Simulation of Feynman-Alpha Measurements from SILENE Reactor Using a Discrete Ordinates Code

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Abstract

In this paper we present the simulation of Feynman-α measurements from SILENE reactor using the discrete ordinates code PANDA. A 2-D cylindrical model of SILENE reactor is designed for computer simulations. Two methods are implemented for variance to mean calculation. In the first method we used the Feynman point reactor formula where the parameters (Diven factor, reactivity, detector efficiency and alpha eigenvalue) are obtained by 2-D PANDA calculations. In the second method the time dependent adjoint equations for the first two moments are solved.

The calculated results are compared to the measurements. Both methods are in excellent agreement with the experimental data.

KEYWORDS: Neutron stochastic transport theory, Feynman-alpha method, SILENE reactor, PANDA code

1. Introduction

Neutron noise techniques like Feynman-α technique [1] are used for nuclear material identification and reactivity measurements of fissile assembly. These experiments are usually interpreted using point reactor analytical models or Monte Carlo numerical methods. However, the factorial moments of the detected neutron count number distribution are solution of coupled adjoint transport equations. For this reason, neutron noise techniques based on moment measurements can be simulated using classical deterministic methods.

In a previous paper [2] we have presented the adaptation of PANDA discrete ordinates code for count number variance calculations in order to simulate the excess of relative variance measured in Feynman-α experiments. For validation purpose we present in this paper the simulation of subcritical measurements performed on SILENE reactor [3].

In a first part we recall the two methods used to calculate the excess of relative variance: The rigorous time-dependent adjoint method and an approximation based on the point model Feynman formula. We present then the experiment and the computational methods. In the last part, calculated results and measurements of the excess of relative variance are compared.

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2. Relative Variance Calculation

2.1 Time Dependent Adjoint Method

The neutron flux φ is solution of a linear Boltzmann equation with initial and zero incoming flux boundary conditions where L is the time dependent transport operator and Q the neutron source.

$$L\varphi = Q \tag{1}$$
$$\varphi(\vec{r},\vec{v},t_0) = 0 \tag{2}$$
$$\varphi(\vec{r}_B,\vec{v},t) = 0 \quad \vec{r}_B.\vec{v} < 0 \tag{3}$$

The adjoint flux φ^+ is solution of the adjoint linear Boltzmann equation with final and zero outcoming flux boundary condition where L^+ is the adjoint time dependent transport operator and Q^+ the adjoint source.

$$L^+\varphi^+ = Q^+ \tag{4}$$
$$\varphi^+(\vec{r},\vec{v},T) = 0 \tag{5}$$
$$\varphi^+(\vec{r}_B,\vec{v},t) = 0 \quad \vec{r}_B.\vec{v} \geq 0 \tag{6}$$

The mean number of neutrons counted by a detector due to a single initial neutron is solution of an adjoint transport equation where the adjoint source is the detector response. For a capture detector σ_D in equation (8) is the capture microscopic cross section of the detector. When neutrons are counted during a ΔT time interval, $h(t-T)$ is equal to one during the time interval and zero outside.

$$\varphi^+ = \bar{c}(T/\vec{r},\vec{v},t) \tag{7}$$
$$Q_1^{\ +} = \sigma_D(\vec{r},v)h(t-T) \tag{8}$$

The second factorial moment of the count number distribution induced by a single initial neutron is also solution of an adjoint transport equation. This equation is coupled to the first moment equation by the adjoint source term $Q_{2F}^{\ +}$ which depends on the mean count number. In equation (11) χ is the fission spectrum, σ_F is the microscopic fission cross section and $\overline{\nu(\nu-1)\sigma_F} = \sum_i i(i-1)f_i$ where f_i is the probability that i neutrons are produced by one fission event.

$$\varphi^+ = \overline{c^2}(T/\vec{r},\vec{v},t) \tag{9}$$
$$Q_2^{\ +}(\bar{c}) = Q_1^{\ +} + Q_{2F}^{+}(\bar{c}) \tag{10}$$
$$Q_{2F}^{\ +}(\bar{c}) = \overline{\nu(\nu-1)\sigma_F}(\vec{r},v)\left(\int \frac{\chi(v)}{4\pi}\bar{c}(T/\vec{r},\vec{v},t)d^3v\right)^2 \tag{11}$$

In presence of a neutron source, the first moment and variance of the count number distribution are given by the inner product of the source by the single neutron first and second moments.

$$\overline{C}(T)=\int_t^T d\tau\int d^3r\int d^3v\,Q(\vec{r},\vec{v},t)\overline{c}(T/\vec{r},\vec{v},\tau)=\left\langle \overline{c},Q\right\rangle \tag{12}$$

$$V_C(T)=\overline{C^2}-\overline{C}^2=\left\langle \overline{c^2},Q\right\rangle \tag{13}$$

2.2 Feynman Formula

The mean and variance can also be expressed in term of direct flux and adjoint source using the direct-adjoint commutation relation.

$$\overline{C}=\left\langle \overline{c},Q\right\rangle=\left\langle Q_1^+,\varphi\right\rangle \tag{14}$$

$$V_C=\left\langle \overline{c^2},Q\right\rangle=\left\langle Q_2^+,\varphi\right\rangle \tag{15}$$

The relative variance is the ratio of the variance to the mean and the Feynman function Y is the excess of relative variance. For Poisson process the relative variance is equal to unity, when one consider the fission process, the neutrons born from one fission event are correlated and the relative variance is greater than unity.

$$V_R=\frac{V_C}{\overline{C}}=1+Y \tag{16}$$

$$Y=\frac{\left\langle Q_{2F}^+,\varphi\right\rangle}{\left\langle Q_1^+,\varphi\right\rangle} \tag{17}$$

In the point reactor model without delayed neutrons, the excess of relative variance is given by the Feynman formula where ε is the detector efficiency, $\rho=\frac{1-k}{k}$ is the reactivity, α is the time absorption eigenvalue (prompt neutron decay constant), $D_\nu=\frac{\overline{\nu(\nu-1)}}{\overline{\nu}^2}$ is the Diven factor and ΔT the detection time interval.

$$Y(\Delta T)=\frac{\varepsilon D_\nu}{\rho^2}\left(1-\frac{1-e^{-|\alpha|\Delta T}}{|\alpha|\Delta T}\right) \tag{18}$$

In more realistic problems where space, energy and angular dependences are taken into account it is possible as an approximation to calculate the excess of relative variance using the Feynman formula with parameters (α, k, ε, D_ν) calculated by a neutron transport code. The detector efficiency and the Diven factor are calculated as the ratio of reaction rates.

$$\varepsilon = \frac{\langle \sigma_D, \varphi \rangle}{\langle \sigma_F, \varphi \rangle} \tag{19}$$

$$D_\nu = \left\langle \frac{\overline{\nu(\nu-1)}}{\overline{\nu}^2} \right\rangle = \frac{\left\langle \frac{\overline{\nu(\nu-1)}}{\overline{\nu}^2} \sigma_F, \varphi \right\rangle}{\langle \sigma_F, \varphi \rangle} \tag{20}$$

3. SILENE Reactor and Measurement System

SILENE facility [4] is an experimental reactor operated by CEA/Valduc since 1974. Initially designed to study the phenomenology of critical accidents, SILENE is also used as a source of gamma and neutron radiation and for studies on subcritical systems.

The core of the reactor is an annular vessel with an outer diameter of 36 cm which contains a fissile solution of uranyl nitrate with a 93% U5 enriched uranium concentration of 71g/l. The reactivity of the system depends on the height of the fissile solution which was equal to 30 cm for this study. The detector is a BF3 counter located in the 7 cm diameter central cavity. Because the internal fission source is very weak an external AmBe neutron source was placed under the core.

The acquisition system registers the time arrival of each detected neutron. During the experiment 1327120 neutrons where measured during 663s. These experimental data are processed to produce the variance to mean ratio for time interval ranging from 0.1 to 100ms.

4. Description of Computational Methods

4.1 PANDA Code

The simulations were performed with PANDA, a general purpose deterministic transport code developed at CEA/Bruyères-le-Châtel. The code is based on the multigroup S_N discrete ordinates method and solves the static and time dependent direct and adjoint linear Boltzmann equations on one, two and three dimensional orthogonal structured spatial meshes. Multigroup nuclear data are produced from standard continuous energy libraries using the cross section processing system developed at CEA/Bruyères-le-Châtel.

4.2 Adjoint Calculations

For the time-dependent variance calculation we applied the following numerical solving methodology. The first two moments of the count number probability distribution induced by a single initial neutron satisfy a system of two coupled, time-dependent, adjoint inhomogeneous transport equations. The adjoint source term of the first moment equation is the detector response. The number of registered counts is equal to the number of captured neutrons in the detector (σ_D=$\sigma_{Capture}$).

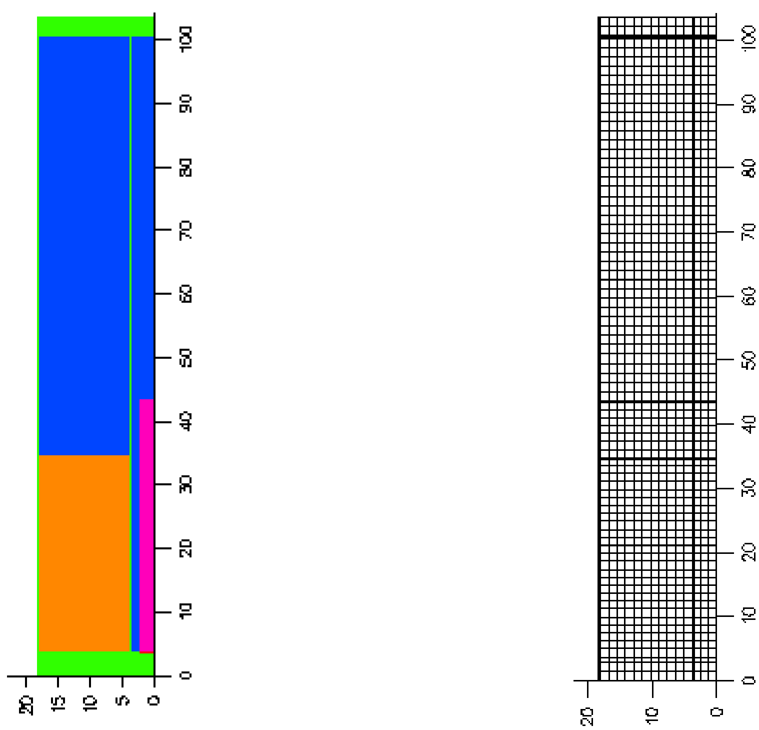
The solution starts from the terminal time. For each time step, the fist moment equation is solved and the solution is used to calculate the second moment equation adjoint source term. The mean value and variance of the count number distribution induced by a neutron source are updated by quadrature as the inner product of the single initial neutron moments with the direct neutron source term. A separate calculation is performed for each point of the Feynman curve.

4.3 SILENE Reactor Computer Model

A two-dimensional cylindrical (2D-RZ) computer model of the SILENE reactor with the BF3 detector was designed (cf. figure 1). This model was discretized on a 23x81 Cartesian mesh. The calculations were performed using an S_8 equal-weight quadrature and thermalized 30 energy group P_3 nuclear data file produced by our cross sections processing system with the ENDF/BVI continuous energy library. The time discretization scheme is implicit with an adaptive time-step ranging from 10µs to 0.5ms.

We first performed an accurate MCNP reference calculation of the effective multiplication factor. This reference calculation is then used to adjust the solution height in the discrete ordinates multigroup PANDA calculation in order to have the same K_{eff}. For SILENE computer model with a 30 cm solution the MCNP result is K_{eff}=0.95302 (σ=0.00076). About the same multiplication factor is obtained with PANDA, K_{eff}=0.95311, when the solution height is set to 31.1 cm. In the experiment the time width is lower than 100 ms, so the delayed neutrons are neglected and prompt neutron nuclear data are used.

Figure 1: SILENE reactor computer model with Uranyl nitrate solution fuel and BF3 detector computer model. The spatial Cartesian mesh for deterministic calculations (23x81 grid) is presented on the left part.



4.4 Neutron Source

Because the AmBe source is placed near the fission solution and is not in direct view of the detector the external neutron source was modeled as a fission source in the solution. We have also verified with MCNP calculations that the energy spectrum in the detector remains almost the same in both cases. It is also worth of noting that the magnitude of the source intensity is not relevant in this problem because the fluctuation excess is independent of source strength.

5. Results

The excess of relative variance is calculated by two methods. In the first one we have applied the Feynman analytic formula valid for a point model reactor.

$$Y(\Delta T) = Y_\infty \left(1 - \frac{1 - e^{-|\alpha|\Delta T}}{|\alpha|\Delta T} \right) \text{ with } Y_\infty = \frac{\varepsilon D_\nu}{\rho^2} \tag{21}$$

The parameters of the formula are determined by 2-D PANDA calculations and presented in table 1.

Table 1: PANDA results for the parameters used in the Feynman formula for the excess of relative variance.

K_{Prompt}	ρ	α	ε	D_ν	Y_∞
0.94494	0.05826	-1690.1 s^{-1}	4.098 10^{-3}	0.793	0.9573

The corresponding Feynman curve is plotted in figure 2. This plot is in excellent agreement with the experimental results.

The second method is more rigorous, it does not need point reactor model results. Each relative variance point is obtained by a time dependent adjoint PANDA calculation. These points are also in good agreement with the measurements and the Feynman curve.

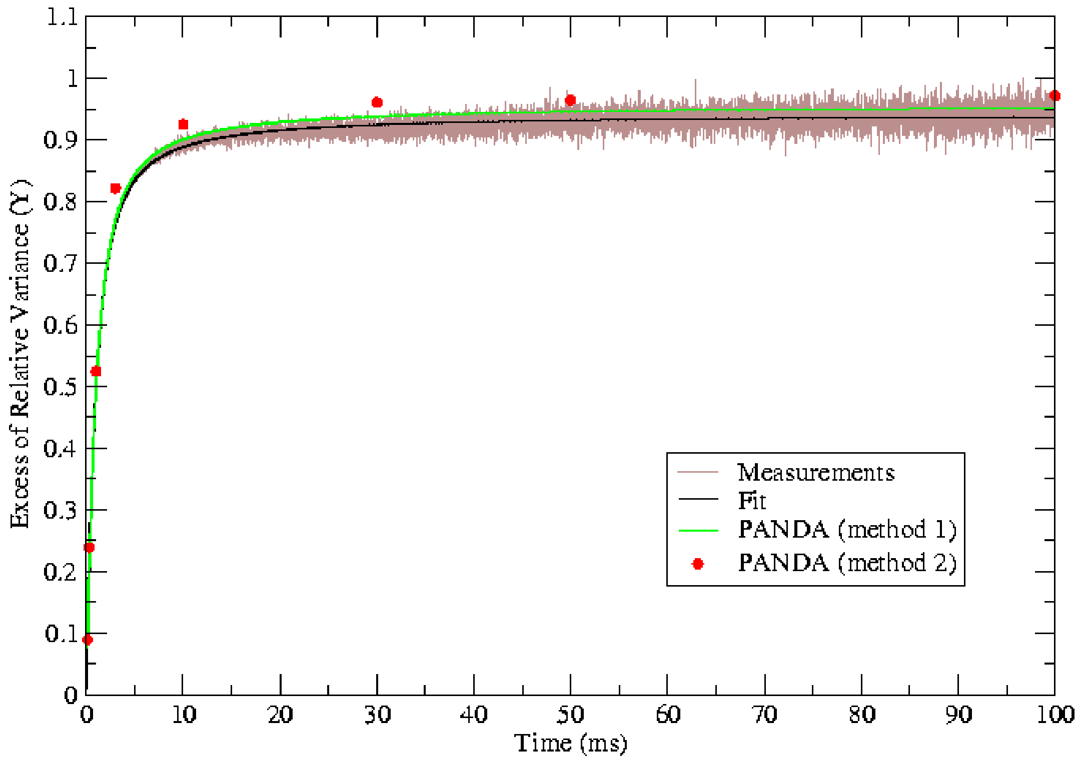
The Y_∞ and α coefficients are obtained by non-linear curve fitting of the experimental and PANDA calculation with the Feynman formula. The results are presented in table 2.

Table 2 Measured and calculated Y_∞ and α coefficients.

	Y_∞	α (s^{-1})
Measurements (fit)	0.943	-1738
Calculation, Method 1	0.957	-1690
Calculation, Method 2 (fit)	0.977	-1888

It is worth of noting that in this particular configuration where the detector is surrounded by the fissile material, the infinite medium approximation given by the Feynman formula is appropriate.

Figure 2: Excess of relative variance plot. The experimental (brown) and two methods of calculation are compared: The point model Feynman formula with parameters calculated with PANDA (green, Method 1) and the adjoint time dependent PANDA calculations (red circles, Method 2).



6. Conclusion

Feynman subcritical measurements performed on SILENE reactor were used to validate the numerical methodology of PANDA deterministic code for stochastic neutronics problems. The application of Feynman point reactor model formula with parameters determined with 2D PANDA calculations is in excellent agreement with the experimental results.

A more rigorous approach based on time dependent adjoint calculations of the first two moments coupled equations was implemented. This method is also in good agreement with the experimental results with a slight overestimation of the relative variance.

As a future work we will carry on the validation process with various subcritical level measurements on SILENE reactor. We will also improve our modelisation of the external neutron source and perform intercode validation with analog Monte-Carlo methods.

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