Interest of the non linear fitting method for reactivity assessment using flux transient experiments

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ABSTRACT

Flux transient measurements, meaning rod drop or source jerk experiments, are useful to estimate subcritical core reactivity or control rod worth. Among numerous analysis methods, the most widely used is the so called “inverse kinetics” method (IK). Based on the inversion of the counting rates, this method gives very good results when counting rates are high. When assessing far subcritical levels with low counting rates, it appears that results are biased and very imprecise. In order to overcome those problems in the case of measurements performed in the framework of the first phase of the RACE-T program, we used a non linear fitting method (NF) to analyse transient experiments.

In this paper, we present the NF method reactivity estimator and study its behaviour, in terms of bias and uncertainties, on simulated transients. Then, RACE-T results on experimental source jerk measurements, obtained using IK and NF, are compared and discussed.

KEYWORDS: Source jerk, rod drop analysis, non linear fit method, X-MODE instrumentation

1 INTRODUCTION

In the framework of the RACE-T experimental program on ADS study, formerly named TRADE [1], source jerk experiments were conducted on the TRIGA reactor facility of the ENEA centre at Casaccia, Italy. Three core configurations, namely SC0, SC2 and SC3, were investigated. The experimental multiplication factors were respectively about 0.997, 0.977 and 0.959.

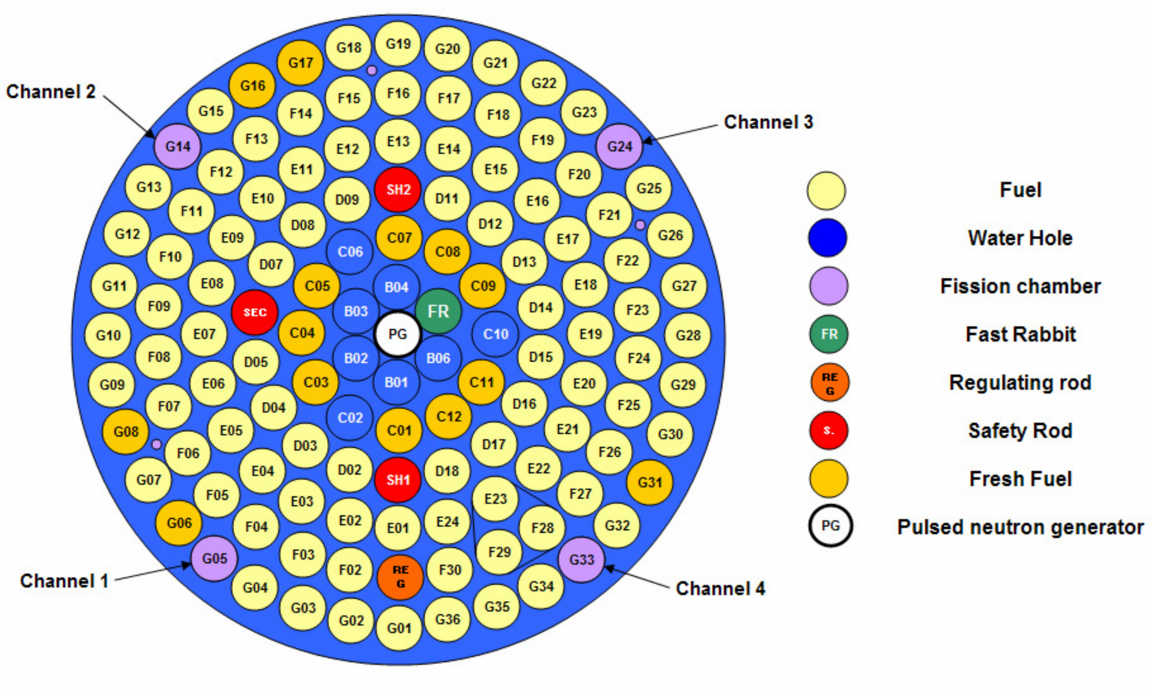
Using a Californium source of about 0.4 Ci, we observed very low counting rates in the case of the SC2 configuration: initial counting rates were about 600 counts per second (cps) and final counting rates were about 20cps. As it is demonstrated here, the inverse kinetics method (IK) is not the best estimator of the reactivity when analyzing such transients. Instead, a non linear fitting method (NF) based on a standard least squares algorithm was used and provided more precise results in that case.

Moreover, the use of the least squares theory provides means of calculating the reactivity uncertainty. Compared to the IK method, for which the uncertainty can only be estimated by Monte Carlo, the NF uncertainty estimation is more precise.

1.1 Experimental setup

Four Photonis CFUM-18 fission chambers were used to monitor the flux. Their sensitivity is about 0.01 cps.n.cm^{-2}s^{-1}. Signals were acquired and stored using an X-MODE acquisition platform running in time marking mode [2]. Finally, the Californium source was inserted and withdrawn from the core using a Fast Rabbit (FR) instrumentation [3]. Figure 1 show the location of the detectors and the source in the SC2 configuration.

Figure 1: RACE-T SC2 configuration



2 THE NON LINEAR FITTING METHOD

The NF method is based on the resolution of the point kinetics equations. Let P be the constant parameters of the reactor (such as the nuclear data parameters) and θ the q parameters to be estimated (e.g. the reactivity), our model calculating the coutning rates $\hat{n}$ versus time is written:

$$\hat{n} = f\left(t, P, \theta\right) \tag{1}$$

Knowing the N experimental samples of the counting rates versus time $n(t)$, a standard non linear least squares optimization gives an estimation $\tilde{\theta}$ of θ. The goodness of fit can be assessed using the standard *RMSE* value (Root Mean Squarred Error).

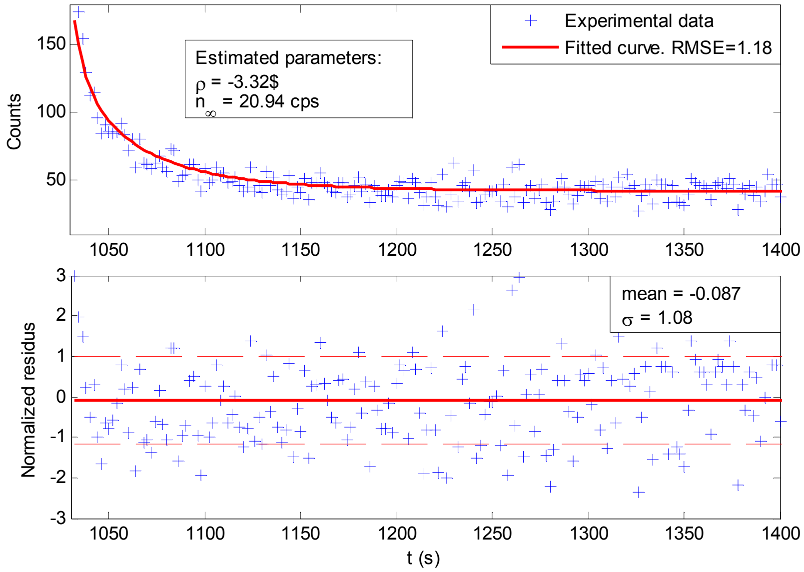
$$RMSE = \frac{1}{N-q}\sqrt{\sum_{i=1}^{N}\frac{\left(n_i - \hat{n}_i\right)^2}{\hat{n}_i}} \tag{2}$$

Our algorithm is based on an analytical solution of the kinetic equations with the hypothesis that the source extraction is instantaneous. The analytical solution is calculated using a well known reactor transfer function decomposition [4]. Two parameters to be estimated were chosen, namely the reactivity ρ and the final counting rate n_∞. It is worth noticing that those parameters are correlated. Indeed, if S is the effective neutron source, they are related to each other by the source multiplication formula:

$$S = -\frac{\rho}{\Lambda} n_{\infty} \tag{3}$$

Figure 2 shows the results of the NF method on an experimental transient.

Figure 2: Experimental fit and residus from NF method



2.1 Uncertainty estimation

By linearizing the problem in the vicinity of the solution $\tilde{\theta}$, one can demonstrate that the covariance matrix of the estimated parameters can be written:

$$\mathrm{V}\left[\tilde{\theta}\right] = \left[J\left(\tilde{\theta}\right)^{T} \mathrm{V}\left[n\right]^{-1} J\left(\tilde{\theta}\right) \right]^{-1} \tag{3}$$

$\mathrm{V}[n]^{-1}$ is the covariance matrix of the counting rates and $J(\tilde{\theta})$ is the so-called jacobin matrix, which is expressed as following:

$$J\left(\tilde{\theta}\right) = \left.\frac{\partial f}{\partial \theta}\right|_{\theta=\tilde{\theta}} \tag{4}$$

The uncertainty due to nuclear data is obtained using the error propagation formula:

$$\sigma_{\rho}^{[ND]} = \sqrt{J_{ND}^{t}\,\mathrm{V}\left[ND\right] J_{ND}} \tag{6}$$

where ND stands for nuclear data. $\mathrm{V}[ND]$ is then the covariance matrix of nuclear data. Its value was taken from [5]. J_{ND} is calculated by finite differences.

2.2 Method validation

Before analysing experimental transients with the NF method described previously, we used simulated transients to study its behaviour in terms of uncertainty and bias. In particular, our purpose was to identify the most influencial parameters. Simulations were conducted using the kinetics resolution algorithm (1). When needed, the measurement noise was simulated by adding poissonian noise to calculated transients.

2.2.1 Uncertainty calculations

First of all, uncertainty calculations were tested on transients obtained for varied reactivities. The following figures show the good agreement between measured and calculated uncertainties up to -5$. Measured uncertainties refer to values obtained by Monte Carlo simulation.We consider the measured values as “reference value” in order to validate our calculations.

Figure 3: Comparison between measured and calculated statistical uncertainty

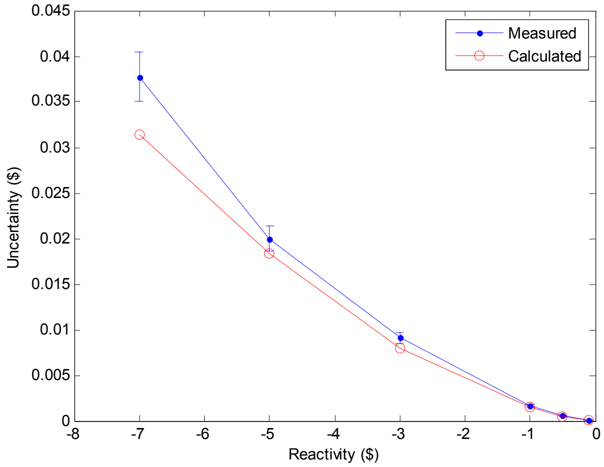
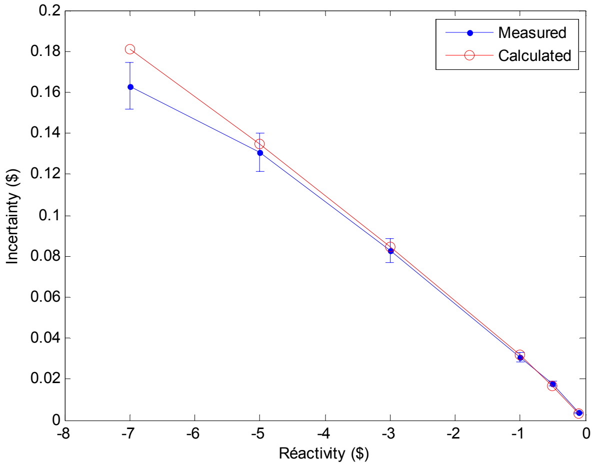


Figure 4: Comparison between measured and calculated nucelar data uncertainty



2.2.2 Influential parameters

In the case of the measurements presented in this paper, we have identified three very influential analysis parameters, namely the starting time of the transient t_0, the length of the fitting range T and the fit starting time t_1.

It is unfortunate that we didn't have any experimental information on the starting time of the source jerk. So, there is an uncertainty on that parameter that must be taken into account in the estimation of the results overall uncertainty. t_0 is chosen visually by the experimentor on the counting rates curve. Then, we consider that the error on t_0 follows a uniform law of probability with a width equal to the measurement bin width dt. The standard deviation of t_0 is then equal to $dt/\sqrt{12}$. If $S_{\rho/t0}$ is the sensitivity of the reactivity to t_0, the uncertainty due to t_0 is given by:

$$\sigma_{\rho}^{[t_0]} = \left| S_{\rho/t_0} \right| \sigma_{t_0} \qquad (7)$$

We measured a strong sensitivity of -0.2\$.s^{-1}. In the case of $dt = 1$s, that implies an additional uncertainty of about 1.6%. In order to limit the uncertainty on the results, we had to chose t_0 as precisely as possible.

The parameter T is influencial on the bias of the results. In order to estimate that influence, we simulated a transient close to the experimental ones (ρ = -3.5\$) and studied the evolution of the mean estimation versus T. Figure 5 shows that the fit range should not be taken greater than 300s. That limit is consistent with our calculation of the reactivity sensitivity to measured counting rates. Indeed, we see on Figure 6 that only the first 200s are really influential on the reactivity estimation. Finally, we made sure (Figure 7) that the uncertainty did not suffer from the limitation on T.

Figure 5: Evolution of the mean estimation versus the length of the fitting range

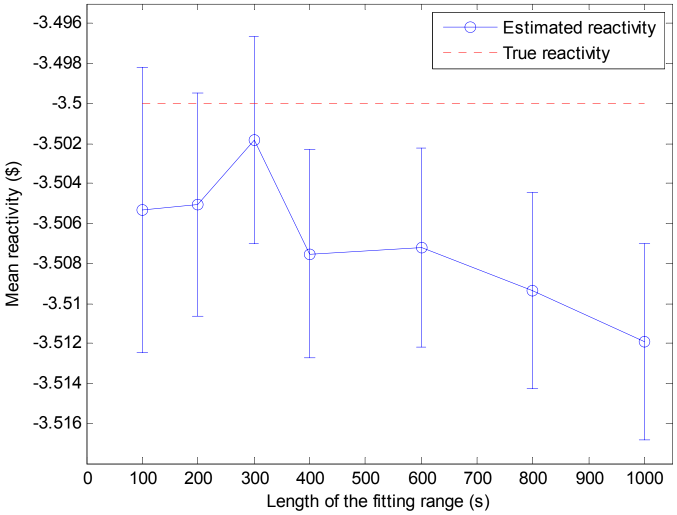


Figure 6: Reactivity sensitivity to measured counting rates

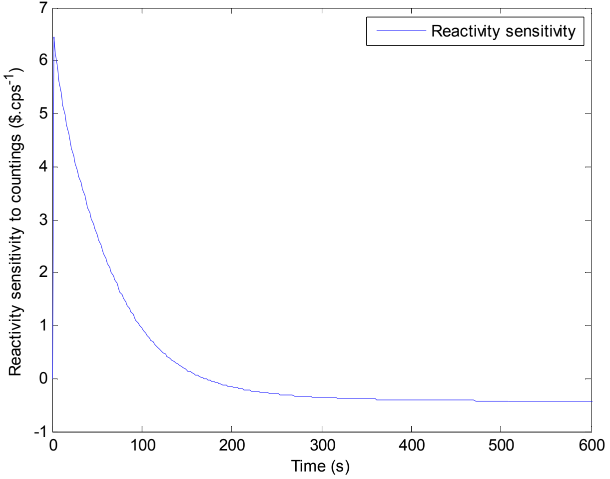


Figure 7: Reactivity uncertainty versus length of the fit

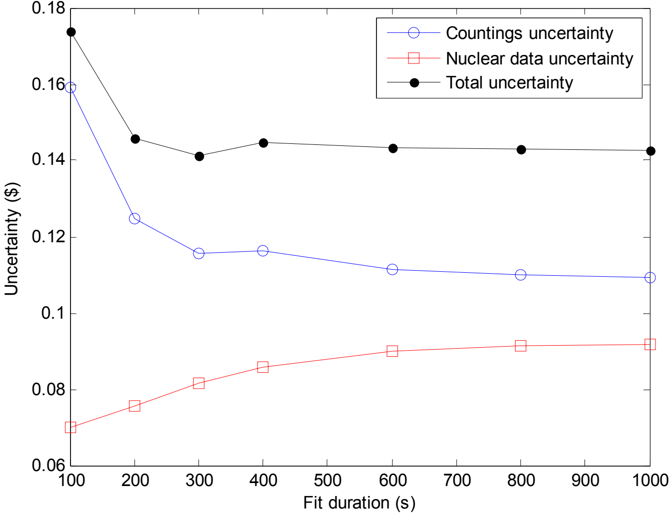
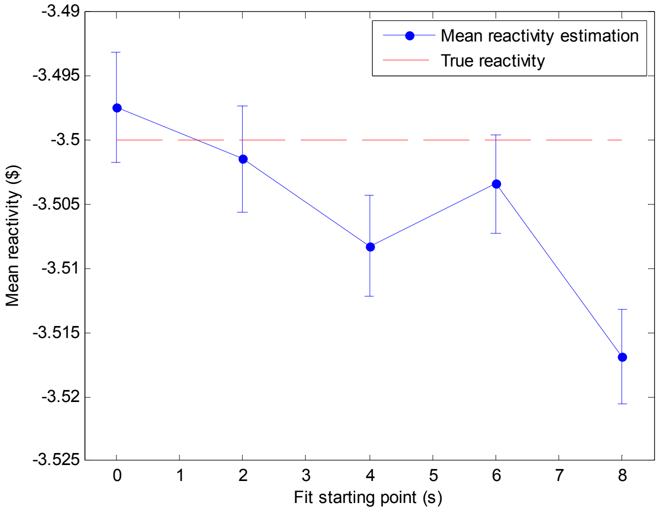


Figure 8: Influence of the fit starting point on reactivity estimation



The last influential analysis parameters is the fit starting point t_1. We used once again Monte Carlo simulations in order to study its influence on results. Like T, t_1 can induce biased results (Figure 8). The simulations allowed us to chose t_1 to be equal to t_0+2s.

3 RESULTS

3.1 Results comparison

Seven source jerk measurements performed in the SC2 core configuration are presented here. Although the core reactivity is only about -3.5 $, the counting rates are quite low. For this configuration, initial counting rates were nearly equal to 600cps and final counting rates were about 20cps.

In order to get the best results from the NF method, the following methodology was applied to each transient:

- t_0 was precisely chosen on a 10ms bin width curve, but the fit was applied on a 2s bin width curve,
- t_1 was taken such as t_1=t_0+2s,
- the fitting range was chosen to be about two times the length of the transient (about 300s).

For the analysis using the IK method, influential parameters have also been studied the same way as for the NF method. It is not our purpose to detail here their influence but we had to adopt a compromise between uncertainty and bias induced mostly by t_1 and T. The study leads to the following optimum parameters:

- the bin width was chosen equal to 0.5s
- t_1 was taken such as t_1=t_0+5s,
- T was chosen nearly equal to 300s.

In the following tables, $\sigma_{\rho_S}^{[C]}$ refers to the uncertainty on the reactivity due to countings and $\sigma_{\rho_S}^{[ND]}$ refers to the uncertainty due to nuclear data. σ_{ρ_S} is the total uncertainty of the results.

As one can see, the results dispersion is better for NF results than for IK results. Dispersion is about 4% for the former and 10% for the latter. Moreover, NF results are more consistent with reactivity estimations obtained by pulsed neutron source experiments [6], which were about -3.35 $.

Table 1: Reactivity estimation by NF for channel 1

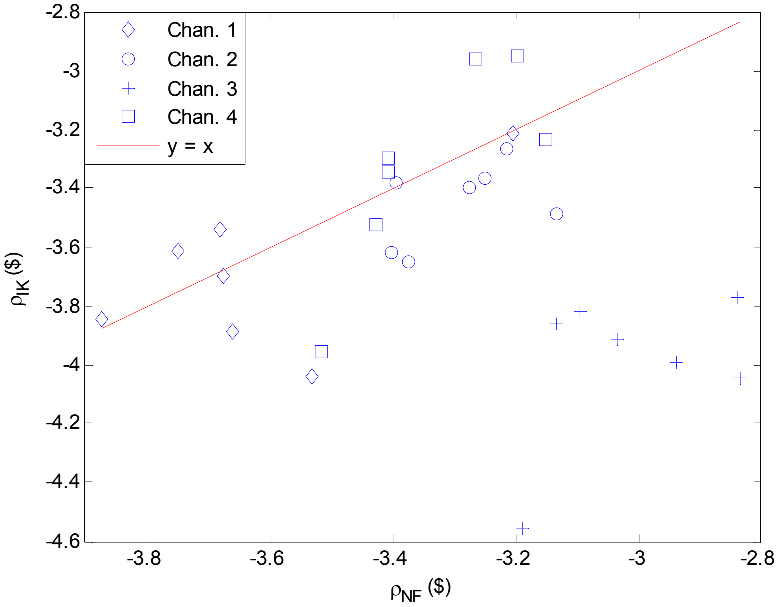
Exp. n°	ρ ($)	$\sigma_{\rho_S}^{[C]}$ (%)	$\sigma_{\rho_S}^{[ND]}$ (%)	σ_{ρ_S} (%)
1	-3.66	3.3	2.7	4.3
2	-3.87	3.5	2.6	4.4
3	-3.20	3	2.7	4.1
4	-3.67	3.3	2.9	4.3
5	-3.75	3.2	2.8	4.3
6	-3.53	3.2	2.8	4.2
7	-3.68	3.7	2.2	4.3
Mean	-3.62	3.3	2.7	4.3

Table 2: Reactivity estimation by IK for channel 1

Exp. n°	ρ ($)	$\sigma_{\rho_s}^{[C]}$ (%)	$\sigma_{\rho_s}^{[ND]}$ (%)	σ_{ρ_s} (%)
1	-3.89	6.1	3.4	7
2	-3.85	6.2	3.4	7.1
3	-3.21	5.6	3.4	6.5
4	-3.7	5	3.2	6
5	-3.61	5.6	3.4	6.6
6	-4.04	6	3.3	6.9
7	-3.54	5.1	3.2	6
Mean	-3.7	5.7	3.3	6.6

Finally, by plotting IK versus NF results (Figure 9), it appears clearly that the formers are biased compared to the latters. IK estimations are in average below NF estimation, especially for channel 3. But it appears that detector 3 is the least sensitive, so that transients from channel 3 show the lower counting rates (about 300cps before the jerk and 5cps after). This tends to prove that the IK method provides biased results when counting rates are very low.

Figure 9: IK results versus NF results



3.2 Discussion

As stated before, NF results are consitent with PNS results, especially for channel 2 and 4 (Table 4 and Figure 10). The reactivity gap between source jerk results and PNS results for the other two channels comes from space effects in the reactor. Indeed, as one can see on Figure 1, channel 2 and 4 are at the same distance of the source, whereas channel 1 is further and channel 3 is closer. On the contrary, PNS estimation has been estimated using the neutron generator placed at the center of the core. That explains why channel 3 overestimates the reactivity whereas channel 1 underestimates it.

Moreover, thanks to the simulation study, we have taken into account all sources of uncertainty: the reproduce uncertainty is of the order of the calculated uncertainty (Table 3).

Table 3: Comparison between results dispersion and calculated uncertainty. Uncertainties is expressed in cents.

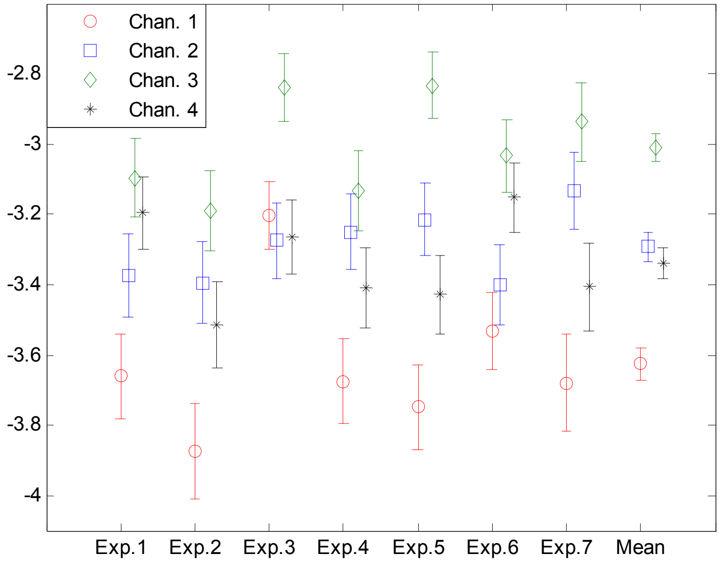
Channel	$\sigma_\rho^{[E]}$ (c)	$\sigma_\rho^{[C]}$ (c)
1	21 ± 6	12
2	10 ± 3	11
3	14 ± 4	11
4	13 ± 4	11

We can now precisely calculate the uncertainty on the mean reactivity over our 7 experiments. It turns out that the statistical uncertainty about 1.3% is negligible compared to that due to nuclear data (about 3.3%).

Table 4: Final reactivity estimation

Channel	PNS reactivity estimation ($)	Mean reactivity ($)	Statistical uncertainty (%)
1	-3.39	-3.62	1.27
2	-3.38	-3.30	1.27
3	-3.32	-2.99	1.33
4	-3.32	-3.33	1.27

Figure 10: SC2 configuration reactivity estimations



4 CONCLUSION

In this paper, we demonstrated that the inverse kinetics method is not the best estimator of the reactivity in the case of source jerk experiments conducted at the ENEA TRIGA reactor. Instead,

we used a non linear fitting method which provides more precise and reproducible results, which are consistent with PNS measurements. Indeed, while uncertainty due to nuclear data is almost constant about 3%, statistical uncertainty was decreased from about 5.7% to 3.3%. As a result, the statistical uncertainty of the mean reactivity over 7 experiments is negligible compared to nuclear data uncertainty.
Moreover, the NF method was presented here and the uncertainty calculations were validated using point kinetics simulated transients. Those precise uncertainty calculations are a great asset of the NF method compared to the IK method. We proposed a methodology in order to limit the uncertainty ans bias due to several analysis parameters.

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