LINEAR SURFACE CHARACTERISTIC SCHEME FOR THE NEUTRON TRANSPORT EQUATION IN UNSTRUCTURED GEOMETRIES

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ABSTRACT

In this paper we present a new numerical scheme for the Method Of Characteristic (MOC) in unstructured geometries for the neutron transport equation. The MOC has become a familiar tool for transport calculations in reactor physics [1], and its use will probably increase in the future. One of the major drawbacks of the MOC is the difficulty to implement higher-order integration schemes to improve spatial convergence. We present here a higher-order schemes for the MOC. We define a conservative and linear characteristic scheme based on linear interpolation on the surface's values of the collisions sources. We have called it Conservative Linear Surface (CLS) scheme in contrast to a preceding non conservative version. Results comparisons of the well-known Stepanek benchmark show CLS faster convergence over the standard step characteristic scheme. A generalization of the synthetic DP_N acceleration scheme provides an efficient method to accelerate the internal transport iterations.

KEYWORDS: Method of characteristics, Higher order schemes

1. INTRODUCTION

The method of characteristic in unstructured meshes (MOC) has become a familiar tool for transport calculations in reactor physics [1]. This method uses numerical trajectories to construct an iterative solution of the transport equation based on the exact cell-balance and propagation equations. The drawback of the method is the difficulty of implementing positive high-order source approximations in arbitrarily shaped regions. The use of polynomial flux expansions does not yield a positive scheme and, therefore, robust methods are based on the flat-flux approximation. Exponential interpolation can be used to restore positivity [3], to the prize of introducing a non-linearity that requires extra tabulations (in regular meshes) or Newton iterations (in unstructured meshes). Moreover, the exponential approximation does not respect the basic linearity of the transport equation (i.e., a linear combination of solutions must be a solution) and, therefore, poses problems in its utilization in multigroup calculations.

In this paper we present a new MOC conservative linear surface (CLS) scheme that has a linear spatial representation, and that is a generalization of a preceding Linear Surface (LS) method [7] that was not conservative. The central idea is to replace the flat source region approximation with a linear interpolation between surfaces values. Volume quantities are computed using balance equations, but this does not assure the conservativeness of the method, even if the method is positive. To force conservation we apply a linear rebalancing technique, by allowing the spatial flux representation to be discontinuous at cell interfaces. Finally we do not alter the linear nature of the problem and preserve conservation. Unfortunately the scheme is not granted to be positive anymore, even if the eventuality of negative fluxes is relegated to far unconverged meshes. We have also generalized the ASA acceleration technique, developed for the flat source scheme, to accelerate the LS internal iterations. A Fourier analysis for heterogeneous slabs, not presented in this paper, proved that this new acceleration is stable and efficient.

2. THE STEP CHARACTERISTICS SCHEME (SC)

The MOC for unstructured meshes applies the discrete ordinate approximation to obtain a numerical iterative solution of the one group transport equation in a geometrical domain *D* of boundary ∂D:

$$\left.\begin{aligned} &\left(\Omega\cdot\nabla+\Sigma\right)\psi^{(n)}=\frac{\sigma_s}{4\pi}\vec{A}\left(\Omega\right)\vec{\phi}^{(n-1)}+q_{ext}, \quad x\in X,\\ &\psi^{(n)}=\beta\psi^{(n-1)}+\psi_0, \qquad x\in\partial_{-}X, \end{aligned}\right\} \tag{1}$$

where *n* denotes the iteration index, x=(r,Ω) stands for a generic point in phase space $X=\left\{x;r\in D,\Omega\in(4\pi)\right\}$, Σ is the total cross section and

$$q=H\psi+S \tag{2}$$

is the emission density, where $\left(H\psi\right)\left(x\right)=\int\limits_{(4\pi)}d\Omega'\Sigma_s\left(r,\Omega\cdot\Omega'\right)\psi\left(r,\Omega'\right)$ is the scattering operator and S the external source.

In the method of characteristics the discrete ordinates scheme is used for the angular approximation. [5] In the classic Step Characteristics (SC) method the geometrical domain *D* is decomposed into a set of homogeneous regions $\left\{D_i,i=1,N_{reg}\right\}$ on which we use the flat source approximation. As it is shown for example in [2] we can give the following representation to the collision term:

$$q\left(x\right)\sim\sum_i\theta_i\left(r\right)q_i\left(\Omega\right)=\sum_i\theta_i\left(r\right)\vec{A}\left(\Omega\right)\cdot\vec{q}_i\left(\Omega\right),\ x\in X, \tag{3}$$

where θ_i is the characteristics function of homogeneous region D_i and

$$\vec{q}_i=\Sigma_{si}\vec{\phi}_i+\vec{S}_i \tag{4}$$

is the average value of the emission density in region *i*. In a multigroup setting, the external source accounts for fissions and scattering from the other groups.

A set of parallel numerical trajectories is constructed for each angular direction in the S_N angular quadrature formula and an iterative solution of the transport equation is then obtained from boundary flux values by direct integration along each numerical trajectory. In the SC method, the integral transport equation can be directly integrated, thanks to the flat source approximation, to give:

$$\psi_{out,i}^{(n)}(i) = e^{-\Sigma_i R_i(t)}\psi_{in,i}^{(n)}(t) + \underbrace{\frac{1-e^{-\Sigma_i R_i(t)}}{\Sigma_i}}_{\beta(R_i)} q_i^{(n-1)}(\Omega), \tag{5}$$

where $R_i(t)$ is the length of the trajectory within the region i and Ω is the direction of trajectory t. This equation is used to compute iteratively the angular flux across the cells along each trajectory.

The updating of the flux moments is made via the cell mean angular fluxes that are defined by the cell balance equation:

$$\Sigma_i \psi_i^{(n)}(\Omega_n) = q_i^{(n-1)}(\Omega_n) - \frac{1}{V_i}\sum_{t\|\Omega, t\cap i} w_\perp(t)\left[\psi_{out,i}^{(n)}(t) - \psi_{in,i}^{(n)}(t)\right]. \tag{6}$$

Here V_i is the volume of the cell and the sum in t is done for all trajectories with direction Ω that intersect cell i. The $w_\perp$ is the spatial integration weight, geometrically representing the area affected to the trajectory. We avoid, in this brief discussion, to discuss the treatment of boundary conditions that can be found in [6].

3. THE PROJECTOR OPERATOR "P"

To introduce the numerical approximation we have studied, we would like to introduce a projector operator "P" that is at the heart of our methodology. This operator interpolates a generic function $\phi \in C^0(D), \phi \in C^\infty(D_i) \quad \forall i \in \{1,..,n\}$. That is to say a continue function on the entire domain and whose derivatives are also continue inside each homogeneous sub regions. "*P*" is defined from the following three integral quantities:

$$\phi_\alpha = \frac{1}{4\pi S_\alpha}\int_{(4\pi)} d\Omega \int_{S_\alpha} dS\, \psi \quad , \tag{7}$$

$$\bar{\phi}_i^G = M(\phi) = \sum_\alpha \Gamma_\alpha \phi_\alpha = \frac{1}{4\pi V}\int_{(4\pi)} d\Omega \int_{S_\alpha} dS_\perp\, l\, \frac{\phi^+ + \phi^-}{2}, \tag{8a}$$

$$\bar{\phi}_i^C = \frac{1}{4\pi V_i}\int_{(4\pi)} d\Omega \int_{V_i} dV\, \psi\,. \tag{8b}$$

In the equation (8a) $dS_\perp = dS\left|\Omega n\right|$ and "*l*" is the length of the chord intersected over region "*i*" by the trajectory passing on a point over the surface *S* and having direction $\boldsymbol{\Omega}$. The quantities in (7,8a,b) are respectively the flux surface average, the flux volume average in hypothesis of linear variation inside the region and the real volume average. The suffixes "G"

and "C" are to denote the important difference between the two types of volume averages. The interpolation is done over each trajectory *t* crossing cell *i* in direction Ω so that we write:

$$\tilde{\phi}(r) = \left(\phi(0)\frac{x}{l} + \phi(l)\frac{l-x}{l} \right) \tag{9}$$

where *x* is the distance along the trajectory and *l* the chord length in region *i*. This interpolation is done via the "parametric" presence of **Ω** that is not a variable the flux depends on. This presence will turn to be useful when the scalar flux appear inside direction integral in characteristic neutron equation. Even if the expression (9) does not appear to be completely orthodox, and in fact flux values do not necessarily coincide in an arbitrary point for different angles **Ω**, we think it is an acceptable numerical approximation. It is easy to see that (4) respects both average surface value and conservative volume average.
As first step we apply the projection (7) to (1a) to obtain the following expression for the average value of the surface flux moments:

$$\phi_\alpha^\rho = \frac{1}{4\pi S} \int_{(4\pi)} d\Omega A^\rho(\Omega) \int dS_\perp \psi(l,\Omega). \tag{1b}$$

Moreover this last expression can be used to define a surface collision source as:

$$\vec{q}_\alpha^{\pm} = \Sigma_{si,\pm}\vec{\phi}_\alpha + \vec{S}_\alpha, \tag{10}$$

that can be directly used inside the integral transport equation.

4. THE LS SCHEME

We recall here the basics of the LS scheme [7], including some recent improvements with respect to the previous version. This scheme is based on a linear interpolation between cell boundary values of the collision source. To perform this integration one needs the value of the sources within each region. For a trajectory *t* crossing cell *i* in direction Ω we write:

$$q_t(x,\Omega) = q_t(R_i(t),\Omega)\frac{x}{R_i(t)} + q_t(0,\Omega)(1-\frac{x}{R_i(t)}). \tag{11}$$

where $R_i(t)$ is the length of the trajectory within the region *i* and Ω is the direction of trajectory *t*. It is worth noting that expression (11) gives rise to a non consistent numerical approximation, in the sense that the same emission source approximated with two different angles does not have the same value in a given point. We have supposed that this does not prevent good convergence behavior. Replacing (11) into the integral transport equation gives:

$$\psi_{out,i}^{(n)}(t) = e^{-\Sigma_i R_i(t)}\psi_{in,i}^{(n)}(t) + \underbrace{\frac{1-e^{-\sigma R_i(t)}}{\Sigma_i}}_{\beta} q_{t,i}^{(n-1)}(0,\Omega) + \underbrace{\frac{R_i(t)-\beta}{\Sigma_i R_i(t)}}_{\beta_1(l)} \left(q_{t,i}^{(n-1)}(R_i(t),\Omega) - q_{t,i}^{(n-1)}(0,\Omega) \right), \tag{12}$$

where the symbol β has been implicitly defined in (6). One can also calculate the average flux over a chord length:

$$\bar{\psi}^{(n)}(t) = \frac{1}{R_i(t)} \int_0^{R_i(t)} ds\,\psi(s)$$
$$= \beta\,\psi_{in,i}^{(n)}(t) + \frac{R_i(t)-\beta}{\Sigma_i} q_{t,i}^{(n-1)}(0,\Omega) + \frac{q_{t,i}^{(n-1)}(R_i(t),\Omega) - q_{t,i}^{(n-1)}(0,\Omega)}{\Sigma_i}[R_i(t)/2 - \beta_1(R_i(t))]. \quad (13)$$

The average chord flux in (13) can be used to express the average angular flux in region *i* by adding the contributions of all trajectories intersecting this region with angle Ω:

$$\Sigma_i V_i \bar{\psi}_{i,C}^{(n)}(\Omega_n) = \sum_{t\|\Omega, t\cap i} w_\perp(t)\Sigma_i R_i(t)\bar{\psi}_i^{(n)}(t) = \sum_{t\|\Omega, t\cap i} w_\perp(t)\times$$
$$[R_i(t)\frac{q^{(n-1)}(R_i(t),\Omega) + q^{(n-1)}(0,\Omega)}{2} + \psi_{in,i}^{(n)}(t) - \psi_{out,i}^{(n)}(t)] = \sum_{\alpha\in i}\left(J_\alpha^- - J_\alpha^+\right)^{(n)}(\Omega) + V_i \bar{Q}_{i,G}^{(n-1)}(\Omega) \quad (14a)$$

the sum in α is done over all currents entering region *i* and we have used the following definitions of partial currents

$$J_\alpha^\pm(\Omega) = \sum_{t\|\Omega, t\cap\alpha} w_\perp(t)\,\psi_{\pm,i}^{(n)}(t) \sim \int_\alpha dS\left|\Omega^\pm\cdot n\right|\psi\left(r,\Omega^\pm\right), \quad (15)$$

where the "+" and "-" stand for outgoing or ingoing angular directions. The following definition of the geometrical source average has also been used:

$$\bar{Q}_{i,G}^{(n-1)}(\Omega) = \frac{1}{4\pi V_i}\int_{(4\pi)} d\Omega \int_{\partial V} dS\left|\Omega\cdot n\right| \int_{(0,l)} ds\, q(s) \approx \frac{1}{V_i}\sum_{t\|\Omega, t\cap i} w_\perp(t)\frac{q_{t,i}^{(n-1)}(l,\Omega) + q_{t,i}^{(n-1)}(0,\Omega)}{2} l = M(q_\alpha). \quad (16)$$

In practice what we need to compute are average angular moments. The average moment operator, applied for example to the vector of flux moments, is therefore:

$$\vec{Q}_{i,G}^{(n-1)} = \sum_{\alpha\in\partial V} \underbrace{\frac{1}{4\pi V_i}\int_{(2\pi)} d\Omega\,\vec{A}(\Omega)\otimes\vec{A}(\Omega)\int_\alpha dS\left|\Omega\cdot n\right| l(r_s,\Omega)\frac{s^\rho s^\upsilon + 1}{2}}_{F_\alpha^{\rho\upsilon}}\vec{q}_\alpha = M(\vec{q}_\alpha), \quad (16a)$$

where "$l(r_s,\Omega)$" is the cord intersected by the trajectory that goes through the position "r_s" and angle Ω, and the term $\frac{s^\rho s^\upsilon + 1}{2}$ takes into account the passage of the integration over the entire sphere to the integration over only half of the sphere.

We have used subscripts "G" and "C" to denote that a quantity is obtained, respectively, from geometrical averaging or directly from the balance equation. Equation (14a) looks like an angular balance equation and can be used to obtain volume average fluxes from surfaces sources. This equation is not a truly balance equation except at the limit when the mesh size goes to 0 and the geometrical mean $\bar{\phi}_{i,G}$ becomes equal to the balance value $\bar{\phi}_{i,C}$. Equation (14a) allows to obtain the average value $\bar{\psi}_{i,C}^{(n)}(\Omega_n)$ of the angular flux over volume i as it results from the integral propagation equation but this value, contrarily to the analytical case,

is non conservative. Equation (14b), even if not numerically coherent with the propagation equation, gives a conservative average value coherent with the computed currents. This matter will be further discussed in the following.

Ultimately the value of the emission density q within a region depends on the inhomogeneous sources, if any, and on the multigroup fluxes via fission and scattering. For the LS scheme we have used a cell linear source approximation obtained by linear interpolation from cell surface values (a surface being the common boundary between two adjacent regions; but it can also be defined by further subdivisions to improve convergence). In practice we compute and store the average flux moments,

$$\phi_\alpha^{kl} = \frac{1}{4\pi S_\alpha}\int_{(\alpha)} dS \int_{(4\pi)} d\Omega A_{kl}(\Omega)\psi(\mathbf{r},\Omega) \sim \frac{1}{S_\alpha}\sum_{\substack{\omega_n \in S_N \\ t\cap S_\alpha}} \omega_n \underbrace{\frac{\omega_\perp(t)}{|\Omega\cdot n|}}_{\delta S^t} A_k(\Omega_n)\psi_\alpha^t(\Omega_n), \quad (17)$$

for each surface α and angular direction $\boldsymbol{\Omega}$. Here A_{kl} is the spherical harmonic of order (kl) and S_α is the area of α, and δS^t is the measure of the actual geometrical surface intersected by trajectory t. Formula (14) shows that all integrals are numerically evaluated using the angular fluxes along numerical trajectories. These fluxes are obtained by applying formula (9), during the transport sweep, and they are used to update the surface source that can be written as:

$$\vec{q}_\alpha^{\pm} = \Sigma_{si,\pm}\vec{\phi}_\alpha + \vec{S}_{\alpha,\pm}, \quad (18)$$

where this source is computed for each cell because the cross sections are cell dependent.

In conclusion, the LS algorithm proceeds as follows:

1. Start with a guess surface source.
2. Make a transport sweep, calculate for each surface the moments of the flux with (14). Compute the current term in the pseudo-balance equation (11), and update volume fluxes.
3. Test convergence on volume and surface fluxes and eventually return to 1

We end this section with a discussion on the non-conservative nature of the LS algorithm. The "conservation' equation enforced in the LS scheme reads:

$$\Sigma_i V_i \overline{\phi}_{i,C}^{(n)} = \sum_{\alpha\in i}\left(J_\alpha^- - J_\alpha^+\right)^{(n)} + V_i\left[\Sigma_{s,i}\overline{\phi}_{i,G}^{(n-1)} + Q_{ext,i}\right], \quad (19)$$

whereas the true conservation equation requires the two average fluxes, $\overline{\phi}_{i,C}$ and $\overline{\phi}_{i,G}$, to be the same.

Even at convergence of the iteration we are not granted that the "conservative" cell-average flux will equal the "geometrically" averaged one appearing in (19). What we have here is a solution where the sources are not coherent with the averaged values of fluxes. In a fully conservative scheme $\overline{\phi}_{i,C}^{(n)}, \overline{\phi}_{i,G}^{(n)}$ should converge to the same quantity, and this would satisfy the balance equation (19). However, even if the LS scheme is not conservative, it exhibits other interesting properties. First of all, since it uses a linear cell approximation for the sources, it is expected to be a high-order scheme. Secondly, the LS scheme is positive.

This property is easy to see since the LS interpolation of the source is strictly positive when obtained by linear interpolation of positive surface sources.. The later sources are positive because they are computed from surface angular fluxes that are positive.

4. LINEAR CONSERVATIVE SCHEME

In order to force the conservation into the linear surface scheme we give a slight modified definition of the emission source:

$$\vec{q}_{\alpha}^{\pm} = \Sigma_{si,\pm}\vec{\phi}_{\alpha,\pm} + \vec{S}_{\alpha,\pm}, \tag{16b}$$

where this time the surface flux has been defined per side. Equation (16a) gives rise to a linear discontinuous spatial representation of the source. The new degree of freedom will be used to force conservation. Actually we will define the side surface flux as:

$$\vec{\phi}_{\alpha,\pm} = \vec{\phi}_{\alpha} + \vec{\phi}_{t(\pm,\alpha),C} - \vec{\phi}_{t(\pm,\alpha),G}, \tag{20}$$

where the symbol $t(\pm,\alpha)$ denotes the region that is on the positive/negative side of the surface. In the equation (16a) $\vec{\phi}_{\alpha}$ denotes the surface flux (17), while $\vec{\phi}_{t(\pm,\alpha),C}$ comes from (14b) and $\vec{\phi}_{t(\pm,\alpha),G} = M\left(\vec{\phi}_{\alpha}\right)$. It is easy to see that, thanks to (16b) applied to the flux, the geometrical average of the side surface fluxes is equal to the conservative value:

$$M_i(\vec{\phi}_{\alpha,\pm}) = \vec{\phi}_{i,C}, \tag{21}$$

for every region "i".

So finally the algorithm proceeds by computing the three quantities at the right of equation (20), by assuming a given source emission density per surface and then it computes a new set of flux surfaces moments through (20) and updates the sources by (16b).

The drawback of forcing conservation is that now the scheme is not positive anymore, since the corrective difference in (20) can lead to negative surface values, and also to negative volume average values. This last eventuality is nevertheless limited to cases where the spatial approximations are quite crude. We will comment further this point by looking at some explicit calculation.

5. DP_N SYNTHETIC ACCELERATION OF THE LS SCHEME

The free iterations of the LS scheme can be accelerated with a synthetic acceleration based on uniform DP_N surface angular fluxes [1]. To ensure numerical stability the LS scheme is carried over into the acceleration equations and all matrix coefficients are evaluated with the numerical trajectories. Since the idea of the DP_N-type accelerations has been described elsewhere [2] for the standard SC scheme, we will give here only a brief description. The basic assumption is to suppose that the flux entering region *i* from a given surface β is given by the expression:

$$\psi_{\beta}(r,\Omega) \sim \vec{A}(\Omega)\delta\left(\Omega,\beta^{+}\right)\vec{\psi}_{\beta^{+}} + \vec{A}(\Omega)\delta\left(\Omega,\beta^{-}\right)\vec{\psi}_{\beta^{-}}, \tag{23}$$

where notation

$$\delta\left(\Omega,\alpha^{\pm}\right)=\begin{cases}=1, if \text{ current } \alpha^{\pm} \text{ has the same direction as } \Omega,\\ =0 \text{ otherwise,}\end{cases}$$

has been used. This assumption permits to express the average flux moments on the surface α as:

$$\vec{\phi}_{\alpha}=\frac{1}{4\pi S_{\alpha}}\int_{\alpha}dS\int_{(2\pi+)}d\Omega\,\vec{A}\otimes\vec{A}\,\vec{\psi}_{\alpha^{+}}+\frac{1}{4\pi S_{\alpha}}\int_{\alpha}dS\int_{(2\pi-)}d\Omega\,\vec{A}\otimes\vec{A}\,\vec{\psi}_{\alpha^{-}}=A_{V}^{\alpha}\,\vec{\psi}_{\alpha}, \quad (24)$$

where a DP_N geometrical averaging operator A_V has been defined. The DP_N problem is posed for the current defined and this quantity is obtained from the integral transport equation:

$$\hat{J}_{\alpha}^{+}=\frac{1}{\pi S_{\alpha}}\int_{S_{\alpha}}dS\int_{(2\pi+)}d\Omega\left|\Omega\cdot n\right|\psi\left(r,\Omega\right)=A_{\alpha^{+}}\vec{\psi}_{\alpha^{+}}=\sum_{\beta\in i}\left(T_{\alpha\beta}\vec{\psi}_{\beta^{-}}+E_{\alpha\beta}q_{\beta}\right), \quad (25)$$

where the following definitions for the transmission probabilities have been used:

$$\begin{aligned}T_{\alpha\beta}&=\int_{S_{\alpha}}dS\int_{(\alpha\leftarrow\beta)}d\Omega\left|\Omega\cdot n\right|\vec{A}\otimes\vec{A}\,e^{-\Sigma_{t}l} \quad , \quad A=\int_{S_{\alpha}}dS\int_{\left(2\pi^{+}\right)}d\Omega\,\vec{A}\otimes\vec{A}\\ E_{\alpha\beta}&=\int_{S_{\alpha}}dS\int_{(\alpha\leftarrow\beta)}d\Omega\left|\Omega\cdot n\right|\vec{A}\otimes\vec{A}\left(\beta-\beta_{1}\right)+\delta_{\alpha\beta}\int_{S_{\alpha}}dS\int_{\left(2\pi^{+}\right)}d\Omega\left|\Omega\cdot n\right|\vec{A}\otimes\vec{A}\,\beta_{1},\end{aligned} \quad (26)$$

where all angular modes indices have been neglected. All these quantities are calculated with the same numerical tracking used for the transport sweep in order to assure complete coherence and to grant stability. The transmission and escape factors of equations (26) respect also the following reciprocity relations, that are enforced into the numerical scheme:

$$T_{\alpha\beta}^{\rho\upsilon}=T_{\alpha\beta}^{\upsilon\rho}=s^{\rho}s^{\upsilon}T_{\alpha\beta}^{\rho\upsilon}\ , \quad E_{\alpha\beta}^{\rho\upsilon}=E_{\alpha\beta}^{\upsilon\rho}=s^{\rho}s^{\upsilon}E_{\beta\alpha}^{\rho\upsilon}.$$

The system of equations (25) is coupled with a system of equations for flux moments, where one has to take into account, as shown by (21), that fluxes have two volume components (geometrical and conservative averages) and two surface components. By applying the DP_N hypothesis one can write the balance equation (14a), as:

$$V_{i}\vec{\phi}_{i,C}=\left\{\sum_{\alpha\in i}\vec{A}_{\alpha,-}\left(\vec{\psi}_{\alpha,-}-s^{\upsilon}s^{\rho}\vec{\psi}_{\alpha,+}\right)+\vec{q}_{i,G}V_{i}\right\}/\Sigma_{i}V_{i} \quad (27)$$

while the geometrical average operator takes the form::

$$\vec{\phi}_{i,G}=F_{i}A_{V}\vec{\psi}_{i}, \quad (28)$$

where definitions (24) and (16a) have been used, and where $\vec{\psi}_{i}$ represents all DP_N flux components relative to region i. With the definitions (24), (27) and (28) one can re-write (21) in the DP_N framework as:

$$\vec{\phi}_{\alpha,-}=(I-F)A_{V}^{\alpha}\vec{\psi}_{\alpha}+c_{i}F\phi_{i}^{-}+\sum_{\beta\in i}\vec{A}_{\alpha,-}\left(\vec{\psi}_{\beta^{-}}-s^{\rho}s^{\upsilon}\vec{\psi}_{\beta^{+}}\right)+\frac{F_{i}q_{ext}^{-}}{\Sigma_{i}}, \quad (29)$$

where ϕ_{i}^{-} is the vector of all DP_N components entering region i. A suitable rearrangement of eqs. (29) and (25) allows us to obtain a response-matrix form that is the same we use for the Step Characteristics scheme. We will explain all these details in future works.

6. RESULTS FOR THE STEPANEK BENCHMARK

To test the performance of the new LS and CLS schemes we have considered the one-group source version of the Stepanek benchmark [3]. The geometry of this benchmark is that of a small reactor core comprising four central regions and an external moderator with vacuum boundary conditions. Isotropic and uniform sources are located in the first and third regions.

Because of the large flux gradients, this is a very challenging benchmark for spatial discretization schemes. Table 1 gives our results for the average fluxes in media from 1 to 5 and for the total leakage, which is the quantity more difficult to get. As for the LS and the CLS, a subdivision of each surface into 4 sub-surfaces has been carried out in order to have a better approximation for the surface sources and, therefore, a well converged result. The results shown below do not claim to be reference results. All calculations have been done with the same angular quadrature formula (a product formula with 12 uniform horizontal angles between 0 and π, and 2 Gauss-Legendre vertical angles) and the same transverse spatial integration parameter δ=0.01 cm. To full understand the meaning of these parameters and evaluate their incidence on convergence we refer to [6]. What is important here is that none of these parameters can affect the convergence of the integration over a characteristic. This convergence is due only to the spatial representation of the source. Here below we will test this convergence, without asking for a fully converged result with respect to the other integration parameters.

The results in the tables confirm the faster convergence of the LS scheme over the standard SC scheme. The tables show that the n×n LS calculation is as precise as the 2n×2n SC one, which proves that for the same precision the LS scheme is faster. The best method is the CLS one. Actually, the CLS is the only method that gives converged values up to three significant figures for all quantities with a 128×128 mesh refinement. Its rate of convergence is much more satisfying than that of the other two methods and its n×n results are roughly as goods as the 4n×4n SC results.

7. CONCLUSIONS

A new spatial discretization methods has been developed for the MOC, and the synthetic ASA acceleration has been modified to work with one of these methods (LS). Our numerical tests prove that the new CLS method is efficient for reactor applications. For slab geometry the LS method is alike to the familiar linear continuous (LC) method [4] used in regular meshes. Therefore, this new method can be viewed as a generalization to non-structured meshes of the LC scheme. On the other hand, the CLS is a linear conservative method with higher-order convergence properties. In this paper we show that a suitable generalization of the DP_N technique can be adapted to the CLS scheme. We think that the new scheme will be of practical interest in reactor calculations, and we aim to extend its applications to realistic multigroup calculations.

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Table 1: Results for the Stepanek benchmark. The relative % errors are computed with respect to a reference CLS 128x128. SC = step characteristics, LS= Linear Surface, CLS= Conservative Linear Surface.

Quantity	Calculation Type	Mesh (nxn)				Reference Value (CLS128x128)
		16x16	32x32	90x90	128x128	
Flux medium 1	SC	-8.5	-4.12	-0.78	-0.23	11.94
	LS	6.7	1.9	0.4	0,16	
	CLS	0.63	0.23	0.09	0.	
Flux medium 2	SC	35.26	17.91	4.09	1.57	0.54
	LS	-4.3	-3.8	-1.8	-0,7	
	CLS	-2.74	-0.46	-0.41	0.	
Flux medium 3	SC	-14.56	-7.26	-1.49	-0.01	19.22
	LS	-13.5	-4.7	-1.2	-0,4	
	CLS	0.69	0.18	0.02	0	
Flux medium 4	SC	41.99	21.15	4.71	2.01	0.833
	LS	29.3	9.1	1.7	0,13	
	CLS	-3.14	-0.6	-0.37	0	
Flux medium 5	SC	16.88	13.82	3.94	1.88	1.527
	LS	27.9	16.6	5.6	1.2	
	CLS	-0.41	0.06	0.04	0	
Total leakage	SC	522.32	205.66	32.99	13.74	8.75
	LS	111.	55.5	18.7	3.7	
	CLS	0.49	2.2	0.5	0	