cells.

2. The LOQD Equations

We consider a few-group k-eigenvalue transport problem for 1D slab geometry with vacuum boundary conditions, $0 \le x \le X$, $g = 1, ..., M_g$. The few-group LOQD equations [8, 9] for the group scalar flux ϕ^g and current J^g are

$$\frac{d}{dx}J^g + \Sigma_t^g \phi^g = \sum_{p=1}^{M_g} \Sigma_{s,0}^{p\to g}\phi^p + \frac{1}{k}\chi^g \sum_{p=1}^{M_g} \nu_f^p \Sigma_f^p \phi^p \ , \tag{1}$$

$$\frac{d}{dx}\left(E^g\phi^g\right)+\Sigma_t^g J^g=0\ , \tag{2}$$

$$J^g(0)=C_L^g\phi^g(0)\ ,\quad J^g(X)=C_R^g\phi^g(X)\ . \tag{3}$$

The functionals E^g, C_L^g and C_R^g are calculated by means of the few-group transport solution

$$E^g=\int_{-1}^{1}\mu^2\psi^g d\mu\Big/\int_{-1}^{1}\psi^g d\mu\ ,\ C_L^g=\int_{-1}^{0}\mu\psi^g d\mu\Big/\int_{-1}^{0}\psi^g d\mu\Big|_{x=0}\ ,\ C_R^g=\int_{0}^{1}\mu\psi^g d\mu\Big/\int_{0}^{1}\psi^g d\mu\Big|_{x=X}\ , \tag{4}$$

where ψ^g is the group angular flux. The LOQD problem (1)-(3) exactly reproduces the transport scalar flux and current provided that the functionals are exact.

3. The Discretization Procedure

In general the procedure of derivation of discretized equations can be described in abstract form in terms of projections operators. Let us consider an equation in the following form:

$$L\varphi=S\varphi\ . \tag{5}$$

Here L and S are operators of the equation, and φ is the solution. The procedure of discretization of Eq. (5) consists of several major steps that require certain choices to be made. These steps are the following:

1. To define basis functions g_k for the expansion of the solution φ (in case of finite element methods)
$$\varphi=\sum_{k=0}^{K}c_k g_k. \tag{6}$$

2. To define a set of projection operators $\mathcal{P}_n$, $n=1,..,N$ for the equation (5) to get
$$\mathcal{L}_n\varphi=\mathcal{S}_n\varphi\ ,\quad \text{where}\quad \mathcal{L}_n=\mathcal{P}_nL\ ,\quad \mathcal{S}_n=\mathcal{P}_nS\ . \tag{7}$$

3. To specify a set of projection operators $\mathcal{F}_m$, $m=1,..,N$ for the solution φ to define $\varphi_m=\mathcal{F}_m\varphi$, for which the final equations will be formulated.

4. To define homogenized cross cross section data and transform the equations (7) into equations for φ_m
$$\tilde{\mathcal{L}}_n\tilde{\varphi}=\tilde{\mathcal{S}}_n\tilde{\varphi}\ ,\quad \text{where } \tilde{\varphi}=\{\varphi_m, m=1,..,N\}\ , \tag{8}$$
$\tilde{\mathcal{L}}_n$ and $\tilde{\mathcal{S}}_n$ are the operators of the final form of the equations. Then, the expansion (5) is substituted in Eq. (8) to get a set of algebraic equations for the expansion coefficients c_k.

For the neutron diffusion equation, there is a family of coarse-mesh discretization methods that is based on the expansion of the solution in polynomials [10]. The balance equation is used as the basis equation (Eq.(5)). The same projection operators are used for both equations and solution ($\mathcal{P}_n=\mathcal{F}_n$) that are defined as spatial polynomial moments over coarse-mesh cells. The resulting equations are formulated in terms of spatial polynomial moments of the scalar flux. To derive the final set of equations (Eq. (8)), the homogenized cross sections are specified as cross sections averaged over assembly with the fine-mesh scalar flux as an averaging function. Note that such definition of averaged cross sections does not permit to transform equivalently higher polynomial

moments of reaction rates. In terms of moments of the transport scalar flux, the numerical solution of such method is able to preserve only the average value of the fine-mesh transport scalar flux over each coarse cell because the higher moments of the balance equation in the resulting method are not consistent with the corresponding moments of the fine-mesh balance equation. There are two reasons of such inconsistency. First, the discretization of the spatial polynomial moments of the divergence of the current. Second, the definition of assembly-homogenized cross sections and the transformation of the spatial polynomial moments of each type of reaction rates.

Similar discretization method for coarse grids was developed for the LOQD equations [4, 7]. To preserve high-order spatial Legendre moments of the fine-mesh transport scalar flux, a strongly consistent discretization method was developed. The main idea behind this method is to apply the described above steps in discretized form to the discretized balance equation (i.e. in discrete space). This is similar to the approach of the algebraically consistent discretization that, for example, is used to discretize transport acceleration methods [11, 12].

To derive a method that preserves average values of the fine-mesh transport scalar flux over subregions of coarse cells, we use a different projection operator for the solution ($\mathcal{F}_n$) so that the final form of the equation (Eq. (8)) will be formulated in terms of the desired subregion averaged values of the scalar flux. These projection operators are defined as a set of zeroth moments of the scalar flux over subregions of coarse cells. This method provides also a different way for eliminating the second source of inconsistency, in which we change the structure of the equations and formulate them for the moments of the scalar flux that are always positive.

4. Coarse-Mesh Method for Discretization of the LOQD Equations

4.1 Basic Elements of the Method

Let us define the following coarse mesh $\{x_{j-1/2},\ j = 1, ..., N_j + 1\}$. The LOQD equations are approximated by means of a coarse-mesh finite-element (CMFE) method based on the following expansion of the coarse-mesh scalar flux (*Step 1*)

$$\Phi_j^g(x) = \sum_{l=0}^{2}(2l+1)\varphi_j^{(l),g}P_l(\zeta_j(x)) + \ \varphi_j^{(3),g}\sinh(\varkappa_j^g(x-x_j)) + \varphi_j^{(4),g}\cosh(\varkappa_j^g(x-x_j)) \quad (9)$$

$$\zeta_j(x) = 2(x-x_j)/H_j\,, \quad H_j = x_{j+1/2} - x_{j-1/2}\,, \quad x_j = 0.5(x_{j+1/2} + x_{j-1/2})\,,$$

where P_l are the Legendre polynomials,

$$\varkappa_j^g = \sqrt{(\langle\Sigma_t\rangle_j^g - \langle\Sigma_{s,0}\rangle_j^{g\to g})\langle\Sigma_t\rangle_j^g \Big/ \langle E\rangle_j^g}\,. \quad (10)$$

Note that we use brackets $\langle\bullet\rangle$ for quantities spatially averaged over coarse cells and defined as

$$\langle\mathcal{A}\rangle_j^g = \int_{x_{j-1/2}}^{x_{j+1/2}}\mathcal{A}^g(x)\phi^{g,fm}(x)dx \Bigg/ \int_{x_{j-1/2}}^{x_{j+1/2}}\phi^{g,fm}(x)dx\,, \quad (11)$$

where $\phi^{g,fm}$ is the fine-mesh transport scalar flux. We integrate the balance equation (1) over jth coarse cell ($x_{j-1/2} \le x \le x_{j+1/2}$) with Legendre polynomials $P_l(\zeta_j(x))$ ($l = 0, 1, 2$) as a weight function (*Step 2*) to get weighted balance equations. Then, we define the group scalar fluxes averaged over subregions inside each coarse-mesh cell (*Step 3*)

$$\hat{\Phi}_{j,m}^g = \frac{1}{\Delta\hat{x}_{j,m}}\int_{\hat{x}_{j,m}}^{\hat{x}_{j,m+1}}\phi^g(x)dx\,, \quad m = 1, 2, 3\,, \quad (12)$$

where $\Delta\hat{x}_{j,m} = \hat{x}_{j,m+1} - \hat{x}_{j,m}$ and $\{\hat{x}_{j,m} : x_{j-1/2} = \hat{x}_{j,1} < \hat{x}_{j,2} < \hat{x}_{j,3} < \hat{x}_{j,4} = x_{j+1/2}\}$ specify subregions inside jth coarse-mesh cell. Next we derive the weighted balance equations in terms of the coarse-cell edge currents and subregion averaged scalar fluxes (*Step 4*). To obtain equations for $\hat{\Phi}^g_{j,m}$, we define the subregion averaged spatial polynomial moments of cross sections

$$\langle\!\langle \Sigma \rangle\!\rangle^{g(l)}_{j,m} = \int_{\hat{x}_{j,m}}^{\hat{x}_{j,m+1}} P_l(\zeta_j(x))\Sigma^g(x)\phi^{g,fm}(x)dx \Big/ \int_{\hat{x}_{j,m}}^{\hat{x}_{j,m+1}} \phi^{g,fm}(x)dx\,, \tag{13}$$

$m = 1, 2, 3$, and equivalently transform the Legendre moments of reaction rates in the following way:

$$\int_{x_{j-1/2}}^{x_{j+1/2}} P_l(\zeta_j(x))\Sigma^g(x)\phi^{g,fm}(x)dx = \sum_{m=1}^{3} \langle\!\langle \Sigma \rangle\!\rangle^{g(l)}_{j,m} \hat{\Phi}^g_{j,m} \Delta\hat{x}_{j,m}\,. \tag{14}$$

The equation (2) is approximated at edges of the jth coarse cell

$$\{E\}^{g,\pm}_j \left.\frac{d\Phi^g_j}{dx}\right|_{x=x_{j\pm1/2}} + \left\{\frac{dE}{dx}\right\}^{g,\pm}_j \Phi^g_j(x_{j\pm1/2}) + \{\Sigma_t\}^{g,\pm}_j J^g_{j\pm1/2} = 0\,, \tag{15}$$

where the coefficients of (15) are calculated by means of data averaged over boundary pin cells. Such quantities are denoted by $\{\bullet\}$.

4.2 Consistent Weighted Balance Equations

To develop a method that preserves the subregion averaged scalar fluxes

$$\tilde{\phi}^{g,fm}_{j,m} = \frac{1}{\Delta\tilde{x}_{j,m}} \sum_{i\in\tilde{\omega}_{j,m}} \phi^{g,fm}_i h_i\,, \quad m = 1, 2, 3\,, \tag{16}$$

the described above derivation of the proposed method is performed in discrete form. Here $\tilde{\omega}_{j,m}$ is a set of indices of fine-mesh cells that belong to the mth subregion of the jth coarse cell. After equivalent manipulations, we get the weighted balance equations given by

$$J^g_{j+1/2} - J^g_{j-1/2} + \langle\Sigma_t\rangle^g_j \sum_{m=1}^{3} \tilde{\Phi}^g_{j,m}\Delta\tilde{x}_{j,m} = \sum_{p=1}^{M_g} \left(\langle\Sigma_{s,0}\rangle^{p\to g}_j + \frac{1}{k}\langle\chi\nu_f\Sigma_f\rangle^{p,g}_j\right) \sum_{m=1}^{3} \tilde{\Phi}^p_{j,m}\Delta\tilde{x}_{j,m}\,, \tag{17}$$

$$\begin{aligned} &J^g_{j+1/2} + J^g_{j-1/2} + \frac{2}{\langle\Sigma_t\rangle^g_j H_j}\left(\{E\}^{g,+}_j \Phi^g_j(x_{j+1/2}) - \{E\}^{g,-}_j \Phi^g_j(x_{j-1/2})\right) \\ &\qquad + \sum_{m=1}^{3} \left(\langle\!\langle \Sigma_t \rangle\!\rangle^{(1)g}_{j,m} + \bar{\alpha}^{(1),g}_j\right) \tilde{\Phi}^g_{j,m}\Delta\tilde{x}_{j,m} \\ &= \sum_{p=1}^{M_g}\sum_{m=1}^{3} \langle\!\langle \Sigma_{s,0} \rangle\!\rangle^{(1)p\to g}_{j,m} \tilde{\Phi}^p_{j,m}\Delta\tilde{x}_{j,m} + \frac{1}{k}\sum_{p=1}^{M_g}\sum_{m=1}^{3} \langle\!\langle \chi\nu_f\Sigma_f \rangle\!\rangle^{(1)p,g}_{j,m} \tilde{\Phi}^p_{j,m}\Delta\tilde{x}_{j,m}\,, \end{aligned} \tag{18}$$

$$\begin{aligned} &J^g_{j+1/2} - J^g_{j-1/2} + \frac{6}{\langle\Sigma_t\rangle^g_j H_j}\left(\{E\}^{g,+}_j \Phi^g_j(x_{j+1/2}) + \{E\}^{g,-}_j \Phi^g_j(x_{j-1/2}) - \frac{2}{H_j}\sum_{m=1}^{3} \langle\!\langle E \rangle\!\rangle^g_{j,m} \tilde{\Phi}^g_{j,m}\Delta\tilde{x}_{j,m}\right) \\ &\qquad + \sum_{m=1}^{3} \left(\langle\!\langle \Sigma_t \rangle\!\rangle^{(2)g}_{j,m} + \bar{\alpha}^{(2),g}_j\right) \tilde{\Phi}^g_{j,m}\Delta\tilde{x}_{j,m} \end{aligned} \tag{19}$$

$$= \sum_{p=1}^{M_g} \sum_{m=1}^{3} \langle\!\langle \Sigma_{s,0} \rangle\!\rangle_{j,m}^{(2)p\to g} \tilde{\Phi}_{j,m}^{p} \Delta\tilde{x}_{j,m} + \frac{1}{k} \sum_{p=1}^{M_g} \sum_{m=1}^{3} \langle\!\langle \chi\nu_f\Sigma_f \rangle\!\rangle_{j,m}^{(2)p,g} \tilde{\Phi}_{j,m}^{p} \Delta\tilde{x}_{j,m} \,.$$

Here we defined special compensation factors $\bar{\alpha}_j^{(l),g}$ to discretize consistently the spatial polynomial moments of the divergence of the current

$$\bar{\alpha}_j^{(1),g} = \left[\sum_{i\in\omega_j} \bar{\mathcal{P}}_i^{(1)j} \left(J_{i+1/2}^{g,fm} - J_{i-1/2}^{g,fm} \right) - J_h^{g,fm}(x_{j+1/2}) - J_h^{g,fm}(x_{j-1/2}) \right.$$

$$\left. - \frac{2}{\langle\Sigma_t\rangle_j^g H_j} \left(\{E\}_j^{g,+} \frac{\phi_h^{g,fm}(x_{j+1/2})}{\bar{G}_j^{g,+}} - \{E\}_j^{g,-} \frac{\phi_h^{g,fm}(x_{j-1/2})}{\bar{G}_j^{g,-}} \right) \right] \Bigg/ \sum_{i\in\omega_j} \phi_i^{g,fm} h_i \,, \tag{20}$$

$$\bar{\alpha}_j^{(2),g} = \left[\sum_{i\in\omega_j} \bar{\mathcal{P}}_i^{(2)j} \left(J_{i+1/2}^{g,fm} - J_{i-1/2}^{g,fm} \right) - J_h^{g,fm}(x_{j+1/2}) + J_h^{g,fm}(x_{j-1/2}) \right.$$

$$- \frac{6}{\langle\Sigma_t\rangle_j^g H_j} \left(\{E\}_j^{g,+} \frac{\phi_h^{g,fm}(x_{j+1/2})}{\bar{G}_j^{g,+}} + \{E\}_j^{g,-} \frac{\phi_h^{g,fm}(x_{j-1/2})}{\bar{G}_j^{g,-}} \right. \tag{21}$$

$$\left. \left. - \frac{2}{H_j} \sum_{m=1}^{3} \langle\!\langle E \rangle\!\rangle_{j,m}^{g} \tilde{\phi}_{j,m}^{g,fm} \Delta\tilde{x}_{j,m} \right) \right] \Bigg/ \sum_{i\in\omega_j} \phi_i^{g,fm} h_i \,,$$

where ω_j is a set of indices of fine-mesh cells that belong to the jth coarse cell, $G_j^{g,\pm}$ are the discontinuity factors.

4.3 Discontinuity Factors

The resulting system of this method consists of the balance equation (17), two weighted balance equations (18) and (19), two cell-edge equations (15), and the discontinuity conditions

$$\bar{G}_j^{g,+} \Phi_j^g(x_{j+1/2}) = \bar{G}_{j+1}^{g,-} \Phi_{j+1}^g(x_{j+1/2}) \,. \tag{22}$$

The discontinuity factors are defined as

$$\bar{G}_j^{g,\pm} = \phi_h^{g,fm}(x_{j\pm1/2}) \Big/ \bar{\Phi}_j^g(x_{j\pm1/2}) \,, \tag{23}$$

where an auxiliary function $\bar{\Phi}_j^g(x)$ is calculated for the jth coarse cell from equations (15) with the given fine-mesh transport currents at the edges of this coarse cell

$$\{E\}_j^{g,\pm} \left. \frac{d\bar{\Phi}_j^g}{dx} \right|_{x=x_{j\pm1/2}} + \left\{ \frac{dE}{dx} \right\}_j^{g,\pm} \bar{\Phi}_j^g(x_{j\pm1/2}) = -\{\Sigma_t\}_j^{g,\pm} J_h^{g,fm}(x_{j\pm1/2}) \,, \tag{24}$$

and the conditions

$$\frac{1}{\Delta\tilde{x}_{j,m}} \int_{\tilde{x}_{j,m}}^{\tilde{x}_{j,m+1}} \bar{\Phi}^g(x) dx = \tilde{\phi}_{j,m}^{g,fm} \,, \quad m = 1,2,3 \,. \tag{25}$$

Note that $\bar{\Phi}_j^g(x)$ is approximated by means of the same expansion as the solution $\Phi_j^g(x)$ (Eq. (9)).

4.4 Consistent Coarse-Mesh Solution

The coarse-mesh discretized low-order QD equations (15), (17)-(19), and (22 with discontinuity factors defined by (23) are consistent with the given transport discretization method that generates the reference fine-mesh transport solution $\phi_h^{g,fm}$ and $J_h^{g,fm}$ in the sense that the coarse-mesh solution $\Phi_j^g(x)$ preserves the average value of the fine-mesh scalar flux and reaction rates over each coarse-mesh cell, values of the fine-mesh scalar flux averaged over subregions of coarse intervals, fine-mesh currents at edges of coarse cells, and the fine-mesh k-eigenvalue.

5. Numerical Results

We present numerical results of the Kord Smith's test problem in 1D slab geometry with two energy groups [7]. In these tests, model uranium and MOX assemblies are used. There are two half-assemblies next to each other with reflective boundary conditions on the outside. A MOX half-assembly is on the left and a uranium half-assembly is on the right. A fuel pin cell is 1.25 cm wide with fuel pin (0.625 cm) located in its center and surrounded by water. Test 1 and 2 consists of assemblies with the same type of fuel pins. Test 3 and 4 differ from Test 2 by design of MOX assemblies that have spatial variation in enrichment near the interface with the uranium assembly. The detailed definition of the tests is given in [7]. The most difficult issue in this type of computational problems is to mimic the large scale behavior of the thermal scalar flux in MOX assembly.

The fine-mesh transport solutions were calculated by the QD method using a second order finite-volume scheme for the LOQD equations and step characteristic method for the transport equation to calculate the QD functionals [15]. The fine spatial mesh consists of 128 equal cells. The angular mesh has 10 equal intervals. In all test problems, the multiplication factor equals 1.5. The coarse mesh consists of one cell per half-assembly.

Tables 1 presents the relative difference in pin-cell average values of the fine-mesh transport solution and coarse-mesh LOQD solution calculated by the CMFE method. In these calculations we used the CMFE method that was formulated for the group scalar fluxes averaged over three subregions, namely, over two pin cells next to each coarse-cell edge and the rest central area (4 pin cells wide) that is located between these two subregions. We denote this submesh as [2&4&2][2&4&2], where the left and right sets of numbers indicate the subregions in MOX and uranium assemblies, respectively. The results of Tests 1 and 2 demonstrate that the proposed method generates very accurate coarse-mesh solutions, if the assemblies consist of the same type of fuel pins. The results of calculations of Tests 3 and 4 indicate that the CMFE method is also accurate in case of spatial variation of fuel enrichment. The presented results show that the numerical solution of the proposed method captures complicated large-scale behavior of the transport solution within coarse cells (i.e. assemblies). Table 2 demonstrates the results of Test 4 calculated using different combinations of subregions within coarse cells. The configuration of subregions [2&4&2][2&4&2] provides the best accuracy minimizing errors across MOX assembly.

6. Conclusions

We have developed the high-order coarse-mesh finite-element method for discretization of the low-order quasidiffusion equations that is consistent with the given fine-mesh transport differencing method in the sense that it preserves the fine-mesh values of cell-average scalar fluxes, cell-edge currents, multiplication factor, reaction rates, as well as extra several average values of the fine-mesh transport scalar flux over subregions of each coarse cell.

The proposed finite-element method is based on the equations for the scalar fluxes averaged over subregions of coarse cells. The definition of the subregion averaged spatial moments of cross sections enabled us to transform exactly the Legendre spatial moments of the reaction rates. The

Table 1: Relative errors in pin-cell average values for the consistent CMFE method

		Test 1		Test 2		Test 3		Test 4	
Pin	Fuel	g=1	g=2	g=1	g=2	g=1	g=2	g=1	g=2
1	MOX	$-2.4\cdot10^{-4}$	$6.7\cdot10^{-4}$	$-3.4\cdot10^{-4}$	$-8.1\cdot10^{-5}$	$3.9\cdot10^{-4}$	$-2.4\cdot10^{-2}$	$6.4\cdot10^{-4}$	$-1.6\cdot10^{-2}$
2	MOX	$2.4\cdot10^{-4}$	$-6.7\cdot10^{-4}$	$3.4\cdot10^{-4}$	$8.1\cdot10^{-5}$	$-3.9\cdot10^{-4}$	$2.4\cdot10^{-2}$	$-6.4\cdot10^{-4}$	$1.6\cdot10^{-2}$
3	MOX	$5.6\cdot10^{-4}$	$-1.7\cdot10^{-3}$	$7.9\cdot10^{-4}$	$1.6\cdot10^{-4}$	$-1.3\cdot10^{-3}$	$5.5\cdot10^{-2}$	$-1.8\cdot10^{-3}$	$3.7\cdot10^{-2}$
4	MOX	$3.6\cdot10^{-4}$	$-1.3\cdot10^{-3}$	$4.8\cdot10^{-4}$	$1.5\cdot10^{-4}$	$-1.6\cdot10^{-3}$	$4.3\cdot10^{-2}$	$-1.7\cdot10^{-3}$	$2.6\cdot10^{-2}$
5	MOX	$-2.3\cdot10^{-4}$	$4.2\cdot10^{-4}$	$-3.5\cdot10^{-4}$	$6.2\cdot10^{-5}$	$-3.0\cdot10^{-4}$	$-1.2\cdot10^{-2}$	$1.4\cdot10^{-4}$	$-1.2\cdot10^{-2}$
6	MOX	$-6.9\cdot10^{-4}$	$2.4\cdot10^{-3}$	$-9.2\cdot10^{-4}$	$-3.5\cdot10^{-4}$	$3.1\cdot10^{-3}$	$-7.5\cdot10^{-2}$	$3.3\cdot10^{-3}$	$-4.2\cdot10^{-2}$
7	MOX	$-4.0\cdot10^{-4}$	$1.2\cdot10^{-3}$	$-4.2\cdot10^{-4}$	$-1.5\cdot10^{-3}$	$7.5\cdot10^{-3}$	$-6.7\cdot10^{-2}$	$1.4\cdot10^{-3}$	$-1.5\cdot10^{-2}$
8	MOX	$4.0\cdot10^{-4}$	$-9.9\cdot10^{-4}$	$4.2\cdot10^{-4}$	$1.2\cdot10^{-3}$	$-7.8\cdot10^{-3}$	$4.9\cdot10^{-2}$	$-1.4\cdot10^{-3}$	$1.2\cdot10^{-2}$
1	UO_2	$2.4\cdot10^{-4}$	$2.5\cdot10^{-3}$	$4.1\cdot10^{-4}$	$5.3\cdot10^{-5}$	$4.5\cdot10^{-5}$	$3.4\cdot10^{-4}$	$1.8\cdot10^{-4}$	$1.4\cdot10^{-4}$
2	UO_2	$-2.4\cdot10^{-4}$	$-2.2\cdot10^{-3}$	$-4.2\cdot10^{-4}$	$-4.6\cdot10^{-5}$	$-4.6\cdot10^{-5}$	$-3.2\cdot10^{-4}$	$-1.9\cdot10^{-4}$	$-1.3\cdot10^{-4}$
3	UO_2	$-2.2\cdot10^{-5}$	$-2.6\cdot10^{-3}$	$-6.1\cdot10^{-5}$	$-6.1\cdot10^{-4}$	$1.0\cdot10^{-4}$	$-3.6\cdot10^{-4}$	$3.5\cdot10^{-5}$	$-4.0\cdot10^{-4}$
4	UO_2	$7.2\cdot10^{-5}$	$-5.9\cdot10^{-4}$	$1.1\cdot10^{-4}$	$-2.3\cdot10^{-4}$	$7.1\cdot10^{-5}$	$-8.4\cdot10^{-5}$	$8.4\cdot10^{-5}$	$-1.3\cdot10^{-4}$
5	UO_2	$7.9\cdot10^{-6}$	$1.3\cdot10^{-3}$	$2.6\cdot10^{-5}$	$3.0\cdot10^{-4}$	$-5.7\cdot10^{-5}$	$1.8\cdot10^{-4}$	$-2.3\cdot10^{-5}$	$2.0\cdot10^{-4}$
6	UO_2	$-5.8\cdot10^{-5}$	$1.7\cdot10^{-3}$	$-7.9\cdot10^{-5}$	$4.8\cdot10^{-4}$	$-1.2\cdot10^{-4}$	$2.5\cdot10^{-4}$	$-9.7\cdot10^{-5}$	$3.1\cdot10^{-4}$
7	UO_2	$-2.8\cdot10^{-5}$	$7.2\cdot10^{-4}$	$-3.9\cdot10^{-5}$	$2.1\cdot10^{-4}$	$-4.4\cdot10^{-5}$	$9.9\cdot10^{-5}$	$-4.0\cdot10^{-5}$	$1.3\cdot10^{-4}$
8	UO_2	$2.8\cdot10^{-5}$	$-7.1\cdot10^{-4}$	$3.9\cdot10^{-5}$	$-2.1\cdot10^{-4}$	$4.4\cdot10^{-5}$	$-9.9\cdot10^{-5}$	$4.0\cdot10^{-5}$	$-1.3\cdot10^{-4}$

Table 2: Relative errors in pin-cell average values for Test 4 in case of different subregions

		[5&2&1][1&2&5]		[4&2&2][2&2&4]		[3&3&2][2&3&3]		[2&4&2][2&4&2]	
Pin	Fuel	g=1	g=2	g=1	g=2	g=1	g=2	g=1	g=2
1	MOX	$8.3\cdot10^{-3}$	$-1.3\cdot10^{-1}$	$3.8\cdot10^{-3}$	$-7.8\cdot10^{-2}$	$1.8\cdot10^{-3}$	$-4.2\cdot10^{-2}$	$6.4\cdot10^{-4}$	$-1.6\cdot10^{-2}$
2	MOX	$3.5\cdot10^{-3}$	$-4.3\cdot10^{-2}$	$1.1\cdot10^{-3}$	$-1.5\cdot10^{-2}$	$-8.0\cdot10^{-6}$	$3.5\cdot10^{-3}$	$-6.4\cdot10^{-4}$	$1.6\cdot10^{-2}$
3	MOX	$-2.0\cdot10^{-3}$	$4.3\cdot10^{-2}$	$-1.8\cdot10^{-3}$	$3.9\cdot10^{-2}$	$-1.8\cdot10^{-3}$	$3.8\cdot10^{-2}$	$-1.8\cdot10^{-3}$	$3.7\cdot10^{-2}$
4	MOX	$-5.2\cdot10^{-3}$	$7.6\cdot10^{-2}$	$-3.1\cdot10^{-3}$	$5.1\cdot10^{-2}$	$-2.2\cdot10^{-3}$	$3.6\cdot10^{-2}$	$-1.7\cdot10^{-3}$	$2.6\cdot10^{-2}$
5	MOX	$-4.6\cdot10^{-3}$	$5.1\cdot10^{-2}$	$-1.8\cdot10^{-3}$	$2.0\cdot10^{-2}$	$-5.6\cdot10^{-4}$	$1.4\cdot10^{-3}$	$1.4\cdot10^{-4}$	$-1.2\cdot10^{-2}$
6	MOX	$-3.1\cdot10^{-4}$	$3.3\cdot10^{-3}$	$1.8\cdot10^{-3}$	$-1.9\cdot10^{-2}$	$2.8\cdot10^{-3}$	$-3.3\cdot10^{-2}$	$3.3\cdot10^{-3}$	$-4.2\cdot10^{-2}$
7	MOX	$3.1\cdot10^{-4}$	$-2.7\cdot10^{-3}$	$8.9\cdot10^{-4}$	$-8.0\cdot10^{-3}$	$1.2\cdot10^{-3}$	$-1.2\cdot10^{-2}$	$1.4\cdot10^{-3}$	$-1.5\cdot10^{-2}$
8	MOX	0	0	$-9.0\cdot10^{-4}$	$6.5\cdot10^{-3}$	$-1.2\cdot10^{-3}$	$9.8\cdot10^{-3}$	$-1.4\cdot10^{-3}$	$1.2\cdot10^{-2}$
1	UO_2	0	0	$2.0\cdot10^{-4}$	$5.0\cdot10^{-5}$	$1.9\cdot10^{-4}$	$1.0\cdot10^{-4}$	$1.8\cdot10^{-4}$	$1.4\cdot10^{-4}$
2	UO_2	$-2.0\cdot10^{-4}$	$1.2\cdot10^{-5}$	$-2.0\cdot10^{-4}$	$-4.5\cdot10^{-5}$	$-1.9\cdot10^{-4}$	$-9.3\cdot10^{-5}$	$-1.9\cdot10^{-4}$	$-1.3\cdot10^{-4}$
3	UO_2	$2.0\cdot10^{-4}$	$-1.1\cdot10^{-5}$	$-1.7\cdot10^{-5}$	$-1.6\cdot10^{-4}$	$4.8\cdot10^{-6}$	$-3.0\cdot10^{-4}$	$3.5\cdot10^{-5}$	$-4.0\cdot10^{-4}$
4	UO_2	$3.6\cdot10^{-4}$	$3.4\cdot10^{-4}$	$1.8\cdot10^{-5}$	$1.6\cdot10^{-4}$	$4.6\cdot10^{-5}$	$-8.7\cdot10^{-6}$	$8.4\cdot10^{-5}$	$-1.3\cdot10^{-4}$
5	UO_2	$2.1\cdot10^{-4}$	$5.4\cdot10^{-4}$	$-7.2\cdot10^{-5}$	$4.1\cdot10^{-4}$	$-5.1\cdot10^{-5}$	$2.9\cdot10^{-4}$	$-2.3\cdot10^{-5}$	$2.0\cdot10^{-4}$
6	UO_2	$-2.8\cdot10^{-5}$	$3.3\cdot10^{-4}$	$-9.8\cdot10^{-5}$	$3.2\cdot10^{-4}$	$-9.8\cdot10^{-5}$	$3.1\cdot10^{-4}$	$-9.7\cdot10^{-5}$	$3.1\cdot10^{-4}$
7	UO_2	$-2.1\cdot10^{-4}$	$-2.8\cdot10^{-4}$	$2.1\cdot10^{-5}$	$-1.2\cdot10^{-4}$	$-4.9\cdot10^{-6}$	$2.1\cdot10^{-5}$	$-4.0\cdot10^{-5}$	$1.3\cdot10^{-4}$
8	UO_2	$-3.3\cdot10^{-4}$	$-9.0\cdot10^{-4}$	$1.5\cdot10^{-4}$	$-6.0\cdot10^{-4}$	$1.0\cdot10^{-4}$	$-3.3\cdot10^{-4}$	$4.0\cdot10^{-5}$	$-1.3\cdot10^{-4}$

resulting method generates accurate coarse-mesh numerical solution. This method uses efficiently the information about the shape of cross sections inside each cell by means of Legendre spatial moments of cross sections without explicit approximation of this shape by interpolation functions as it is done in the current methods for core-level calculations. The proposed methodology gives an option of developing a method with adaptive features that allows one to preserve average scalar flux over desired subregions to minimize errors in certain domains and improve the accuracy of the coarse-mesh numerical solution. It can be especially important in multidimensional geometry.

In the proposed method, the assembly-averaged cross sections, quasidiffusion functionals, consistency terms, discontinuity factors, etc. are generated by means of fine-mesh transport solutions that account for neighboring assemblies. The necessary transport solution is available from single-assembly calculations with special ablbedo boundary conditions [2, 6]. However, in realistic calculations the assembly group data are arranged in form of tables, and necessary quantities are determined by means of interpolation of tabulated data. A similar consistent coarse-mesh discretization method demonstrated the stability of the solution to variation in cross sections and functionals [7], and it gives a reason to apply the proposed homogenization procedure for core-level reactor physics calculations.

The presented method can be extended to multidimensional geometries, multigroup case, finite-element methods based on higher order expansions of the coarse-mesh scalar flux that creates an option of preserving average values of the fine-mesh transport scalar flux over even more subregions within coarse cells. The elements of the proposed methodology can be also used for improving reactor physics methods based on the diffusion equations and for efficient spatial collapsing of discretized diffusion equations on coarse meshes in multidimensional problems to perform calculation of a set of grids.

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