Estimation of 6 Groups of Effective Delayed Neutron Fraction Based on Continuous Energy Monte Carlo Method

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Abstract

New method is proposed to estimate effective fraction of delayed neutrons radiated from precursors categorized into 6 groups of decay constant. Instead of adjoint flux ϕ^*, an expected number of fission neutrons in next generations, M, is applied as a weight function [1]. Introduction of M enables us to calculate the fraction based on continuous energy Monte Carlo method. For the calculation of the fraction, an algorism is established and implemented into the MCNP-5 code. The method is verified using reactor period data obtained in reactivity measurements.

KEYWORDS: $\beta_{eff,ij}$, continuous energy Monte Carlo method, expected number of fission neutrons in next generation, reactivity, reactor period

1. Introduction

Effective fraction of delayed neutrons radiated from precursors of *j*th group generated by fission reactions of nuclide *i*, $\beta_{eff,ij}$, is indispensable to deduce reactivity of a perturbed core from its reactor period T based on an inhour equation (eq. (25)). $\beta_{eff,ij}$ is conventionally defined using adjoint flux ϕ^* of unperturbed system as follows [2].

$$\beta_{eff,ij} = \frac{\int dr^3 dE d\Omega \phi^* \left\{ \int dE' d\Omega' \chi_{d,ij} \nu_{d,ij} \sigma_{f,i} N_i \phi \right\}}{Sum_i \int dr^3 dE d\Omega \phi^* \left\{ \int dE' d\Omega' \chi_{t,i} \nu_{t,i} N_i \sigma_{f,i} \phi \right\}} , \quad (1)$$

where subscripts t, d and f mean total, delayed and fission, respectively. σ, χ, ν, and N mean microscopic cross section, fission neutron spectrum, number of neutron emission per fission and number density of nuclide, respectively. $Sum_i g$ means summation of g over all i. To calculate $\beta_{eff,ij}$ defined by eq. (1), we have to introduce approximations, such as discrete modeling of energy E, angle Ω and position r, because it is difficult to calculate ϕ^* without those approximations.

Recently, new methods have been proposed to estimate $\beta_{eff}=Sum_{ij}\,\beta_{eff,ij}$ based on continuous energy Monte Carlo method [1,3,4,5]. Among them, the authors have proposed to estimate kinetic parameters using expected number of fission neutrons in next generation, M, which converges on "importance for neutron production", instead of ϕ^* [1,6]. By the proposal, we can estimate kinetic parameters, β_{eff} and Λ, in eigenvalue calculations of MCNP-4C [7] without multi group cross sections, etc.

In this work, we proposed to estimate $\beta_{eff,ij}$ in the same manner. We implemented the functions to calculate $\beta_{eff,ij}$ into the MCNP-5 code [8]. Verification of the estimation was performed with data obtained in reactivity measurements.

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2. Theory

2.1 Point Kinetic Equation with Parameters Weighted by Arbitrary Function

The time dependent neutron transport equation is

$$\frac{1}{V}\frac{d\Psi}{dt} = -A\Psi(r,E,\Omega,t) + B_p\Psi(r,E,\Omega,t) + Sum_{ij}\frac{\chi_{d,ij}}{4\pi}C_{ij}\lambda_{ij} \;. \qquad (2)$$

Density of precursor of *j*th group delayed neutrons from fissions of nuclide *i*, $C_{ij}=C_{ij}(r,t)$, is expressed as

$$\frac{dC_{ij}(r,t)}{dt} = F_{d,ij}\Psi(r,E,\Omega,t) - C_{ij}(r,t)\lambda_{ij} \;. \qquad (3)$$

Here, V is velocity of a neutron and Ψ is time dependent angular neutron flux. λ_{ij} is the decay constant of the precursor. A, B, F are removal, neutron production, fission operators for an function f.

$$Af(r,E,\Omega) = \Omega\nabla f + Sum_i N_i\sigma_{t,i} f - Sum_i N_i\int_{E_{\min}}^{E_{\max}} dE'\int_{4\pi} d\Omega'\sigma_s(E',\Omega' \to E,\Omega) f(r,E',\Omega'), \qquad (4)$$

$$\begin{aligned} Bf(r,E,\Omega) &= Sum_i\frac{1}{4\pi}\int_{E_{\min}}^{E_{\max}} dE'\int_{4\pi} d\Omega'\chi_{t,i}(E,E')\nu_{t,i}(E')N_i\sigma_{f,i}(r,E')f(r,E',\Omega') \\ &= Sum_i\frac{1}{4\pi}\int_{E_{\min}}^{E_{\max}} dE'\int_{4\pi} d\Omega'\chi_{p,i}(E,E')\nu_{p,i}(E')N_i\sigma_{f,i}(r,E')f(r,E',\Omega') \\ &+ Sum_{ij}\frac{1}{4\pi}\int_{E_{\min}}^{E_{\max}} dE'\int_{4\pi} d\Omega'\chi_{d,ij}(E)\nu_{d,ij}(E')N_i\sigma_{f,i}(r,E')f(r,E',\Omega') \\ &= \left(B_p + Sum_{ij}B_{d,ij}\right)f \end{aligned} \;, \qquad (5)$$

$$\begin{aligned} Ff(r,E',\Omega') &= Sum_i\int_{E_{\min}}^{E_{\max}} dE'\int_{4\pi} d\Omega'\nu_{t,i}(E')N_i\sigma_{f,i}(r,E')f(r,E',\Omega') \\ &= Sum_i\int_{E_{\min}}^{E_{\max}} dE'\int_{4\pi} d\Omega'\nu_{p,i}(E')N_i\sigma_{f,i}(r,E')f(r,E',\Omega') \\ &+ Sum_{ij}\int_{E_{\min}}^{E_{\max}} dE'\int_{4\pi} d\Omega'\nu_{d,ij}(E')N_i\sigma_{f,i}(r,E')f(r,E',\Omega') \\ &= \left(F_p + Sum_{ij}F_{d,ij}\right)f \end{aligned} \;. \qquad (6)$$

Here the subscripts *s* and *p* means scattering and prompt, respectively. Since the delayed neutron spectrum $\chi_{d,ij}$ is independent of the energy *E'* of the neutron inducing the fission,

$$B_{d,ij}f = \frac{\chi_{d,ij}}{4\pi}F_{d,ij}f \;. \qquad (7)$$

Assuming that the Ψ is expressed by an amplitude function $P(t)$ and flux $\phi=\phi(r,E,\Omega,t)$,

$$\Psi(r,E,\Omega,t) = P(t)\phi(r,E,\Omega,t). \qquad (8)$$

By substituting Ψ of eq. (2) by $P\phi$,

$$\frac{1}{V}\phi\frac{dP}{dt} + P\frac{d}{dt}\left(\frac{1}{V}\phi\right) = -A\phi P(t) + B_p\phi P(t) + Sum_{ij}\frac{\chi_{d,ij}}{4\pi}C_{ij}\lambda_{ij} \;. \qquad (9)$$

Here we multiply the eq. (9) by an arbitrary time independent function $W=W(r,E,\Omega)$ and integrate the product over r, E, Ω. Angle bracket <> means the integration.

$$\left\langle W\frac{1}{V}\phi\right\rangle\frac{dP}{dt} + P\frac{d}{dt}\left\langle W\frac{1}{V}\phi\right\rangle = -\langle WA\phi\rangle P(t) + \langle WB_p\phi\rangle P(t) + \left\langle WSum_{ij}\frac{\chi_{d,ij}}{4\pi}C_{ij}\lambda_{ij}\right\rangle. \qquad (10)$$

In the condition where one point reactor model is valid,

$$\frac{\partial}{\partial t}\left\langle W\frac{1}{V}\phi\right\rangle = 0 \;. \qquad (11)$$

Then the eq. (10) is transformed into

$$\left\langle W\frac{1}{V}\phi\right\rangle\frac{dP}{dt}=\langle W(B-A)\phi\rangle P(t)-\langle WSum_{ij}B_{d,ij}\phi\rangle P(t)+Sum_{ij}\left\langle W\frac{\chi_{d,ij}}{4\pi}C_{ij}\lambda_{ij}\right\rangle, \tag{12}$$

$$\left\{\frac{\langle WV^{-1}\phi\rangle}{\langle WB\phi\rangle}\right\}\frac{dP}{dt}=\left\{\frac{\langle W(B-A)\phi\rangle}{\langle WB\phi\rangle}-\frac{\langle WSum_{ij}B_{d,ij}\phi\rangle}{\langle WB\phi\rangle}\right\}P(t)+Sum_{ij}\frac{\lambda_{ij}\langle W\chi_{d,ij}C_{ij}\rangle}{4\pi\langle WB\phi\rangle}. \tag{13}$$

By defining kinetic parameters using the arbitrary weight function W,

$$\beta_{W,ij}=\frac{\langle WB_{d,ij}\phi\rangle}{\langle WB\phi\rangle}\quad, \tag{14}$$

$$\Lambda_W=\frac{\langle WV^{-1}\phi\rangle}{\langle WB\phi\rangle}\quad, \tag{15}$$

$$C_{W,ij}=\frac{\langle W\chi_{d,ij}C_{ij}\rangle}{4\pi\langle WV^{-1}\phi\rangle}\quad, \tag{16}$$

a point kinetic equation is derived [9].

$$\frac{dP}{dt}=\frac{1}{\Lambda_w}\left\{\frac{\langle W(B-A)\phi\rangle}{\langle WB\phi\rangle}-Sum_{ij}\beta_{W,ij}\right\}P(t)+Sum_{ij}\lambda_{ij}C_{W,ij}. \tag{17}$$

A static equation of which eigenvalue is k_{eff} and eigen function is ϕ_s, is expressed as

$$A\phi_s=\frac{1}{k_{eff}}B\phi_s\quad. \tag{18}$$

By multiplying the eqs. (18) by W and integrating the product, we can express reactivity ρ as

$$\rho=1-\frac{1}{k_{eff}}=\frac{\langle W(B-A)\phi_s\rangle}{\langle WB\phi_s\rangle}. \tag{19}$$

In the condition where the point reactor model is valid, shape of ϕ in eq. (17) converges on that of the fundamental flux, ϕ_s. From eq. (19), we may choose arbitrary weight function W other than ϕ^* to express ρ. Accordingly, we can rewrite eq. (17) as

$$\frac{dP}{dt}=\frac{\rho-Sum_{ij}\beta_{W,ij}}{\Lambda_w}P(t)+Sum_{ij}\lambda_{ij}C_{W,ij}\quad. \tag{20}$$

Therefore, we can relate the reactor power history $P(t)$ to ρ using the kinetic parameters defined with W under the condition.

Whereas, by multiplying eq. (3) by $W\chi_{d,ij}/4\pi$ and integrating the product over r, E, Ω,

$$\left\langle\frac{d}{dt}\left\{\frac{W\chi_{d,ij}C_{ij}}{4\pi}\right\}\right\rangle=\left\langle\frac{W\chi_{d,ij}F_{d,ij}\Psi}{4\pi}\right\rangle-\lambda_{ij}\left\langle\frac{W\chi_{d,ij}C_{ij}}{4\pi}\right\rangle \tag{21}$$

is obtained. Recalling the condition $\langle WV^{-1}\phi\rangle$ is constant (eq.(11)) and relation of eq.(7),

$$\frac{d}{dt}\left\{\left\langle\frac{W\chi_{d,ij}C_{ij}}{4\pi}\right\rangle\langle WV^{-1}\phi\rangle^{-1}\right\}=\langle WB_{d,ij}(\phi P(t))\rangle\langle WV^{-1}\phi\rangle^{-1}-\lambda_{ij}\left\langle\frac{W\chi_{d,ij}C_{ij}}{4\pi}\right\rangle\langle WV^{-1}\phi\rangle^{-1}, \tag{22}$$

$$\frac{dC_{W,ij}}{dt}=P(t)\langle WB_{d,ij}\phi\rangle\langle WB\phi\rangle^{-1}\left\{\langle WV^{-1}\phi\rangle\langle WB\phi\rangle^{-1}\right\}^{-1}-\lambda_{ij}C_{W,ij}\quad, \tag{23}$$

$$\frac{dC_{W,ij}}{dt}=P(t)\frac{\beta_{w,ij}}{\Lambda_w}-\lambda_{ij}C_{W,ij}\quad. \tag{24}$$

Eqs. (20) and (24) are identical to conventional point kinetic equations in which kinetic parameters are defined with adjoint function ϕ^* instead of W [9].

In the case where constant reactivity is inserted to a core and outer neutron source is negligible, the following inhour equation is derived from eqs. (20) and (24).

$$\rho = \frac{\langle W(B-A)\phi\rangle}{\langle WB\phi\rangle} = \frac{\Lambda_W}{T} + Sum_{ij}\frac{\beta_{W,ij}}{1+\lambda_{W,ij}T} \approx Sum_{ij}\frac{\beta_{W,ij}}{1+\lambda_{W,ij}T}\,. \tag{25}$$

2.2 Number of Fission Neutrons in Next Generation

As described above, we may choose an arbitrary weighting function for the kinetic parameters to relate reactor period T to reactivity ρ. Since adjoint flux ϕ^* is hardly solved by the continuous energy Monte Carlo method [10], we have investigated another functions.

A concept, "generation of neutron" is adopted in the eigenvalue calculation scheme of the method [7]. It spans from neutron birth in a fission to its loss. Succeeding fission neutron emission induced by a neutron in one generation is categorized into events in the next one. The k_{eff} is defined by the number ratio of fission neutrons in the next generation to those in the current one. Here we have introduced a function $M=M(r,E,\Omega)$, which is the expected number of fission neutrons in the next generation yielded by a neutron at r, E, Ω in the current generation. With M and neutron flux in the previous generation, ϕ', k_{eff} is expressed as follows

$$k_{eff} = \frac{\left\langle M\int dE'd\Omega'\left[Sum_i\left(\chi_t\nu_t N\sigma_f\right)_i\phi'\right]\right\rangle}{4\pi\left\langle Sum_i\left(\nu_t\sigma_f N\right)_i\phi'\right\rangle} = \frac{\langle MB\phi'\rangle}{\left\langle Sum_i\left(\nu_t\sigma_f N\right)_i\phi'\right\rangle}\,. \tag{26}$$

Kobayashi and Nishihara discussed a kind of Green function, G, [6] defined by

$$Sum_i\left(\nu_t\sigma_f N\right)_i = A^*G(r,E,\Omega). \tag{27}$$

By multiplying eq. (27) by ϕ_s and integrate the product,

$$\left\langle\phi_s Sum_i\left(\nu_t\sigma_f N\right)_i\right\rangle = \left\langle\phi_s A^*G\right\rangle = \left\langle GA\phi_s\right\rangle \tag{28}$$

is obtained. From eqs. (18) and (28),

$$\left\langle\phi_s Sum_i\left(\nu_t\sigma_f N\right)_i\right\rangle = \frac{1}{k_{eff}}\left\langle GB\phi_s\right\rangle\,. \tag{29}$$

From eqs. (26) and (29), we can see that M converges on G in the case ϕ' does on ϕ_s [1], since

$$k_{eff} = \langle MB\phi'\rangle\left\langle\phi' Sum_i\left(\nu_t\sigma_f N\right)_i\right\rangle^{-1} = \langle GB\phi_s\rangle\left\langle\phi_s Sum_i\left(\nu_t\sigma_f N\right)_i\right\rangle^{-1}\,. \tag{30}$$

Since M has physical meaning described in eq. (27), we have proposed to use M for W to estimate effective delayed neutron fraction β_{eff} and neutron generation time Λ [1]. As the same manner, we propose to estimate $\beta_{eff,ij}$ by $\beta_{nnn,ij}$ defined below.

$$\beta_{nnn,ij} = \frac{\int dr^3 dE d\Omega M\left\{\int dE'd\Omega'\chi_{d,ij}\nu_{d,ij}N_i\sigma_{f,i}\phi'\right\}}{Sum_i\int dr^3 dE d\Omega M\left\{\int dE'd\Omega'\chi_{t,i}\nu_{t,i}N_i\sigma_{f,i}\phi'\right\}}\,. \tag{31}$$

In the case of prompt critical state, the number of fission neutrons in the next generation produced by prompt neutrons is equal to that of all fission neutrons in the current one.

$$\left\langle MB_p\phi'\right\rangle = \langle MB\phi'\rangle - Sum_{ij}\left\langle MB_{d,ij}\phi'\right\rangle = \left\langle Sum_i\left(\nu_t\sigma_f N\right)_i\phi'\right\rangle\,. \tag{32}$$

Then the following relation is derived.

$$1 - Sum_{ij}\left\langle MB_{d,ij}\phi'\right\rangle\langle MB\phi'\rangle^{-1} = 1 - Sum_{ij}\beta_{nnn,ij} = k_{eff}^{-1} \tag{33}$$

Accordingly the sum of $\beta_{nnn,ij}$, β_{nnn}, indicates the reactivity of the state.

Meulekamp and van der Marck has proposed to estimate $\beta_{eff} = Sum_{ij}\beta_{eff,ij}$ using "the ratio of fissions in next generation induced by delayed neutrons to that by all neutrons" [3]. From their description, "number of fission" is expected to be used for the weight function W in eq. (14). They stated their weight function is an approximation of "iterated fission probability" [3] which is considered proportional to adjoint flux ϕ^* [11]. M is not identical to them. Since effective multiplication factor k_{eff} and reactivity ρ are essentially defined by the number ratio of fission neutrons which maintain reaction chain, we proposed to use M to evaluate effectiveness of delayed neutrons in the chain.

Although the fission neutron yield after infinite generations is expected to be proportional to ϕ^* as "iterative fission probability", we have no intension to state M and $\beta_{nnn,ij}$ were approximations of ϕ^* and $\beta_{eff,ij}$, because the G function (eq. (27)) on which M converges is different from adjoint function ϕ^* defined as

$$A^*\phi^* = \frac{1}{k_{eff}} B^*\phi^* \quad . \tag{34}$$

Intension of this paper is to discuss availability of M and $\beta_{nnn,ij}$ for point kinetic equations, (17), (24), (25).

3. Algorism

An eigenvalue is evaluated with MCNP-5 code as follows. Fission positions of current generation are determined as the results of neutron transport in the previous generation. For each position, fission nuclide i is sampled, then either prompt or delayed neutrons are sampled according to the ratio $\nu_{d,i}$: $\nu_{p,i}$. In the case of emission of a delayed one, precursor group j is sampled. After that, neutron emission energy is sampled according to spectrum χ_p or $\chi_{d,ij}$. The position and the energy are stored in a file. Those data in the file are read in as source neutrons for transport calculation in the current generation. Considering the case $S_{current}$ neutrons are stored in the file. The transport calculation is performed for each neutron and the number of fission neutrons in next generation produced by the neutron, M_{source}, is evaluated using three kinds of k_{eff} estimator (collision, analog and track length). The total number of fission neutrons in the next generation, S_{next}, is obtained by accumulating M_{source} for $S_{current}$ neutrons. Consequently, k_{eff} is evaluated as the ratio $S_{next}/S_{current}$ [7,8].

We implemented two functions to the MCNP-5 code. The one is to store the information of i and j for each source neutron, and the other is to accumulate M_{source} for neutrons tagged with the i and j. By the procedure, we can evaluated the number of fission neutrons in next generation produced by delayed ones categorized into the i and j group, $S_{d,ij}$. We can obtain $\beta_{nnn,ij}$ as a ratio of $S_{d,ij}$ to $S_{current}$ ($S_{d,ij}/S_{current}$).

4. Verification

The $\beta_{eff,ij}$ data have not been directly measured. Even the sum of $\beta_{eff,ij}$, β_{eff}, can be measured in relation to other parameters such as ρ or Λ, etc. Since it is also difficult to measure them directly, β_{eff} data are often derived from experimental data of ρ/β_{eff} or β_{eff}/Λ with calculated ρ or Λ. That means almost all the experimental data of β_{eff} are biased by values based on calculations with some approximations and arbitrary cross section libraries.

Whereas, $\beta_{eff,ij}$ is the quantity which relates power history and reactivity. In reactivity measurements, directly measurable data are 1) geometrical data of critical and perturbed cores and 2) power history data of the perturbed core. Using the geometrical data, we can calculate

$\beta_{nnn,ij}$ with the enhanced MCNP-5 code. With the $\beta_{nnn,ij}$ and measured reactor period T, reactivity ρ_{in} is deduced based on the inhour equation (eq. (25)). Besides, k_{eff}s of the both cores can be calculated with MCNP-5 based on the geometrical data. From the two k_{eff}s, the reactivity of perturbed core is derived as

$$\rho_{dir} = \frac{1}{k_{eff-critical}} - \frac{1}{k_{eff-perturbed}} \quad . \tag{35}$$

By the comparison of ρ_{in} to ρ_{dir}, the applicability of $\beta_{nnn,ij}$ can be clarified.

Verification of the proposed method is performed for the reactivity data measured in STACY and TCA cores of Japan Atomic Energy Agency (JAEA).

STACY is a series of critical cores facilitated in NUCEF in JAEA. 280T of STACY is a slab shaped core tank in which uranium nitrate solution is loaded. ^{235}U enrichment of the solution is 9.7%wt. The tank can be immersed in light water or loaded adjacent to reflector slabs of concrete or polyethylene. In this work we use the reactivity data measured for 4 simple bare cores named R0163, R0178, R0182, R0184. The critical solution levels of the 4 cores varied with concentration of the solution. In each core, positive period T was measured for perturbed state where the level of the solution was raised from the critical state [12].

In TCA, a critical lattice of 2.6%-^{235}U enriched UO_2 fuel rods is immersed in light water. Criticality is attained by adjusting the water level. In the critical condition, a dry lattice sticks out the level. We obtained a positive period datum measured for a perturbed 17x17 lattice of which volumetric ratio of moderator to fuel, V_m/V_f=1.83 [13,14].

We calculated $\beta_{nnn,ij}$s of ^{235}U and ^{238}U for those cores with the enhanced MCNP-5 code using FSXLIB-J33 [15] library based on JENDL-3.3. In the calculation, $\beta_{nnn,ij}$s were averaged over 100 generations after skipping 50 generations, in each of which 1000000 source fission neutrons were sampled. The covariance averaged values of $\beta_{nnn,ij}$ evaluated with the k_{eff} estimators were used for verification described below.

In the STACY core, a common data set of $a_j=\beta_{eff,j}/Sum_j\beta_{eff,j}$ of ^{235}U is used for deduction of reactivity ρ/β_{eff} for the 4 cores. We compared $\beta_{nnn,j}/Sum_j\beta_{nnn,j}$ of ^{235}U to a_j in Fig. 1. Both values agree within the accuracy of a_j reported in the literature [12].

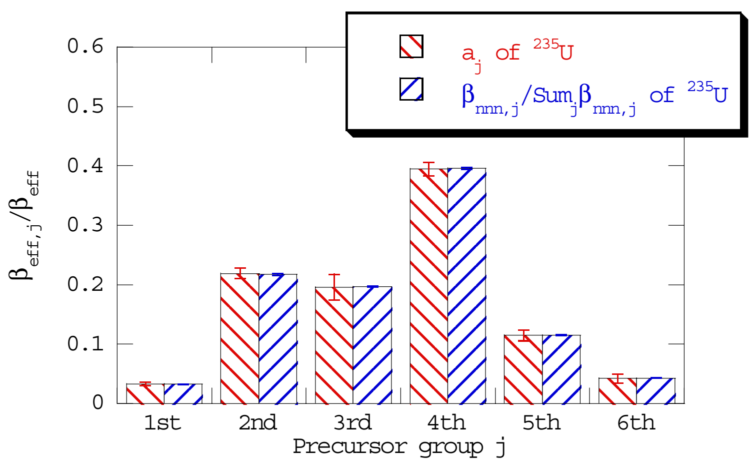

Figure 1: Comparison of calculated $\beta_{nnn,j}/Sum_j\beta_{nnn,j}$ of ^{235}U to a_j used for reactivity meter in STACY 280T core.

In TCA, $a_{eff,j}$s are used for its reactivity meter. The $a_{eff,j}$s are $\beta_{eff,j}/Sum_j\beta_{eff,j}$ data of ^{235}U partially modified taking account of ^{238}U effect [13]. In Fig. 2, we compared period – reactivity curve based on the inhour equation (eq. (25)) with $\beta_{nnn,ij}/Sum_{ij}\beta_{nnn,ij}$ of ^{235}U and ^{238}U

to that with $a_{eff,j}$ data. The reactivity ($) with $\beta_{nnn,ij}$ is 1.7%~2.0% smaller than that with $a_{eff,j}$, but the difference is sufficiently small comparing with the estimation error of β_{eff}.

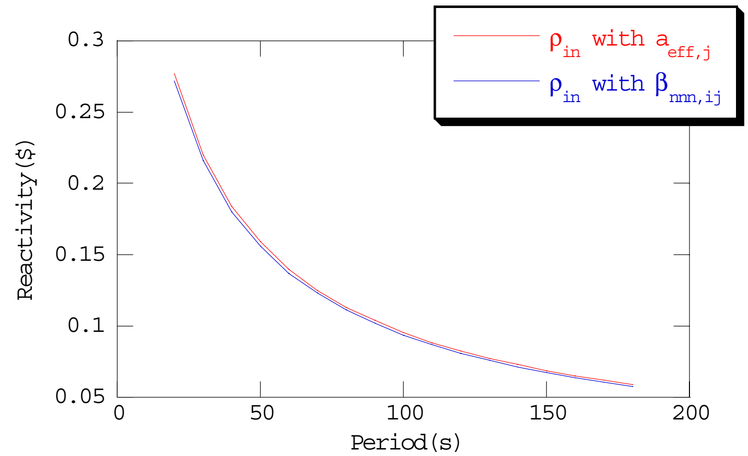

Figure 2: Comparison of Period – Reactivity curve based on inhour equation with $\beta_{nnn,ij}$ to that with $a_{eff,j}$ used in reactivity meter of TCA

From the results, it is clarified that kinetic parameters estimated by the proposed method are applicable for reactivity meters of STACY and TCA

Besides, we calculated ρ_{dir}s for STACY and TCA cores. Since positive reactivity insertion is restricted to be less than 0.2$ for those cores, we increased the number of significant figures of output format of k_{eff} by one. k_{eff}s are obtained by 1000 generations of calculations for the STACY cores and 2600 generations for the TCA core. In each generation, 1000000 neutrons were sampled. We confirmed the convergence of ρ_{dir}s with statistical fluctuations of ~3% as shown in Fig. 3.

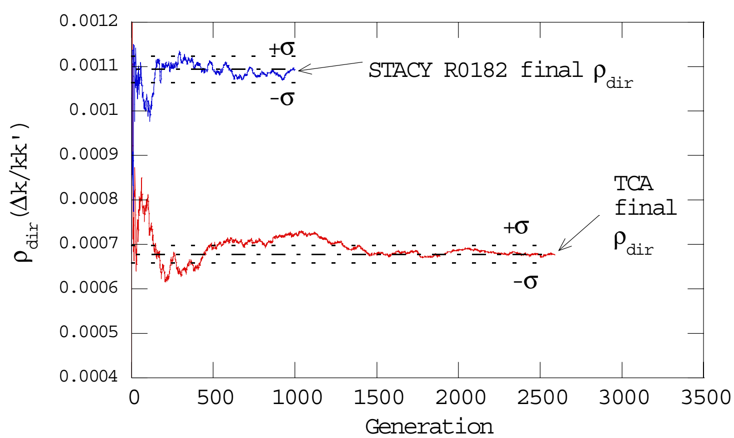

Figure 3: Convergence of calculated ρ_{dir} for STACY R0182 and TCA cores in relation to generations.

Finally we compared ρ_{in} to ρ_{dir} in Table. 1. For those cores except R0179, both reactivities agree within the statistical error (1σ) of ρ_{dir}, 3%. Although ρ_{in} shows 6.3% larger than ρ_{dir} for R0179 core, the difference is still within the range of 2σ.

As the results, the applicability of $\beta_{nnn,ij}$ is well confirmed. The good agreement is attributed not only to the proposed method using M for kinetic parameters, but also to accuracy of

JENDL-3.3, rigorous calculation method of MCNP-5 and precise experimental data.

Table 1: Comparison of ρ_{in} derived from inhour equation using $\beta_{nnn,ij}$ and measured T to ρ_{dir} derived from direct calculations of difference of $1/k_{eff}$

Facility	Run No	Period s	error	ρin	err	ρdir	err	ρin/ρdir	err
STACY	R0163	53.2	1.0	0.001152	0.000009	0.001147	0.000030	1.0046	0.0274
	R0179	66.8	0.7	0.000979	0.000005	0.000921	0.000031	1.0630	0.0339
	R0182	58.1	0.6	0.001082	0.000005	0.001094	0.000029	0.9891	0.0273
	R0184	43.4	0.7	0.001031	0.000007	0.001049	0.000031	0.9827	0.0300
TCA		106.5	#	0.000692	0.000002	0.000678	0.000020	1.0209	0.0293

5. Conclusion

A new method is proposed to estimate effective fraction of delayed neutrons radiated from precursors categorized into 6 groups of decay constant with $\beta_{nnn,ij}$ which is defined with the expected number of fission neutrons in next generation, *M,* instead of adjoint flux ϕ^*. The proposal is based on point kinetic equations with parameters weighted by an arbitrary function, derived from the space-dependent kinetic equation [9]. Introduction of *M* enables us to calculate $\beta_{nnn,ij}$ based on the continuous energy Monte Carlo method. For the calculation of $\beta_{nnn,ij}$, an algorism is established and implemented into the MCNP-5 code. The method is verified using reactor period data obtained in reactivity measurements in STACY and TCA. The reactivity ρ_{in}s deduced by period data *T*s and proposed $\beta_{nnn,ij}$s agree well with reactivity ρ_{dir}s done by difference of k_{eff}^{-1}s. From the results, it is concluded that kinetic parameters weighted by *M* can be applied to the point kinetic equations.

As the future work, the authors will continue to verify the proposed method by comparison of ρ_{in} to ρ_{dir} for various kinds of core. In the verification, we will also investigate the accuracy of reaction cross sections and fundamental data of ν_d, χ_d, etc.

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