

Methodology of 3-D Neutron-Physical Calculation Analysis of Axial-symmetrical Nuclear Reactor Cell with Finite Step along the Axis

Tamara S. Poveschenko* and Nickolay Laletin

Russian Research Center "Kurchatov Institute", 1, Kurchatov Squ., Moscow, 123182, Russia

Abstract

Methodology for 3-D calculation analysis of nuclear reactor cell with axial symmetry and finite mesh step is described. This methodology is based on the axial leakage calculation analysis method that has been developed for nuclear reactor with closed lattice like VVER-type. The trial functions that are used at full core level of nuclear reactor calculation analysis are defined. Analytical solutions of two-group diffusion equation with "symmetrical" and "cross" boundary conditions are given. Two-group cell characteristics that are matrixes of boundary values of these functions have been calculated. Calculated values of these matrixes have been analyzed in dependence on the step mesh along Z(axis) value. These results show that if step mesh along Z-axis is enough small then diagonal elements are closed to initial diffusion parameters of the cell and not diagonal elements that are corrections caused by curvature of neutron flux can be neglected. But in contrary, if step mesh is more than diffusion length then these corrections are essential and they should be taken into account at full core calculation analysis level. These methodology can be used in full core nuclear reactor calculation with Surface Value System with finite step mesh along Z-axis.

KEYWORDS: *nuclear reactor cell, axial leakage, neutron transport equation, diffusion coefficient, boundary conditions, two-group equations*

1. Introduction

Method of nuclear reactor core calculation analysis called as Surface Value System (SVS) has been developed since last century, eighty in Russia. General principles of this method are formulated in the paper [1]. This method is based on the principle of two levels of full core calculation analysis: the first one is cell (assembly) level and the second one is full core level. In this context boundary conditions for nuclear reactor cell problem have been formulated. Importance from physical meaning point of view of the neutron transport equation solution with entered current ("symmetrical") solution and neutron transport equation solution with cross current ("cross") solution has been stressed. "Symmetrical" solution is responsible for neutron absorption and multiplication in the cell, and "cross" solution is responsible for neutron diffusion in the cell.

Physical structure of nuclear reactor with axial symmetry leads to difference of radial leakage calculation analysis and axial leakage calculation analysis. Method for calculation analysis of

* Corresponding author, Tel. (095) 196 9409, Fax. (095) 196 7016, E-mail: povesch@dhtpkia.ru

axial leakage of nuclear reactor cell with axial symmetry has been described. This method is based on the approach of Russian authors [2,3] that has been developed to take into account correctly the neutron escape through coolant zones. If material composition along Z-axis is uniform then neutron distribution function is slowly changed along this direction so linear axial dependence of the flux vs. z is directly assumed. So decomposition of 3-D problem had been assumed and as consequence 2-D axial problem and 1-D axial problem have been considered. Axial diffusion coefficient is defined with direction probabilities P_{ij}^{zg} . Note that these coefficients differ from Benoist direction probabilities [4] and it is needn't to assume isotropy of angular flux to calculate them. Diffusion coefficients calculated with this methodology take into account influence of multi group neutron spectrum of 2-D problem into axial leakage value. Calculation analysis of nuclear reactor with closed lattice (VVER-type) has confirmed that this effect is enough essential. It has already mentioned that this approach supposes linear approximation of neutron flux along Z-axis so mesh step along this direction has to be enough small (neutron free path only). Thus, in order to be able to solve 3-D neutron transport equation with finite (large) step along Z-axis for nuclear reactor with axial symmetry, correct formulation of boundary conditions on the butt-ends of the cell is needed

In this context focus of our paper is distribution of above described approach into 3-D nuclear reactor cell with finite (large) step along Z-axis. So this approach is used as the base for methodology which takes into account effect of neutron flux into axial leakage and admits using of finite mesh step along Z-axis. It is possible to achieve this purpose if boundary conditions on the butt-ends of the cell are formulated correctly.

2. Methodology of 3-D Calculation Analysis

2.1 Brief description of The Method for Axial Leakage Calculation Analysis in Nuclear Reactor Cell with Axial Symmetry.

Integral equation for neutron flux $\Phi(\vec{r}, E)$ with continual energy dependence in case of isotropic scattering is considered as the basic stage of this calculation analysis method

$$\Phi(\vec{r}, E) = \int dV' \frac{\exp(-\tau(\vec{r}, \vec{r}'))}{4\pi|\vec{r} - \vec{r}'|^2} \left[\int \sigma(\vec{r}', E' \rightarrow E) \Phi(\vec{r}', E') dE' + Q(\vec{r}', E', E) \right] \quad (1)$$

Following designates are used here

$\tau(\vec{r}', \vec{r})$ is the optical distance between (\vec{r}', r)

$Q(\vec{r}, E', E) = \chi(E) \nu \sigma^f(\vec{r}, E') \Phi(\vec{r}', E')$

$\chi(E)$ is fission neutron spectrum

ν is average fission neutron number

$\sigma^f(\vec{r}, E)$ is fission cross section

$\sigma(\vec{r}, E' \rightarrow E)$ is differential scattering cross section

Subdivision of energy value into multi-group intervals comes to multi-group approximation.

$$\Phi_g(\vec{r})ds = \left[\int d\vec{r}' \frac{e^{-\tau_g(\vec{r},\vec{r}')}}{4\pi|\vec{r}-\vec{r}'|^2} \left(\sum_{g'} \sigma_{g' \rightarrow g}(\vec{r}')\Phi_{g'}(x',y',z') + Q(\vec{r}') \right) \right] ds$$

Main approximation of this method is linear approximation of the flux along Z direction; in other words $\Phi^g(\vec{r})$ is slowly changed function within neutron free path along Z. Then one can use Taylor expansion along Z-axis and take into account two terms only; and α is supposed the same for all groups:

$$\Phi_g(x,y,z) = \Phi_{0g}(x,y) + \alpha z \Phi_{1g}(x,y) \tag{2}$$

It is not difficult to prove [2] that

$$\frac{d\Phi_g}{dz}(x,y,0) = \Phi_g(x,y,0) \tag{3}$$

Further we are interesting in group current along Z-axis. To take into account equation (1) we can write the equation for this value:

$$I_g(\vec{r})ds = \left\{ \int_V d\vec{r}' \frac{e^{-\tau_g(\vec{r},\vec{r}')}}{4\pi|\vec{r}-\vec{r}'|^2} \left[\sum_{g'} \sigma_{g' \rightarrow g}(\vec{r}')\Phi_{g'}(x',y') + Q_g(x',y') \right] |\vec{r}-\vec{r}'| \cos^2 \theta \right\} ds \tag{4}$$

So we can naturally define effective multi-group axial diffusion coefficient as following:

$$D_g = \frac{\bar{I}_g}{\bar{\Phi}_g} \tag{5}$$

Here \bar{I}_g means neutron current value along Z for group “g” averaged over the cell; and $\bar{\Phi}_g$ means neutron flux for group “g” averaged over the cell. To calculate averaged current subdivision of the cell into a set of uniform meshes is used and neutron flux as well the current are supposed to be constant within every mesh. Then following equations are coming

$$I_g^i V^i = \sum_{j=1}^N P_{ij}^{zg} \sum_{g'=1}^G (\Phi_{g'}^j \sigma_{g' \rightarrow g} + Q_g^j); \tag{6}$$

The coefficients are designated:

$$P_{ij}^{zg} = \int_{V_i} d\vec{r}' \int_{V_j} d\vec{r}'' \frac{e^{-\tau(\vec{r},\vec{r}'')} \cos^2 \theta}{4\pi|\vec{r}-\vec{r}''|} \tag{7}$$

These coefficients P_{ij}^{zg} connect the group components of the current along Z-axis with the group components of neutron flux over (X,Y) so they are called as “direction probabilities”. Note that these direction probabilities differ from those ones defined in Benoist leakage method.[4]

2.2 Definition of trial function for calculation

Method of calculation analysis of axial diffusion coefficients has been described.[2,3]. These diffusion coefficients are used in methodology for calculation of trial function that are used for

full core reactor calculation. These trial functions are defined in [1]. They are as following:

$\bar{\phi}(x)$ is “symmetrical” trial functions that is responsible for absorption and multiplication processes in the cell;

$\bar{\psi}(x)$ is “cross” trial functions that is responsible for diffusion processes in the cell.

:On this stage it is supposed that material composition along Z is uniform so diffusion approximation is true.

$$\Delta\Phi_1 - \frac{1}{\tau}\Phi_1 + \frac{D_2}{D_1} \frac{k}{L^2}\Phi_2 = 0 \quad (8)$$

$$\Delta\Phi_2 - \frac{1}{L^2}\Phi_2 + \frac{D_1}{D_2} \frac{1}{\tau}\Phi_1 = 0 \quad (9)$$

Boundary conditions are defined for these functions accordingly as

$$\bar{\phi}(a) = \bar{\phi}(-a) \quad (10)$$

$$\bar{\psi}(a) = -\bar{\psi}(-a) \quad (11)$$

The components of “symmetrical” matrix are defined as boundary values of the functions with following conditions

$$D_1 \frac{d\phi_1}{dz}(-a) = 0 \quad \text{and} \quad D_1 \frac{d\phi_1}{dz}(-a) = 1 \quad (12)$$

$$D_2 \frac{d\phi_2}{dz}(-a) = 1 \quad D_2 \frac{d\phi_2}{dz}(-a) = 0 \quad (13)$$

The components of “cross” matrix are defined as the boundary values of the function with following conditions

$$D_1 \frac{d\psi_1}{dz}(-a) = 1 \quad \text{and} \quad D_1 \frac{d\psi_1}{dz}(-a) = 0 \quad (14)$$

$$D_2 \frac{d\psi_2}{dz}(-a) = 0 \quad D_2 \frac{d\psi_2}{dz}(-a) = 1 \quad (15)$$

2.3 Analytical solution of the system of diffusion equation with symmetrical and “cross” boundary conditions.

In this paragraph analytical solution of the equation system will be obtained. For this purpose uniform system of diffusion equations is considered.[5]

$$\Delta\Phi_1 + B^2\Phi_1 = 0 \quad (16)$$

$$\Delta\Phi_2 + B^2\Phi_2 = 0. \quad (17)$$

This is standard procedure that defines significance of B^2 that admits existence of the decision of system of the equations:

$$\mu^2 = \frac{1}{2} \left[-\left(\frac{1}{\tau} + \frac{1}{L^2}\right) + \sqrt{\left(\left(\frac{1}{\tau} + \frac{1}{L^2}\right)^2 + \frac{4(k-1)}{\tau L^2}\right)} \right] \quad (18)$$

$$-v^2 = \frac{1}{2} \left[-\left(\frac{1}{\tau} + \frac{1}{L^2} \right) - \sqrt{\left(\left(\frac{1}{\tau} + \frac{1}{L^2} \right)^2 + \frac{4(k-1)}{\tau L^2} \right)} \right] \quad (19)$$

The solutions that are satisfied to symmetrical conditions:

$$X = \cos \mu z ; \quad Y = ch \nu z \quad (20)$$

So the group solutions are as linear combinations of these basic solution.

$$\varphi_1 = AX + CY \quad (21)$$

$$\varphi_2 = AS_1 X + CS_2 Y \quad (22)$$

The constants A,C are defined with boundary conditions;

$$S_1 = \frac{D_1}{\tau D_2} \frac{1}{1/L^2 + \mu^2} ; \quad S_1 = \frac{D_1}{\tau D_2} \frac{1}{1/L^2 + \mu^2} \quad (23)$$

Appropriately the solutions that are satisfied to “cross” boundary conditions

$$Z = \sin \mu z ; \quad W = sh \nu z \quad (24)$$

And the group solutions are also as linear combination of these basic solutions

$$\psi_1 = BZ + DW \quad (25)$$

$$\psi_2 = AS_1 X + CS_2 Y \quad (26)$$

After appropriate transformations we have as result two sets of the constants for symmetrical matrix $\hat{\phi}$. The first one is as following:

$$A = \frac{1}{D_1 \sin \mu a \mu \left(\frac{S_1}{S_2} - 1 \right) \mu \sin \mu a} ; \quad C = A \frac{S_1}{S_2} \frac{\mu \sin \mu a}{vsh \nu a} \quad (27)$$

The second one is as following:

$$A = \frac{1}{D_2 (S_2 - S_1) \mu \sin \mu a} ; \quad C = A \frac{\mu \sin \mu a}{vsh \nu a} \quad (28)$$

Also after transformations we have two sets of the constants for matrix $\hat{\psi}$

$$B = \frac{1}{D_1 \left(1 - \frac{S_1}{S_2} \right) \mu \cos \mu a} ; \quad D = -B \frac{S_1}{S_2} \frac{\mu \cos \mu a}{vch \nu a} \quad (29)$$

Elements of symmetrical matrix are as following expressions:

$$\begin{aligned}
 \varphi_{11} &= \frac{2}{a\Sigma_1} \frac{1}{k-1} \frac{L^2}{\mu^2 + \nu^2} \left[\nu^2 \left(\mu^2 - \frac{1}{L^2} \right) \frac{z_1}{thz_1} - \mu^2 \left(\frac{1}{L^2} + \nu^2 \right) \frac{z_2}{tgz_2} \right] \\
 \varphi_{21} &= -\frac{2}{a\Sigma_1} \frac{1}{k-1} \frac{1}{\mu^2 + \nu^2} \left(\nu^2 \frac{z_1}{thz_1} + \mu^2 \frac{z_2}{tgz_2} \right) \\
 \varphi_{12} &= -\frac{2}{a\Sigma_2} \frac{1}{k-1} \frac{1}{\mu^2 + \nu^2} \left(\nu^2 \frac{z_1}{thz_1} + \mu^2 \frac{z_2}{tgz_2} \right) \\
 \varphi_{22} &= \frac{2}{a\Sigma_2} \frac{1}{k-1} \frac{\tau}{\mu^2 + \nu^2} \left[\nu^2 \left(\mu^2 - \frac{1}{\tau} \right) \frac{z_1}{thz_1} - \mu^2 \left(\nu^2 + \frac{1}{\tau} \right) \frac{z_2}{tgz_2} \right]
 \end{aligned} \tag{30}$$

Further elements of “cross” matrix ψ are as following expressions:

$$\begin{aligned}
 \psi_{11} &= \frac{a}{2} \frac{1}{\mu^2 + \nu^2} \frac{1}{D_1} \left[\left(\frac{1}{\tau} + \nu^2 \right) \frac{thz_1}{z_1} - \left(\frac{1}{\tau} - \mu^2 \right) \frac{tgz_2}{z_2} \right] \\
 \psi_{21} &= -\frac{a}{2} \frac{1}{\mu^2 + \nu^2} \frac{1}{D_2} \left(\frac{thz_1}{z_1} - \frac{tgz_2}{z_2} \right) \\
 \psi_{12} &= -\frac{a}{2} \frac{1}{\mu^2 + \nu^2} \frac{k_\infty}{D_1 L^2} \left(\frac{thz_1}{z_1} - \frac{tgz_2}{z_2} \right) \\
 \psi_{22} &= \frac{a}{2} \frac{1}{\mu^2 + \nu^2} \frac{1}{D_2} \left[\left(\frac{1}{L^2} + \nu^2 \right) \frac{thz_1}{z_1} - \left(\frac{1}{L^2} - \mu^2 \right) \frac{tgz_2}{z_2} \right]
 \end{aligned} \tag{31}$$

We can calculate the determinates of these matrixes

$$Det(\varphi) = -\frac{2}{a\Sigma_1} \frac{2}{a\Sigma_2} \frac{z_1}{thz_1} \frac{z_2}{tgz_2} \frac{1}{k-1} \tag{32}$$

$$Det(\psi) = \left(\frac{a}{2} \right)^2 \frac{1}{D_1} \frac{1}{D_2} \frac{thz_1}{z_1} \frac{tgz_2}{z_2} \tag{33}$$

The designates used here are as following:

$$z_1 = \frac{\mu a}{2}; \quad z_2 = \frac{\nu a}{2}; \quad \Sigma_1 = \frac{D_1}{\tau}; \quad \Sigma_2 = \frac{D_2}{L^2} \tag{34}$$

Further we give elements of recursive matrixes φ^{-1} and ψ^{-1} because of they allow to estimate the corrections that are concerned just to large mesh step along Z axis. For matrix $\hat{\varphi}$ they are as following

$$\begin{aligned}
 (\varphi^{-1})_{11} &= \frac{a}{2} \frac{\Sigma_1 \tau}{\mu^2 + \nu^2} \left[\mu^2 \left(\frac{1}{\tau} + \nu^2 \right) \frac{thz_1}{z_1} + \nu^2 \left(\frac{1}{\tau} - \mu^2 \right) \frac{tgz_2}{z_2} \right] \\
 (\varphi^{-1})_{12} &= -\frac{a}{2} \frac{\Sigma_2 k_\infty}{\mu^2 + \nu^2} \left(\mu^2 \frac{thz_1}{z_1} + \nu^2 \frac{tgz_2}{z_2} \right) \\
 (\varphi^{-1})_{21} &= -\frac{a}{2} \frac{\Sigma_1}{\mu^2 + \nu^2} \left(\mu^2 \frac{thz_1}{z_1} + \nu^2 \frac{tgz_2}{z_2} \right) \\
 (\varphi^{-1})_{22} &= \frac{a}{2} \frac{\Sigma_2 L^2}{\mu^2 + \nu^2} \left[\mu^2 \left(\frac{1}{L^2} + \nu^2 \right) \frac{thz_1}{z_1} + \nu^2 \left(\frac{1}{L^2} - \mu^2 \right) \frac{tgz_2}{z_2} \right]
 \end{aligned} \tag{35}$$

Further, for matrix ψ they are as following

$$\begin{aligned}
 (\psi^{-1})_{11} &= \frac{2}{a} \frac{D_1}{\mu^2 + \nu^2} \left[\left(\frac{1}{L^2} + \nu^2 \right) \frac{z_2}{tgz_2} - \left(\frac{1}{L^2} - \mu^2 \right) \frac{z_1}{thz_1} \right] \\
 (\psi^{-1})_{12} &= \frac{2}{a} \frac{D_2}{\mu^2 + \nu^2} \frac{k_\infty}{L^2} \left(\frac{z_2}{tgz_2} - \frac{z_1}{thz_1} \right) \\
 (\psi^{-1})_{21} &= \frac{2}{a} \frac{1}{\mu^2 + \nu^2} \frac{D_1}{\tau} \left(\frac{z_2}{tgz_2} - \frac{z_1}{thz_1} \right) \\
 (\psi^{-1})_{22} &= \frac{2}{a} \frac{D_2}{\mu^2 + \nu^2} \left[\left(\frac{1}{\tau} + \nu^2 \right) \frac{z_2}{tgz_2} - \left(\frac{1}{\tau} - \mu^2 \right) \frac{z_1}{thz_1} \right]
 \end{aligned} \tag{36}$$

2.4 Two-group cross sections of diffusion equations

In this section we consider the procedure of calculation of the coefficients of 1-D two group diffusion equations that is used to calculate neutron flux distribution along Z-axis in the frame of linear approximation of neutron flux. The equations are written as following:

$$\begin{aligned}
 D_1 \Delta \Phi_1 - \Sigma_{1 \rightarrow 2} \Phi_1 - \Sigma_1 \Phi_1 + \Sigma_{2 \rightarrow 1} \Phi_2 + \nu \Sigma_1^f \Phi_1 + \nu \Sigma_2^f \Phi_2 &= 0 \\
 D_1 \Delta \Phi_1 - \Sigma_{1 \rightarrow 2} \Phi_1 - \Sigma_1 \Phi_1 + \Sigma_{2 \rightarrow 1} \Phi_2 &= 0
 \end{aligned} \tag{37}$$

And the designations that are usually used:

D_1, D_2 are the diffusion coefficients of fast and thermal groups.

$\Sigma_{1 \rightarrow 2}$ is the cross section of neutron transfer from fast group to thermal one

$\Sigma_{2 \rightarrow 1}$ is the cross section of neutron transfer from thermal group to fast one

Σ_2 is absorption cross section of thermal group

Σ_1 is absorption cross section of fast group

$\nu \Sigma_1^f$ is fission cross section of fast group

$\nu \Sigma_2^f$ is fission cross section of thermal group

The coefficients of this system of diffusion equations are provided from 2-D calculation analysis of the cell. The diffusion coefficients D_1, D_2 are obtained from direction probabilities

calculation analysis of the cell [3]. Thus influence of the group neutron flux structure is taken into account. Further, multi group neutron transport calculation analysis of 2D cell is carried out. Boundary conditions are formulated as “zero current” at the boundary. Isotropic reflection on the cylindrical boundary of equivalent cylindrical cell or mirror reflection on real geometrical boundary is used. Then the coefficients $\Sigma_{1 \rightarrow 2}, \Sigma_{2 \rightarrow 1}$ are obtained as averaged values of scattering matrix over spatial-energetic neutron flux distribution.

$$\Phi_g = \frac{\sum_{i=1}^N \Phi_g^i V_i}{V}; \quad \Sigma_{g' \rightarrow g}^s = \frac{\sum_{i=1}^N \sigma_{g' \rightarrow g}^{si} \Phi_g^i V_i}{\Phi_{g'} V} \quad (38)$$

$$\Sigma_{1 \rightarrow 2} = \frac{\sum_{g=G_T+1}^G \sum_{g'=1}^{G_T} \Sigma_{g' \rightarrow g}^s \Phi_{g'}}{\sum_{g=1}^{G_T} \Phi_g} \dots \dots \dots \Sigma_{2 \rightarrow 1} = \frac{\sum_{g=1}^{G_T} \sum_{g'=G_T+1}^G \Sigma_{g' \rightarrow g}^s \Phi_{g'}}{\sum_{g=G_T+1}^G \Phi_g} \quad (39)$$

Two group absorption cross sections are obtained as averaged values of multi group absorption cross sections over spatial-energetic neutron flux distribution

$$\Sigma_1 = \frac{\sum_{g=1}^{G_T} \Phi_g \sigma_g^a}{\sum_{g=1}^{G_T} \Phi_g}; \quad \Sigma_2 = \frac{\sum_{g=G_T+1}^G \Phi_g \sigma_g^a}{\sum_{g=G_T+1}^G \Phi_g} \quad (40)$$

Two-group fission cross sections are obtained as averaged multi group fusion cross sections over spatial-energetic neutron flux distribution

$$\nu \Sigma_1^f = \frac{\sum_{g=1}^{G_T} \nu \Sigma_g^f \Phi_g}{\sum_{g=1}^{G_T} \Phi_g}; \quad \nu \Sigma_2^f = \frac{\sum_{g=G_T+1}^G \nu \Sigma_g^f \Phi_g}{\sum_{g=G_T+1}^G \Phi_g} \quad (41)$$

For the system of diffusion equation written in form (37) we can define neutron age and diffusion length square [5]:

$$\tau = \frac{D_1}{\Sigma_{1 \rightarrow 2} + \Sigma_1^a - \nu \Sigma_1^f}; \quad L^2 = \frac{D_2}{\Sigma_2^a + \Sigma_{2 \rightarrow 1}}; \quad (42)$$

and multiplication factors:

$$K_1 = \frac{\nu \Sigma_2^f + \Sigma_{2 \rightarrow 1}}{D_1}; \quad K_2 = \frac{\Sigma_{1 \rightarrow 2}^s}{D_2} \quad (43)$$

In these designations the roots of characteristic equation are written:

$$B_{1,2} = \frac{1}{2} \left[- \left(\frac{1}{\tau} + \frac{1}{L^2} \right) \pm \sqrt{\left(\frac{1}{\tau} + \frac{1}{L^2} \right)^2 + 4 \left(K_1 K_2 - \frac{1}{\tau L^2} \right)} \right] \quad (44)$$

3. Calculation Results

Calculation analysis of matrixes $\hat{\phi}$ and $\hat{\psi}$ have been carried out for the cell like VVER-type nuclear reactor cell. Material compositions and geometrical data are given in table 1

Table 1 Material composition and geometric parameters for the cell

	Fuel	Clad	Moderator
Radius, cm	0.413	0.474	0.668
Nuclear densities	235-U 8.255 E-04	Zr 4.259 E-02	H 6.671E-02
	238-U 2.210 E-04	Hf 6.597 E-06	O 3.336E-02
	O 4.586 E-02	Nb 4.225 E-04	

For this cell the constants of two-group system of diffusion equation are the following:

$$D_1 = 1.183 \text{ cm}$$

$$D_2 = 0.345 \text{ cm}$$

$$\Sigma_{1 \rightarrow 2} = 0.209 \text{ cm}^{-1}$$

$$\Sigma_{2 \rightarrow 1} = 0.022 \text{ cm}^{-1}$$

$$\Sigma_2 = 0.122 \text{ cm}^{-1}$$

$$\Sigma_1 = 0.0244 \text{ cm}$$

Appropriate significances of the age and the diffusion length square are the following

$$\tau = 48.43 \text{ cm}^2; L^2 = 2.83 \text{ cm}^2$$

The elements of above defined matrixes $\hat{\phi}$ and $\hat{\psi}$ have been calculated for following value of the mesh step along Z axis a: a=0.2L; a=0.5; a=L; a=M; a=2M

L is the diffusion length path; M is the migration length path.

The results of calculations are given in the tables 2,3

Table 2: Elements of inverse symmetrical matrix in dependence on the mesh step along Z

Step of the mesh	$(\phi^{-1})_{11} \text{ cm}$	$(\phi^{-1})_{12} \text{ cm}$	$(\phi^{-1})_{21} \text{ cm}$	$(\phi^{-1})_{22} \text{ cm}$
0.2L	2.42 E-02	2.08 E-01	2.18 E-02	1.21 E-01
0.5L	2.36 E-02	2.04 E-01	2.14 E-02	1.19 E-01
L	2.15 E-02	1.92 E-01	2.09 E-02	1.12 E-01
M	4.12 E-03	9.51 E-02	9.94 E-03	5.35 E-02
2M	5.36 E-03	5.22 E-02	5.46 E-03	2.72 E-02

Table 3: Elements of inverse “cross” matrix in dependence on the mesh step along Z

Step of the mesh	$(\psi^{-1})_{11} \text{ cm}$	$\varphi \text{ cm}$	$(\psi^{-1})_{21} \text{ cm}$	$(\psi^{-1})_{22} \text{ cm}$
0.2L	1.183 E00	-1.96 E-03	-2.06 E-04	3.463 E-01
0.5L	1.184 E00	-1.22 E-02	-1.28 E-03	3.52 E-01
L	1.189 E00	-4.83 E-02	-5.06 E-03	3.73 E-01
M	1.253 E00	-6.96 E-01	-7.28 E-02	7.48 E-01
2M	1.30 E00	-1.92 E00	-2.00 E-01	1.43 E00

These results show that if the step mesh along Z-axis is enough small then the diagonal elements are closed to the initial diffusion parameters of the cell. Not diagonal elements of the matrixes are the corrections caused by the curvature of the neutron flux and they can be neglected in this case. But in contrary if the step mesh is more than diffusion length then these corrections are essential and they should be taken into account at core calculation level.

4. Conclusion

Methodology for 3-D calculation analysis of nuclear reactor cell with axial symmetry and finite mesh step has been described. The trial functions that are needed at the full core level of nuclear reactor calculation analysis have been defined. Two-group cell characteristics that are matrixes of boundary values of these functions with defined boundary conditions have been calculated. Calculated values of these matrixes have been analyzed in dependence on the step mesh value

In conclusion, the following items must be submitted:

1. These results show that if the step mesh along Z-axis is enough small then the diagonal elements are closed to the initial diffusion parameters of the cell constants and the not diagonal elements that are as corrections caused by curvature of the neutron flux can be neglected. But in contrary if the step mesh is more than diffusion length then these corrections are essential and they should be taken into account at full core calculation
2. This methodology can be used in full core nuclear reactor calculation with Surface Harmonics Method when finite step mesh along Z-axis is used.

References

1. N.I. Laletin “On the Equations of Heterogenous reactors” Voprosy Atomnoi Nauki i tehniki. Ser.: FisikaYadernych Reactorov, 5, 18, 31 (1981)
2. N.I.Laletin “Distribution of Neutrons in Heterogenous Medium” Proceeding of Second International Conference in Peaceful Using Atomic Energy. Vol.2, p.534, Moscow,1958
3. Tamara S. Poveschenko, Nickolay I. Laletin “About Calculation of Axial Diffusion Coefficient in Nuclear Reactor Cells”. Proc. Int. Conf. PHYSOR-2004 April 25-29, 2004 Chicago, Illinois, USA
4. P.Benoist Nuclear Science and Engineering **34**, p.285 (1968),
5. Glasstone S. and Edlund M.C. “Elements of Nuclear Reactor Theory.” D Van Nostrand Co., Inc., 1966