

## On the Analysis Method of Effective Delayed Neutron Fraction at Thermal Neutron Systems

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### Abstract

The effective delayed neutron fraction (beta-effective) was numerically analyzed with different analysis methods, and their effects on the results were investigated. The cores investigated in this study were light-water moderated low enriched UO<sub>2</sub> lattices, of which the beta-effectives had been reported. The effects of transport/diffusion calculation, energy group collapsing, and change of nuclear data library were studied.

The study showed that the diffusion calculation with coarse group cross section gave smaller beta-effectives than the transport one with fine group cross section, although the difference was not so large, about 2%. On the other hand, the change of nuclear data library from JENDL-3.3 to ENDF/B-VI.8 gave a significant difference, over than 4%. In comparisons with the experiments, it was indicated that the delayed neutron data in JENDL-3.3 are more reliable than those in ENDF/B-VI.8.

**KEYWORDS:** *Effective Delayed Neutron Fraction, Analysis Method, Thermal Neutron System, Transport Effect, Energy Collapsing Effect, Library Effect, JENDL-3.3, ENDF/B-VI.8*

## 1. Introduction

The effective delayed neutron fraction (beta-effective) plays an important role in the reactivity evaluation. It is used as a scale of the reactivity in the dk/k unit from the measured reactivity that has the dollar unit. Since the calculated reactivity is generally given in the dk/k unit, the unit of either reactivity has to be converted to the other one in order to compare the measured and calculated reactivities. Thus, the uncertainty of beta-effective directly affects the accuracy of reactivity evaluation.

Recently, experimental data on the beta-effective have been accumulated, and they are used in benchmark calculations for the verifications of delayed neutron data[1-3]. In these calculations, transport codes with cross sections of fine energy group (over than 10-20) are generally used to obtain the results with high accuracy. However, the calculations using a diffusion code with coarse energy group are still widely used in the design or kinetic analysis of the nuclear reactors. Then, it is worthwhile to understand the effects of calculation method on the calculated beta-effective. In this paper, we have investigated the following effects on the beta-effective calculation,

- 1) transport/diffusion (Transport effect),
- 2) energy group collapsing (Collapsing effect), and

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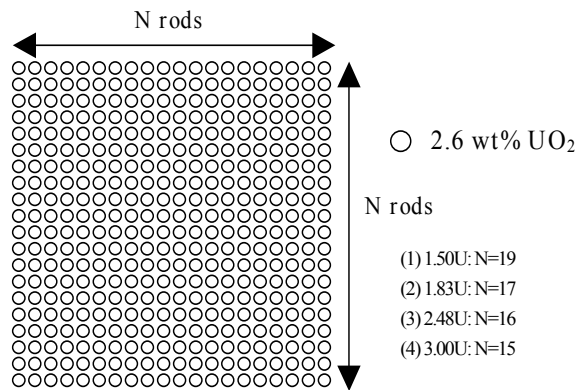
3) change of nuclear data library (Library effect).

## 2. Analysis

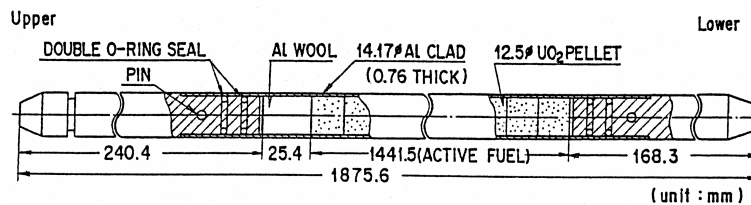
### 2.1 Investigated Cores

The cores investigated in this study were four light-water moderated low enriched  $UO_2$  cores, of which the beta-effectives had been measured[3]. These cores were constructed at the Tank-type Critical Assembly (TCA)[4] of the Japan Atomic Energy Agency (former Japan Atomic Energy Research Institute). These cores were constructed by arraying vertically 2.6wt% enriched  $UO_2$  rods as shown in Fig. 1. The  $UO_2$  rod consisted of 1.25cm diameter pellets that were clad into a 0.76 thick aluminum tube of 14.17mm outer diameter. The effective fuel length of the rod was 144.2cm. The water to fuel volume ratios for each core were 1.50, 1.83, 2.48, and 3.00, respectively. The specifications of the cores are shown in Table 1.

**Figure 1: Core configuration.**



**Figure 2: 2.6 wt% enriched  $UO_2$  fuel rod.**



**Table 1: Core specifications.**

Core name	Water to fuel volume ratio	Lattice pitch (mm)	Core size (rod)	Critical height (cm)	Measured beta-effective
1.50U	1.50	18.49	19 by 19	99.45	0.00774
1.83U	1.83	19.56	17 by 17	114.59	0.00769
2.48U	2.48	21.50	16 by 16	78.67	0.00770
3.00U	3.00	22.93	15 by 15	90.75	0.00755

### 2.2 Analysis Methods

The effective delayed neutron fraction  $\beta_{eff}$  was calculated using the forward and adjoint fluxes, which were the results of an eigenvalue problem, with Eq. (1).

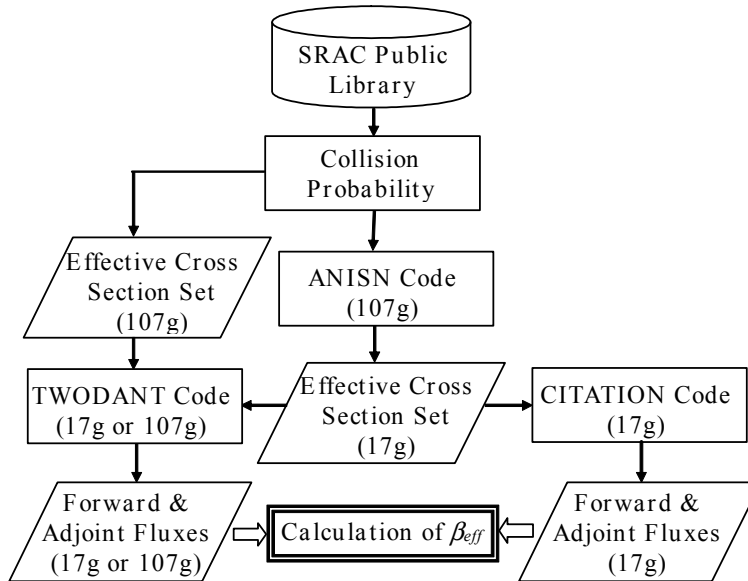
$$\beta_{eff} = \sum_i \beta_{eff,i} = \sum_i \frac{\int dV \int dE \chi_{d,i} \phi^*(\mathbf{r}, E) \int dE' \beta_{d,i} V_d \Sigma_f \phi(\mathbf{r}, E')}{k \int dV \int dE \chi_i \phi^*(\mathbf{r}, E) \int dE' \nu_i \Sigma_f \phi(\mathbf{r}, E')} \quad (1)$$

- where
- $\beta_{eff,i}$  : Effective delayed neutron fraction of the  $i$ -th group,
  - $\beta_i$  : Delayed neutron fraction of the  $i$ -th group,
  - $\chi_i$  : Fission (prompt + delayed) neutron spectrum,
  - $\chi_{d,i}$  : Delayed neutron spectrum of the  $i$ -th group,
  - $k$  : Effective multiplication factor.

The forward and adjoint fluxes were calculated by a two-dimensional transport code, TWODANT[5], or a multi-dimensional diffusion code, CITATION[6], employing an X-Y core model. The cross section set for these codes was prepared by the SRAC code system[7] with the JENDL-3.3 and ENDF/B-VI.8 libraries. In the SRAC calculation, the effective cross section sets of 107-energy group were generated by the collision probability method using 107-energy group public library. Then, this effective cross section set of 107-energy group were collapsed to 17-energy group by using the neutron spectrum calculated with the ANISN code, which is incorporated in SRAC.

Both the 107 and 17-energy group cross section sets were used in the core calculation with TWODANT. For the CITATION calculation, only 17-energy group was employed. As a result of these eigenvalue calculations, the effective neutron multiplication factor was also obtained. The flow diagram of the calculation is shown in Fig. 3. After all, the following four calculations have been conducted.

**Figure 3:** Calculation flow diagram of the effective delayed neutron fraction.



- i) 2D(X-Y) diffusion calculation with 17 energy groups (JENDL-3.3)
- ii) 2D(X-Y) transport calculation with 17 energy groups (JENDL-3.3),
- iii) 2D(X-Y) transport calculation with 107 energy groups (JENDL-3.3), and
- iv) 2D(X-Y) transport calculation with 107 energy groups (ENDF/B-VI.8).

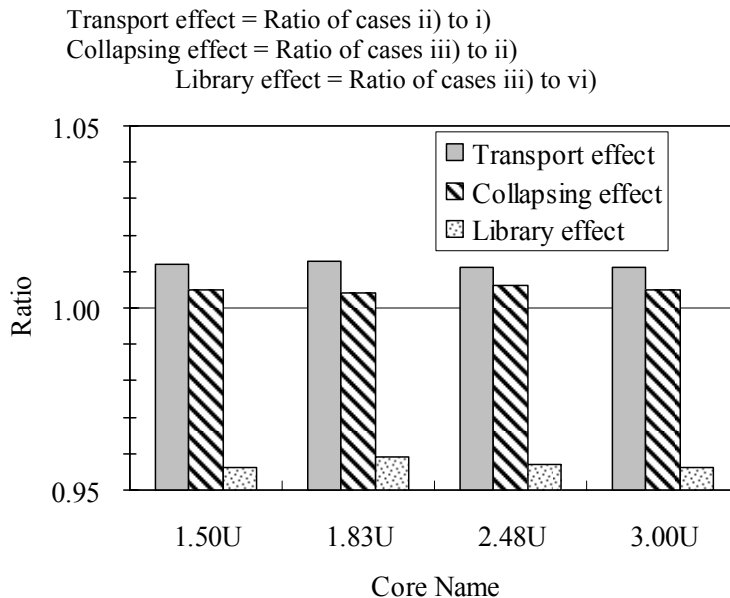
Using those calculated results, the transport effect can be obtained as a ratio of the results of ii) to i), and the collapsing effect as a ratio of iii) to ii). The library effect is a ratio of the results of iii) to iv).

In addition to those calculations, the transport calculations with 4- and 8-energy group were also performed for the 1.83U core, to investigate the collapsing effect in a little more detail.

### 3. Results and Conclusion

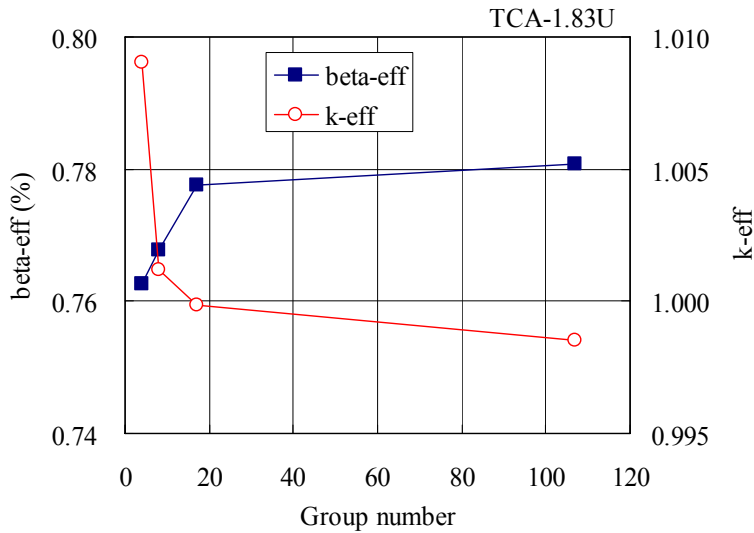
Figure 4 shows the results of investigation. In the present study, both the transport and collapsing effects are greater than unity for the all cores. This means that the diffusion calculation with coarse group cross section gives smaller beta-effective than the transport one with fine group cross section, although the difference is not so large, about 2%. On the other hand, the change of nuclear data library gives significant difference, over than 4%.

**Figure 4:** Transport, collapsing and library effects on beta-effective calculation.



The collapsing effect from 107 to 4 groups for 1.83U was investigated, and the results are shown in Fig. 5. From this figure, it is found that beta-effective increases with decreasing group number. On the other hand, the multiplication factor increases with group number. In particular, the calculation with the group number less than 10-20 gives significant change of beta-effective and multiplication factor.

**Figure 5:** Neutron energy group collapsing effect on effective multiplication factor and beta-effective



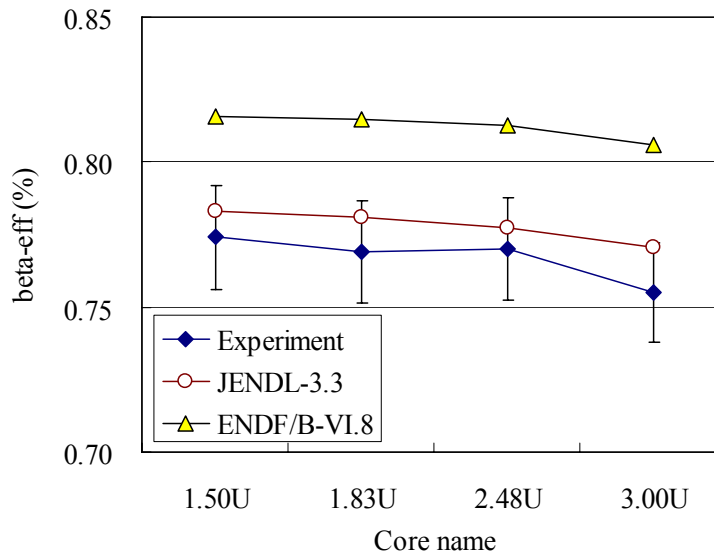
The results of 107-energy group transport calculation with JENDL-3.3 and ENDF/B-VI.8 libraries are compared with the experiments as shown in Table 2 and Fig. 6. The results with JENDL-3.3 show good agreement with experiments, while those with ENDF/B-VI.8 overestimate the experiments about 5-6%. This indicates that the delayed neutron data in JENDL-3.3 are more reliable than those in ENDF/B-VI.8.

**Table 2:** Results of beta-effective calculation.

Core name	Experiment(E)*	Calculation(C)		C/E	
		JENDL-3.3	ENDF/B-VI.8	JENDL-3.3	ENDF/B-VI.8
1.50U	0.00774	0.007828	0.008156	1.011	1.054
1.83U	0.00769	0.007809	0.008145	1.015	1.059
2.48U	0.00770	0.007776	0.008125	1.010	1.055
3.00U	0.00755	0.007707	0.008058	1.021	1.067

\* Experimental error: 2.3%

**Figure 6:** Results of beta-effective calculation



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