

From CANDLE Reactor to Pebble-Bed Reactor

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Abstract

This paper attempts to reveal theoretically, by studying a diffusion-burn-up coupled neutronic model, that a so-called CANDLE reactor and a pebble-bed type reactor have a common burn-up feature. As already known, a solitary burn-up wave that can develop in the common U-Pu and Th-U conversion processes is the basic mechanism of the CANDLE reactor. In this paper it is demonstrated that a family of burn-up wave solution exists in the boundary value problem characterizing a pebble bed reactor, in which the fuel is loaded from above into the core and unloaded from bottom. Among this solution family there is a particular case, namely, a partial solitary wave solution, which begins from the fuel entrance side and extends into infinity on the exit side, and has a maximal burn-up rate in this family. An example dealing with the ^{232}Th - ^{233}U conversion chain is studied and the solutions are presented in order to show the mechanism of the burn-up wave.

KEYWORDS: *Diffusion model, burn-up equations, solitary wave solution, CANDLE reactor, pebble-bed reactor*

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1. Introduction

It has been reported that there are self-propagating nuclear burning waves in fertile media of ^{238}U and ^{232}Th in [1]. This mechanism was proposed in some detail for an interesting concept of a self controlled nuclear fission reactor [2], in which a nuclear breeding and burning wave is ignited and propagates slowly in the axial direction of the core. Natural thorium and uranium fuel can be used for this type of reactor once the nuclear burning wave has been established. Consequently no fuel enrichment and reprocessing are needed, and a long operation duration and a high fuel burn-up can theoretically be achievable. Since the burning mechanism and the geometry of this reactor are similar to those of a burning candle, it is sometimes called *nuclear candle* or *CANDLE reactor* [3].

Fundamental understanding of this new type of reactor were provided in [4,5,6,7], where solitary wave solutions are obtained both from a single group diffusion equation with burn-up dependent macroscopic coefficients that are either suitably approximated or solved from burn-up equations of a realistic conversion chain. Moreover, feedback effects can be taken into account as well in the solution [4, 8]. Intensive numerical studies of multi-group diffusion and burn-up coupled equations were carried out for this kind of reactor in [3] and the feasibility of this new concept by achieving a quasi-asymptotic solution was demonstrated.

Since the wave propagation is relative with respect to the medium, it is possible, instead of the conventional treatment of a moving wave in a stationary fuel, to make the fuel moving but the wave being fixed with respect to the laboratory coordinate system. This represents the typical case of a pebble-bed reactor. It was shown by numerical simulations in a 1-D case [9] and in a 2-D case [10] that asymptotic solitary waves exist in a realistic core composition of pebble-bed type HTGR. Although the similarity between the candle and pebble-bed reactors is quite clear, the pebble-bed reactor has a finite axial length, the axial neutron flux distribution can not be a solitary wave profile that has to be extended to infinity. In this paper we deal with the boundary value problem of the diffusion equation coupled with simplified burn-up equations and find that this problem is analytically solvable in the one-dimensional case. We call this solution *fundamental burn-up mode*. If the fuel is chosen, there is a relationship between the fuel moving speed and the core length. In the solution family the lowest speed corresponds to a solitary wave solution where the core length is theoretically infinite and the burnup gets its maximum. As an example the ^{232}Th - ^{233}U conversion chain is chosen and results are presented.

This paper is just a conceptual study. The calculated results have not been validated by any experimental data. But it shows an important relation between the candle reactor and pebble-bed reactor, i.e. both of them possess solitary wave solutions. This implies that the solitary burn-up wave concept, where a high burnup can be achieved, is feasible in the existing pebble-bed type reactor [11,12].

2. Neutronic Model

2.1 Diffusion Equation

For the sake of simplicity we consider here only a one-dimensional single group steady state problem without any external source. For the neutron balance in the core the diffusion equation reads

$$\frac{d}{dx} \left(D \frac{d}{dx} \phi \right) - \Sigma_a \phi + \nu \Sigma_f \phi = 0, \quad (1)$$

where ϕ is the neutron flux, D the diffusion coefficient, ν the average number of generated neutrons per fission and Σ_a and Σ_f are the macroscopic absorption and fission cross sections, respectively. In the above equation a convective term of first x-derivative has been neglected, because of its coefficient is of the order of the ratio the fuel drift speed to the neutron average velocity, which is negligibly small.

Unlike the CANDLE reactor, see e.g. [3, 4], that might be theoretically infinitely long, the pebble-bed reactor has a finite length. Therefore we have to pose a suitable boundary condition for the pebble-bed reactor problem. For a naked core, i.e. a core without reflectors, it reads

$$\phi + d \frac{d}{dn} \phi = 0, \quad (2)$$

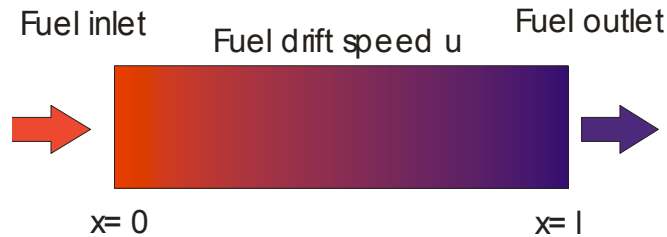
where n is the outward normal vector on the boundary and the extrapolation distance d can be expressed e.g. as

$$d = \frac{2}{3} \frac{1}{\Sigma_{tr}}, \quad (3)$$

where Σ_{tr} is the macroscopic transport cross section. Suppose the fuel is moving from left to right with a speed u . The fresh fuel is fed in at $x = 0$ and the burned fuel gets out at $x = l$, see Fig. 1.

The coefficients in (1) are not constant and they are actually burn-up dependent, which will be discussed in §2.3. Unlike conventionally using an external source or introducing an eigenvalue to make the equation have a nontrivial solution, we adjust the core length l to make the core be critical in an asymptotic burn-up state, that will be discussed later in §3.

Figure 1: Schematic of a one-dimensional pebble-bed reactor.



2.2 Burn-up Equations

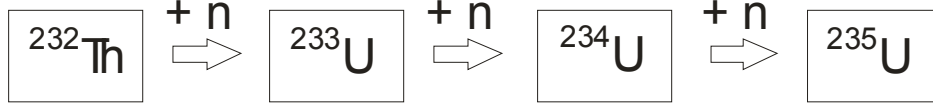
We consider a truncated ^{232}Th - ^{233}U conversion chain for our burn-up calculation assuming a thermal neutron spectrum. This means that only the heavy metals ^{232}Th , ^{233}U , ^{234}U and ^{235}U , characterized by the indices $i = 2, 3, 4$ and 5 , and, in addition, a typical burnable fission product pair (FPP) are taken into account. Because the radioactive decay processes are, in the considered case, either too short or too long with respect to the considered time scale of the order of several years, natural radioactive decay processes and $(n, 2n)$ processes are neglected. Thus the simplified burn-up equations can be written for the conversion chain shown in Fig. 2 as

$$\begin{aligned} \frac{\partial N_2}{\partial t} &= -N_2 \sigma_{a,2} \phi, & \frac{\partial N_i}{\partial t} &= -N_i \sigma_{a,i} \phi + N_{i-1} \sigma_{c,i-1} \phi, & i &= 3,4,5 \\ \frac{\partial N_{FPP}}{\partial t} &= -N_{FPP} \sigma_{a,FPP} \phi + \sum_{i=2,3,4,5} N_i \sigma_{f,i} \phi, \end{aligned} \quad (4)$$

where N_i is the atom number density of isotope i , $\sigma_{a,i}$, $\sigma_{c,i}$, $\sigma_{f,i}$ are the absorption, capture

and fission cross sections of isotope i , respectively.

Figure 2: Simplified Th-U conversion chain.



2.3 Coupling of Diffusion and Burn-up Equations

In the diffusion equation the macroscopic coefficients are neither uniform in space, nor constant in time. They depend in general on the material composition that changes with fuel burn-up. This means that the diffusion equation is coupled by the burn-up equations through the macroscopic coefficients Σ_a , Σ_f , Σ_{tr} and D in the following manner,

$$\Sigma_a = \sum_n N_n \sigma_{a,n}, \quad \nu \Sigma_f = \sum_n N_n \nu_n \sigma_{f,n}, \quad \Sigma_{tr} = \sum_n N_n \sigma_{tr,n}, \quad D = \frac{1}{3 \Sigma_{tr}}. \quad (5)$$

The burn-up equations provide the macroscopic coefficients to the diffusion equation and the diffusion equation provides the neutron flux to the burn-up equations. Tab. 1 below gives typical microscopic cross sections. The given values should not be considered as representative but may illustrate order of magnitude and tendencies, which is sufficient for this theoretical study.

Table 1: Microscopic cross sections for a Maxwellian-averaged spectrum, where the absorption cross section $\sigma_a = \sigma_f + \sigma_c$ is used in this paper.

	²³² Th	²³³ U	²³⁴ U	²³⁵ U	FPP
ν	2.21	2.49	2.37	2.42	0
σ_f [barn]	0	468	0.407	505	0
σ_c [barn]	6.55	41.8	90.5	86.4	35.4

3. Mathematical Solution

3.1 Solution of Burn-up Equations

The burn-up equations (4) can be solved in a straightforward manner. In general, if the natural radioactive decay processes are neglected, all atom number densities N_i can be expressed as functions of the neutron fluence ψ , i.e.,

$$N_i = N_i(\psi) \quad \text{with} \quad \psi = \int_0^t \phi \, dt. \quad (6)$$

For the sake of simplicity, we only write here the solution of the reaction chain until ²³³U, i.e. we consider N_2 as a typical fertile nuclide, N_3 as a typical fissile nuclide and N_{FPP} as associated fission product. This is sufficient for an appropriate physical understanding of this phenomenon. Nevertheless the complete solution of (4) is used later on in this paper for numerical examples.

If the conversion chain is cut after ²³³U, N_2 , N_3 and N_{FPP} are expressed as

$$N_2 = N_{2,0} e^{-\sigma_{a,2}\psi},$$

$$N_3 = N_{3,0} e^{-\sigma_{a,3}\psi} + N_{2,0} \frac{\sigma_{c,2}}{\sigma_{a,3} - \sigma_{a,2}} \left[e^{-\sigma_{a,2}\psi} - e^{-\sigma_{a,3}\psi} \right],$$

$$N_{FPP} = N_{3,0} \frac{\sigma_{f,3}}{\sigma_{a,F} - \sigma_{a,3}} \left[e^{-\sigma_{a,3}\psi} - e^{-\sigma_{a,F}\psi} \right] - N_{2,0} \left(\frac{\sigma_{c,2}}{\sigma_{a,3} - \sigma_{a,2}} \right) \left(\frac{\sigma_{f,3}}{\sigma_{a,F} - \sigma_{a,3}} \right) \left[e^{-\sigma_{a,3}\psi} - e^{-\sigma_{a,F}\psi} \right]$$

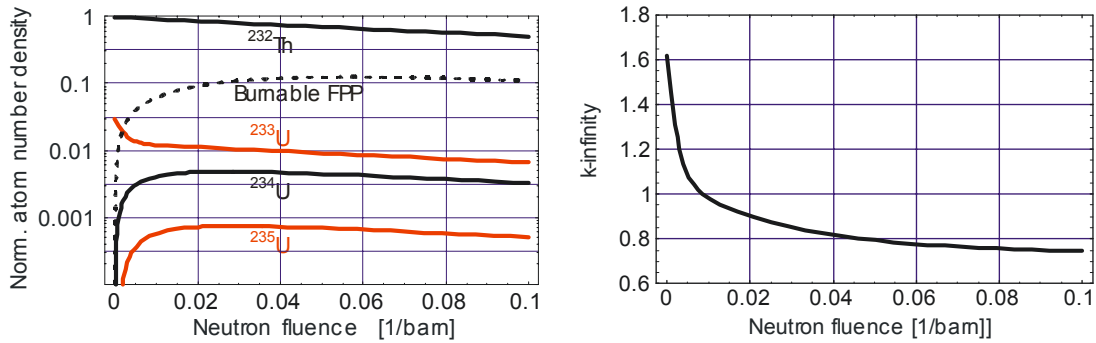
$$+ N_{2,0} \left[\frac{\sigma_{f,2}}{\sigma_{a,F} - \sigma_{a,2}} + \left(\frac{\sigma_{c,2}}{\sigma_{a,3} - \sigma_{a,2}} \right) \left(\frac{\sigma_{f,3}}{\sigma_{a,F} - \sigma_{a,2}} \right) \right] \left[e^{-\sigma_{a,2}\psi} - e^{-\sigma_{a,F}\psi} \right].$$

The fresh fuel consists of only ^{232}Th and ^{233}U as heavy metal. Therefore we can set the total initial heavy metal atom number density N_0 as

$$N_0 = N_{2,0} + N_{3,0}. \tag{7}$$

A complete solution of the burn-up equations (4) is shown in Fig. 3 for $N_{2,0}/N_0 = 0.97$ and $N_{3,0}/N_0 = 0.03$.

Figure 3: Solution of the burn-up equations with $N_{2,0}/N_0 = 0.97$ and $N_{3,0}/N_0 = 0.03$.



3.2 Solution of Diffusion Equation

Because the fuel is moving and the neutron flux is stationary with respect to the core-fixed coordinate system, the fuel residence time t and the fuel position x have a following relation,

$$dx = u dt, \tag{8}$$

where u is the fuel drift speed. Therefore, from (6) we have

$$\psi(x) = \frac{1}{u} \int_0^x \phi dx \tag{9}$$

and the macroscopic net production cross section

$$f(\psi) = \nu \Sigma_f(\psi) - \Sigma_a(\psi) \tag{10}$$

is a known function that can be obtained from the solution of burn-up equations.

Let us write (1) in the following form and begin to solve it.

$$\frac{d}{dx} \left(D \frac{d}{dx} \phi \right) + f(\psi) \phi = 0. \tag{11}$$

Integrating the above equation over $(0, x)$ yields

$$D \frac{d}{dx} \phi - D_0 \frac{d}{dx} \phi \Big|_{x=0} + \int_0^x f(\psi) \phi dx = 0. \tag{12}$$

By using (9) the last integral in (12) can be written as

$$\int_0^x f(\psi) \phi \, dx = u \int_0^\psi f(\psi) \, d\psi \stackrel{\text{def}}{=} u g(\psi). \quad (13)$$

Thus (11) can be rewritten in a form as

$$\frac{d}{dx} \phi = \frac{D_0}{D(\psi)} \left(\frac{d}{dx} \phi \right)_{x=0} - u \frac{g(\psi)}{D(\psi)}. \quad (14)$$

Multiplying ϕ to the above equation and integrating it once more over $(0, x)$ gives

$$\frac{1}{2} \phi^2 - \frac{1}{2} \phi_0^2 - \int_0^x \frac{D_0}{D(\psi)} \left(\frac{d}{dx} \phi \right)_{x=0} \phi \, dx + \int_0^x u \frac{g(\psi)}{D(\psi)} \phi \, dx = 0. \quad (15)$$

Because

$$\int_0^x \frac{D_0}{D(\psi)} \left(\frac{d}{dx} \phi \right)_{x=0} \phi \, dx = u \left(\frac{d}{dx} \phi \right)_{x=0} \int_0^\psi \frac{D_0}{D(\psi)} \, d\psi \stackrel{\text{def}}{=} u \left(\frac{d}{dx} \phi \right)_{x=0} E(\psi),$$

and

$$\int_0^x u \frac{g(\psi)}{D(\psi)} \phi \, dx = u^2 \int_0^\psi \frac{g(\psi)}{D(\psi)} \, d\psi \stackrel{\text{def}}{=} u^2 h(\psi),$$

(14) becomes

$$\phi^2 = \phi_0^2 + 2u \left(\frac{d}{dx} \phi \right)_{x=0} E(\psi) - 2u^2 h(\psi). \quad (16)$$

Since $f(\psi)$, $g(\psi)$, $h(\psi)$, and $E(\psi)$ are known functions, together with the boundary conditions at $x = 0$ and $x = l$ in (2), (14) provides ϕ_x as a known function of ψ and (16) provides ϕ as a known function of ψ . From the viewpoint of dynamic systems, the solution has been already completed, since the solution can be displayed in the phase plane (ϕ, ϕ_x) as parametric functions of ψ . In particular, $\psi(x)$ can be carried out from $u\psi_x = \phi(\psi)$, $\phi(x)$ can be obtained as well. For certain fuel drift speed u , the core length l can be determined through this solution. The nontrivial solution obtained here is essential for the burn-up problem and can be called *fundamental burn-up mode*. Interested readers may assume $f(\psi)$ to be a decreasing linear function of ψ and carry out associated solution in the way described above.

3.3 Variables, Parameters and their Normalization

The spatial variables to be solved are the neutron flux ϕ , the neutron fluence ψ and the atom number density N_i of nuclide i in this problem. The given parameters are D_0 , $\sigma_{a,0}$, and N_0 , where subscript 0 refers to either core inlet or fresh fuel. The freely chosen parameters might be the neutron flux ϕ_0 at the core inlet and the fuel drift speed u . The core length l is a derived parameter. For the sake of easy recognition the corresponding capital letters will be used for the non-dimensional variables in the following.

A suitable normalization makes the formulation more clear and reduces the number of input parameters. The most suitable normalization of ψ for this kind of problem was suggested in [5],

$$\Psi = \psi \sigma_{a,0} = O(1).$$

It is interesting to remark that the non-dimensional fluence Ψ is usually of order of one. A natural way to normalize ϕ is

$$\Phi = \frac{\phi}{\phi_0}.$$

From the two equations above we can derive a typical time scale $t_0 = 1/(\phi_0 \sigma_{a,0})$. The diffusion length

can be used as the length scale: $l_0 = \sqrt{D_0/\Sigma_{a0}}$, where $\Sigma_{a0} = \sum N_{i,0} \sigma_{ai,0} = N_0 \sigma_{a0}$. Therefore we can have a typical drift speed

$$u_0 = l_0/t_0 = \phi_0 \sigma_{a,0} \sqrt{D_0/\Sigma_{a0}}.$$

The coordinate x and the drift speed u are of course normalized by its typical values l_0 and u_0 as

$$X = x/l_0 \quad \text{and} \quad U = u/u_0.$$

As a result of this normalization, e.g., the diffusion equation becomes

$$\frac{d}{dX} \left(\frac{D}{D_0} \frac{d}{dX} \Phi \right) + F(\Psi) \Phi = 0,$$

where F is the normalized net production cross section, i.e. $F = f/(N_0 \sigma_{a0})$.

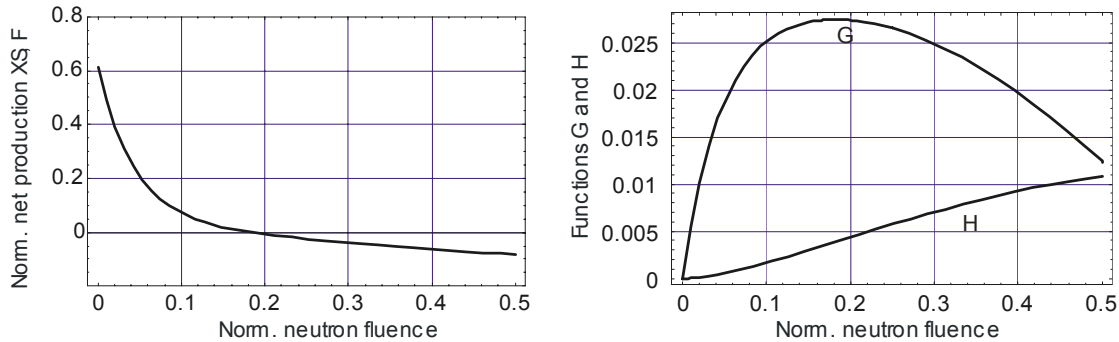
3.4 Numerical Results of the Coupled Solution

Since both the burn-up equations and the diffusion equation are solved analytically, the numerical tasks here are just evaluation and presentation of results. Assume the macroscopic transport cross section to be constant and assign it in particular be $\Sigma_{tr} = c\Sigma_{a0}$, where $c = 3$ for the current example. Then $D = D_0 = 1/(3\Sigma_{tr})$ and $d/l_0 = 2/3$ and the non-dimensional boundary conditions become $(\Phi_x)_0 = K\Phi_0$ and $(\Phi_x)_L = -K\Phi_L$, where $K = l_0/d$. The solution in the non-dimensional form can be written as

$$\frac{d}{dX} \Phi = K - U G(\Psi), \quad \text{with} \quad G(\Psi) = \int_0^\Psi F(\Psi) d\Psi,$$

$$\Phi^2 = 1 + 2UK\Psi - 2U^2H(\Psi), \quad \text{with} \quad H(\Psi) = \int_0^\Psi G(\Psi) d\Psi.$$

Figure 4: The normalized averaged microscopic net production cross section $F(\Psi)$ and its integral functions $G(\Psi)$ and $H(\Psi)$ in the case of $N_{2,0}/N_0 = 0.97$ and $N_{3,0}/N_0 = 0.03$.



Tab. 2 below shows the dimensional variable scale values.

Table 2: Dimensional variable scale values for certain chosen values of ϕ_0 and N_0 .

Variable	ϕ_0	N_0	σ_{a0}	$l_0 = \sqrt{D_0/\Sigma_{a0}}$	$t_0 = 1/(\phi_0 \sigma_{a,0})$	$u_0 = l_0/t_0$
Dimension	1/(cm ² s)	1/cm ³	barn	cm	s	cm/year
Value	5 10 ¹³	3.5 10 ²¹	21.65	4.399	9.238 10 ⁸	0.15017

After normalization, only the non-dimensional drift speed U and the core length L are left in the problem, which will be determined by the solution. The criticality condition, which is actually an

eigenvalue problem of a nonlinear equation, i.e. the non-trivial solution of the nonlinear diffusion equation, will provide a relation between U and L . Physically it is easy to imagine that there are two limit cases. One is U is infinitely large, so that the whole core is filled with fresh fuel homogeneously. The solution in this case tends to be the fundamental mode of a homogeneous core, i.e. sin or cos-function. In this case the core is shortest and the burnup is zero. The other case is that U takes its minimum value and L tends to be infinite. In this case a maximum burnup will be achieved.

For certain U , the solution can be immediately presented in the phase plane (Φ, Φ_x) , as shown for several typical values of U in the left plot of Fig. 5. The corresponding solutions in the physical plane (X, Φ) are shown in the right plot of Fig. 5. It is found that there is a minimum of the drift speed $U_{\min} = 65.145$ i.e. 9.8 cm/year, at which the neutron flux has a shape of solitary wave and the burnup gets its maximum $BU_{\max} = 10.82 \text{ at\%}$. If the fuel drift speed is lower than this minimum, the infinitely long core becomes subcritical, where there exists no nontrivial solution. It is important to remark that the neutron flux at the exit has been shown in Fig. 5, which is presented by the curve cross point with the outlet boundary line at the left plot and by the right curve end at the right plot. The core length L and the burnup vs. the fuel speed U as well as the burnup vs. L are presented in Fig. 6. It is worth to notice that the burnup is not far from its maximum, if L is larger than a certain value, e.g. $L > 15$.

Figure 5: Solutions in the phase plane (Φ, Φ_x) and in the physical plane (X, Φ) in the case of $N_{2,0}/N_0 = 0.97$ and $N_{3,0}/N_0 = 0.03$.

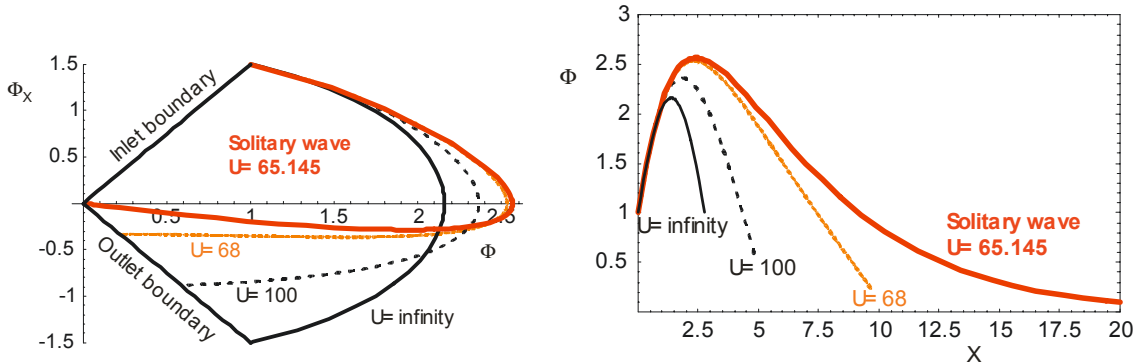
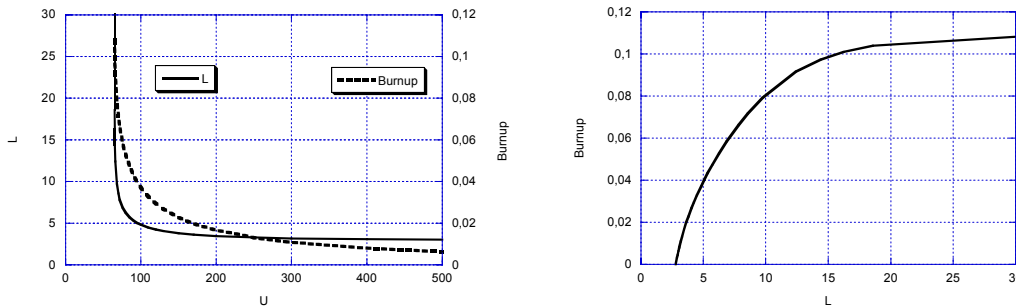
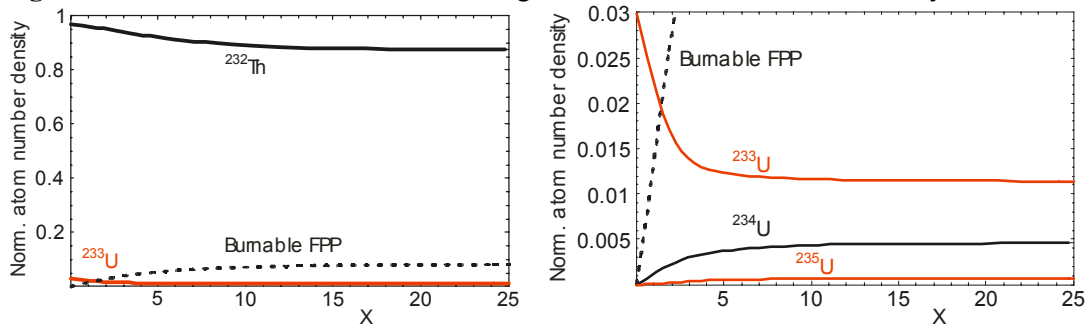


Figure 6: The core length L and the burnup vs. the fuel speed U (left) and the burnup vs. L (right).



Finally, as an example, we show the distribution of nuclide atom number densities along the core axis in the case of the solitary wave solution for $U = 65.145$ and an infinitely long core in Fig. 7 below. The associated neutron flux has been shown in Fig. 5.

Figure 7: Nuclide atom number densities along the core axis in the case of the solitary wave solution.



4. Conclusion

By studying the one-dimensional diffusion-burnup coupled neutron model, a family of analytic solution has been found for the pebble-bed type reactor, in which the fuel drift speed is a free-chosen parameter and the core length is determined by the solution. This may be called *fundamental burn-up solution* for the pebble-bed type reactor. Among this solution family, a partial solitary wave solution exists with a minimum fuel drift speed and a maximum burnup. This shows the same burning wave feature as the CANDLE reactor.

It has been observed that the burnup of the “pebble-bed reactor” considered is around 10at%, which is several times less than the theoretical value for a fast CANDLE reactor. There are two reasons for the lower burnup. First, we have chosen a fairly small enrichment for the fresh fuel. Higher enrichment, maybe together with burnable poison, would lead to higher fuel burnup. Second, the typical cross sections given in Tab. 1 are only valid for a thermal Maxwellian neutron spectrum, with a conversion ratio of about 0.4 for the fresh fuel in our current example. If using more representative cross sections e.g. averaged over a spectrum of an existing HTGR, the conversion ratio for the fresh fuel might be close to or even slightly exceed unity, so that a higher fuel burnup could be achieved.

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