PHYSOR-2006, ANS Topical Meeting on Reactor Physics
Organized and hosted by the Canadian Nuclear Society. Vancouver, BC, Canada. 2006 September 10-14

Optimization of the ACR-700™Fuel Management using Gradient Methods

Richard Chambon, Elisabeth Varin[*] and Daniel Rozon
Institut de Génie Nucléaire, École Polytechnique de Montreal,
P.O. Box 6079, Succ. Centre-ville, Montréal, Québec H3C 3A7, Canada

Abstract

In the process of designing new reactors, fuel management optimization is an important step. Automatic fuel management optimization tools have been developed into the reactor core DONJON. Two gradient based methods are used to solve the optimization of the ACR-700 fuel management at equilibrium. Studies have been conducted to define the more appropriate method to adequately and efficiently reach an optimized exit burnup distribution for the ACR-700 core. Specific studies relevant to safety parameters are also presented.

KEYWORDS: ACR, fuel management problem, optimization, gradient method, augmented Lagrangian

1. Introduction

In the process of designing new reactors, fuel management optimization is an important step. The Advanced CANDU Reactor (ACR™[a]) is currently under development in Canada. [1] This paper concerns a study on the ACR-700 reactor, an early model of the ACR. As all CANDU reactors, ACR-700 will be refueled on-line. However enriched uranium fuel will be used in this new reactor to compensate for absorption in the light water coolant. This new fuel type will result in a small number of bundles shift per refueling (2). Moreover large channel powers will be extracted. For CANDU reactor, fuel management is used at three different steps of the reactor operation. At start-up, the fuel burnup distribution has to be optimized to allow the safe operation of the reactor. Then the on-line refueling of the core proceeds during the approach to equilibrium. Finally the core reaches equilibrium, which means that the refueling frequency in each channel is constant. This is this final step that is studied in the present work.

The first step of our fuel management study for a new reactor design is to find a configuration of the core at equilibrium for which the reactor power remains stable ($k_{eff} = 1$) and under the safety limits ($q_s \leq q_{lim}$). The fuel management optimization problem then consists of minimizing the fuel throughput F_T, while core integrity is maintained. This functional is proportional to the number of fuel bundles required per year to maintain full power. For the design and the fuel management optimization problem, the reactor core parameters are obtained by solving

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[*] Corresponding author, Tel. 514-340-4711, Fax. , E-mail: elisabeth.varin@polymtl.ca

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the diffusion equation. It is thus managed through a time average (TA) model. The minimization of the fuel throughput is then achieved by optimizing the average reactor state over the refueling frequency (i.e. the average exit burnups). Thus, the average exit burnup distribution B^e represents the decision vector $\vec{X}$ of the optimization problem, given as follows:

$$\min_{\vec{X}} f(\vec{X}, \phi(\vec{X})) \text{ with } \left\{ \begin{array}{rcl} h_i(\vec{X}, \phi(\vec{X})) & = & b_i \\ g_j(\vec{X}, \phi(\vec{X})) & \leq & c_j \\ X_n^{INF} \leq & X_n & \leq X_n^{SUP} \end{array} \right. \tag{1}$$

where f represents the objective function F_T, h_i represents the constraint on the neutron multiplication factor k_{eff}, g_j represents the power distribution constraints q_s and ϕ is the neutron flux.

The multigroup code DONJON [2] is used at Ecole Polytechnique for several purposes in full-core simulation. Automatic optimization of fuel management first developed in the code OPTEX [3] has been recently integrated and enhanced into the DONJON code. [4,5]

The aim of the present work is to test the general tools for optimization calculations implemented recently in DONJON on the fuel management problem of a totally new design of reactor: the ACR-700. We first briefly describe the two optimization algorithms used to solve the problem (Eq. (1)). Then, these methods will be applied to the fuel management of the ACR-700. This will give feedback about the pros and cons of the methods for different choices made in the ACR-700 fuel management. Moreover, in the ACR-700, the core void reactivity has to be negative. Thus it has been monitored along the iterations of the optimization process to study its dependence upon the burnup distribution. An analysis of the influence of the channel power limit choice upon the objective function has also been carried out. Finally, conclusions are drawn.

2. Optimization algorithms

The two general algorithms developed to solve the fuel management optimization problem are based on the gradient method. Indeed, functionals which define the problem Eq. (1) are function of the neutron power, while the decision parameters influence the flux. Thus functionals are non-linear with respect to the decision variables. We have decided to linearize the optimization problem using gradient methods. The resulting gradients are computed using the generalized perturbation theory (GPT). [6]

The *classical* gradient method is based on the use of a **simplex** algorithm to solve the linearized optimization problem. The general algorithm can be summarized as : for a specific decision vector, the flux distribution is computed, which gives the functional values and allows the calculation of the corresponding gradients. A quadratic constraint S is usually defined to maintain the linearization approximation valid. A mathematical programming [7] is then used to solve the quasi-linear problem, resulting in a new decision vector. The objective function is then reevaluated and the new decision vector can be validated against the constraints and minimization requirements. If this decision vector is not valid, the quadratic limit is reduced and the new point is rejected. This process is repeated until convergence is reached. Convergence is tested on both the functional and the decision vector. When several iterations are valid in a row (4), the quadratic constraint is relaxed by multiplying the limit S by two to speed up the convergence towards the optimum. All decision vectors must be feasible points, i.e. fulfill all the constraints. Thus the classical gradient may have a major drawback: the initial point of the algorithm has to be feasible. This can be easily achieved when an actual design of reactor is simulated, however, the available engineering expertise may not be sufficient for a new design.

Two general approaches [4,5] have been proposed to overcome the limitations of the classical gradient method. The main goal is to have optimization algorithms which are not restricted by the fact that the starting point (core configuration) has to comply to the problem constraints.

The first method consists in using the mathematical programming proposed previously on a sequence of optimization problems that represent the original constraint requirements in Eq. (1). This method is called the multistep optimization method (MS). During the first step, a configuration corresponding to a critical reactor is found by minimizing $(k_{eff} - 1)^2$ without any constraint. Then, the maximum channel power is decreased until its value reached the prescribed limit with $k_{eff} = 1$ as a single constraint. Finally, the original optimization problem Eq. (1) is solved. Results on CANDU-6 reactor have shown that this approach can be applied to very complex optimization problem (190 burnup zones, i.e. 190 decision variables). [4]

The second method is based on external penalty functions. The basic idea of this method is to include the constraints within the objective function. Constraint weights are introduced to take into account their different order of magnitude within the objective function. We have chosen to couple the augmented Lagrangian method (AL) with the classical gradient method. This method is called the mixed method (MM). This method and the AL itself were used for CANDU-6 reactor fuel management. [5] The AL method was found to be very dependent of the constraint weights within the objective function resulting in solution either too slow or not accurate enough. However, the AL allows to find a feasible point in a small number of iterations when using large constraint weights. Moreover from the point of view of code use, it is simpler to define the AL algorithm than the MS. Actually the Mixed Method was developed to take advantage of the MS and AL methods, minimizing their drawbacks.

During the first step of the MM algorithm, the maximum channel power is decreased, and the core reached a critical state simultaneously using the AL approach. When a feasible point is obtained, the original fuel management optimization problem is solved using the *classical* gradient method. Applied to CANDU-6 reactor, results show that the MM can also be applied to any case up even very complex ones (up to 11 zone cases were tested). [5] This method has also be found less dependent of the initial point than the MS and AL approaches.

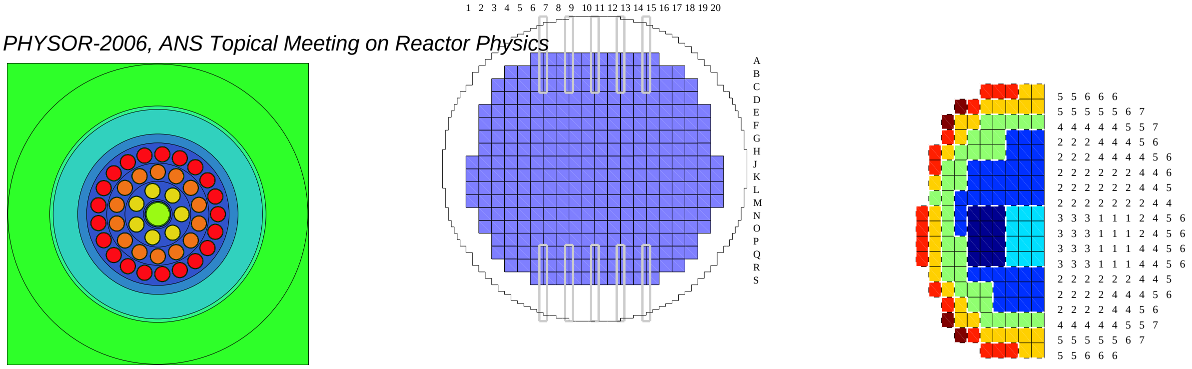
3. Applications to the ACR-700

In this ACR-700 reactor, the 22 cm wide fuel lattice cell is composed of a 43-pin bundle, as illustrated on Fig. 1a, with lightly enriched uranium fuel, except for the central pin which is composed of natural uranium and dysprosium to obtain a negative void reactivity. A front view of the simulated core is illustrated on Fig. 1b, including the control rods (50% inserted, grey boxes), the reflector (approximate round shape) and the labels of the rows and the columns of the channels.

The two gradient-based methods have never been applied to the fuel management problem of the ACR-700. The different advantages and drawbacks observed in [4, 5] were valid for the CANDU-6 reactor. When applied to the ACR-700 case, studies on the dependence of the results on different calculation options were performed. The fuel management of the ACR-700 has been optimized using several sets of optimization parameters.

3.1 Impact of the optimization parameters

Two initial points have been used, an exit burnup distribution based on engineering knowledge provided by AECL [8] and another distribution based on a rough estimated of the core



a) b) c)

Figure 1: ACR-700 model for optimization.

average exit burnup; they are referred to by 'AECL' and 'flat' respectively. The information provided by AECL is composed of 150 different exit burnups.

To study the two burnup distributions, a simpler model representing 7 burnup zones was set up. The choice of the 7 zones is based on the detailed burnup distribution provided by AECL, where channels with similar burnup have been grouped. The 7 zones are illustrated on Fig. 1c. Optimization is then performed for two different burnup zone distributions: 7 zones and 150 zones. Taking into account the left/right symmetry, each zone of the 150-zone case corresponds to two channels.

Results are summarized in Table 1. The optimization options are given by n_z the number of zones, I.P. the initial point (burnup distribution) and $meth$ the method. For the MM method, the constraint weights w_k for core reactivity and w_q for maximum channel power are respectively 50000 and 1000. In Table 1, F_T represents the the objective function in arbitrary units, B_{min} and B_{max} correspond to the minimum and maximum burnup in MWd/t. The effect of each optimization options will be carefully studied.

Table 1: Results for the different initial burnup distributions and optimization methods.

n_z	I.P.	$meth$	F_T	B_{min}	B_{max}
7	AECL	MS	4.4756	15938	27258
7	flat	MS	4.4751	15805	27336
7	AECL	MM	4.4756	15939	27258
7	flat	MM	4.4753	15779	27330
150	AECL	MS	4.3357	16926	27565
150	flat	MS	NA	NA	NA
150	AECL	MM	4.3400	16708	27564
150	flat	MM	4.3058	19466	27769

For the simplest case (7 zones), the initial burnup has almost no influence on the optimized results, indeed objective functions are all equal within a range of 0.01%. The minimum and maximum exit burnups are very close. Actual channel power and burnup distributions obtained

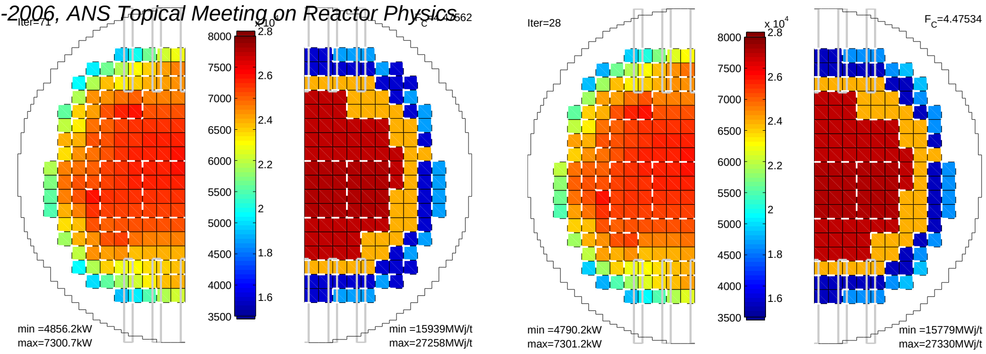

(a) 'AECL' (b) 'flat'

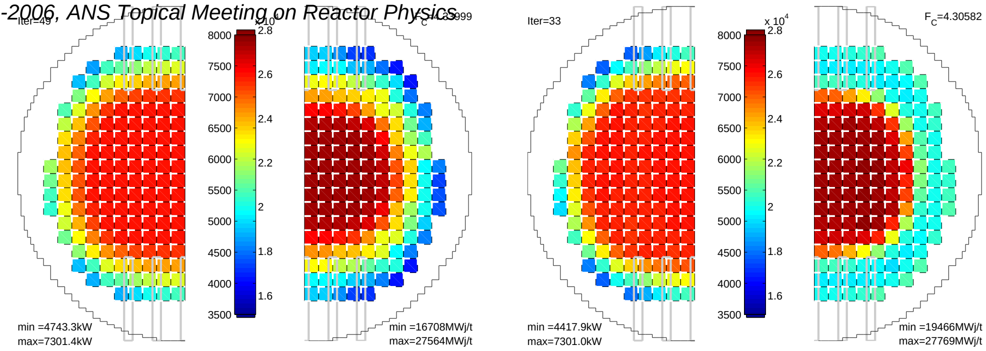
Figure 2: Power and burnup distribution for the 7-zone cases with the MM.

using the MM method are illustrated on Figure 2 for both initial burnup distributions (case (a) AECL) and (case (b) flat). The left part of the figure represents the channel powers and the right part corresponds to the burnup distribution. The thick white dash-line illustrates the burnup zone limits, the grey boxes the control rods (50% inserted), and the outer line the reflector/calandria limit. Results show that no significant difference is obtained with respect to the optimized burnups and consequently the power distribution for the 7-zone case.

In zone number 7 (eight `peripheral` channels), a $3\ GWd/t$ exit burnup difference between results for the two initial burnup distributions is observed for any optimization approach. As explained in [4,5], the definition of the objective function F_T introduces a bias on the results, i.e. the larger a zone is, the more important it is in the objective function, and the more optimized is the burnup of this zone. Zone number 7 is a very small zone compared with the others.(See Figure 1) Thus the exit burnup there is more related to its initial value than to the optimization effort results. Actually, there is a large discrepancy between burnup values in this zone for the AECL-case ($10GWd/t$) and the flat-case ($23GWd/t$).

To avoid any mathematical bias in the objective function, all zones must have the same number of channels, ultimately one zone per channel. Taking into account the right/left symmetry of the ACR-700 core, we used 150 zones (two channels per zone, one on each side). For this complex case, however, limitations of the MS method have been encountered when starting from an uniform exit burnup. Indeed, for the maximum channel power minimization, the problem has not converged before the maximum number of iterations (200) has been reached. The process had thus been cancelled, resulting in the mention (NA) in Table 1.

For the MM method, we have observed that results are dependent of the initial point. Indeed, the objective function is approximately 0.8% lower when starting from an uniform burnup distribution than when using the AECL distribution as an initial point. Channel power and burnup distributions obtained using the MM method for the 150-zone case are illustrated on Figure 3 for both initial burnup distributions. Results show clearly that the MM method is dependent of the initial point for a complex case. The minimum channel powers for the AECL-case is 300 kW larger than the one for the flat-case. This initial point dependence was also pointed out for the CANDU-6 fuel management problem. [5] The burnup difference is mostly in the peripheral region. Actually, in the AECL-case, the exit burnups in that regions are initially around $10GWd/t$, whereas in the flat case an average value of $23GWd/t$ is used.



(a) 'AECL' (b) 'flat'

Figure 3: Power and burnup distribution for the 150-zone cases with the MM.

Thus, the exit burnup values in the peripheral region increases up to a converged average value of $17GWd/t$ (Figure 3a, for AECL-case) while it decreases to reach a converged average value of $19.5GWd/t$ (Figure 3b, for flat-case). Because of the complexity of the case, it is however difficult to know if these differences arise from finding two distinct local minima or from a convergence problem in one of the methods.

Results in Table 1 shows however a significant reduction of the objective function when considering 150 zones. Depending of the initial point, the reduction is between 3 and 4%. For the CANDU-6 reactor, it was lower than 1.5%, even when comparing 2 zone case to the more complex case of 190 zones. This can be explained by the very different ranges of burnups between the two reactors. For the CANDU-6 reactor, the burnup varies between 6.8 and $8.2GWd/t$, i.e. a range of $\sim 15\%$, and for the ACR-700, we observe a variation range of $\sim 45\%$. Moreover, the channel power distributions on Figure 2 show that the burnup in one zone is usually limited by the power in a single channel. Comparison of Figures 2 and 3 shows that the power distribution can be flattened in the center of the core by using 150 zones. Indeed, the channel power and burnup profiles are more *continuous* in the 150-zone case.

Comparison of the MS and MM method results for the AECL initial point shows that both methods are equivalent as regards the objective function and the extremum burnups. (See Table 1) Different constraint weights (w_k for core reactivity and w_q for maximum channel power) have also been tested for the 7-zone case, using both initial points. 4 orders of magnitude of weights for a constant ratio w_k/w_q and 3 orders of magnitude of ratio w_k/w_q have been considered. No significant difference upon the results has been observed.

The objective function results are obviously dependent of the number of decision variables. The larger the number of burnup zones is, the finer the burnup profile can be, and the lower the objective function becomes. Initial point dependence of the results is only significant for complex case. Both methods give similar results.

3.2 Computation requirements

CPU requirements corresponding to the results in Table 1 are given in Table 2. It_e, ϕ and *Grad* give the CPU requirements (number of external iterations, of flux calculations and of gradient computations).

Generally, the MM method needs fewer flux calculations to converge on the optimized burnup

Table 2: Computation requirements.

n_z	I.P.	$meth$	It_e	ϕ	*Grad*
7	AECL	MS		103=7+6+90	82=3+4+75
7	flat	MS		101=11+65+25	82=7+56+19
150	AECL	MS		83=13+10+60	65=7+8+50
150	flat	MS		NA=18+201($^{\alpha}$)+NA	NA=13+176+NA
7	AECL	MM	5=4+1	105=33+72	60=21+39
7	flat	MM	6=5+1	79=50+29	64=40+24
150	AECL	MM	2=1+1	61=11+50	52=8+44
150	flat	MM	10=9+1	118=84+34	91=62+29

distribution. The only exception is the 7-zone case using the AECL initial point. The number of gradient set calculations is always lower for the MM method than for the MS method. As previously noted, the 150-zone case using the ’flat’ initial point with the MS method stopped before any feasible point was found. Detailed studies of the second step of the MS method, i.e. the minimization of the maximum channel power, have shown that the quadratic constraint has to remain small (around 50 compared to initial value of 2000) to maintain the critical reactor constraint valid. This results in a very small advance step at each iteration, which does not allow the process to converge when it uses an initial point very far from the optimized value. Thus the MS method is not divergent but it requires too much iterations and thus CPU requirement.

Now let us compare specifically the CPU requirements to find an initial feasible point. In Table 2, this effort corresponds to the sum of the 2 first values on the RHS of the ’=’ sign under the column ’ϕ’ for the MS method, and directly the first value for the MM method. Numbers of iterations are always lower using the AECL distribution than using an uniform distribution of the burnup. The effort to converge the objective function minimization is given by the last values of the sum for both algorithms. Convergence for the objective function minimization part of both algorithms (MS and MM) is faster when using an uniform burnup (around 30 iterations) than when using the AECL distribution (around 75 iterations).

For the 7-zone case, the MM method starting from the uniform is the fastest. For the 150-zone case, results starting from the AECL distribution are obtained faster than when using an uniform initial burnup. However, accordingly to the fuel throughput F_T, results are less minimized in the AECL case. Thus, either a local minimum is reached in this case, or the convergence criteria are too large. It is however difficult to prove the first hypothesis. Indeed, the number of variables is too large to plot analytically the objective function keeping the dependence of all variables. Even a qualitative appreciation of the coupling of the burnup zones together and with the constraints is out of reach. As concerns the converge criteria, they are limited by the flux precision, which limits directly the functional precision.

Regarding previous results, the MM method is recommended for its speed of convergence. Thus it is used in further calculations. Even if the number of iterations are similar for the 7 and 150-zone cases, time required to compute all gradients is much higher in the complex case. Thus, it takes around 1 hour for the 7-zone case and 1 day for the 150-zone case to solve the optimization problem Eq. (1). Thus specific studies have been performed using the 7-zone case.

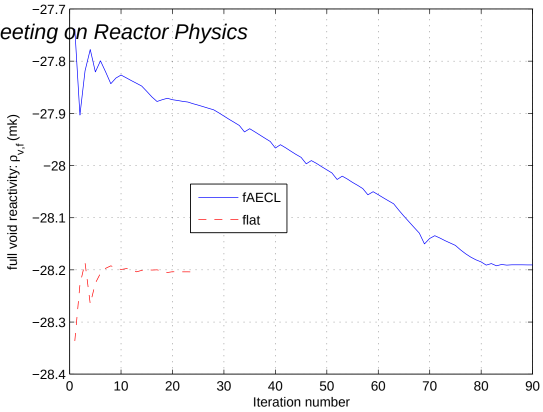


Figure 4: Full core void reactivity with respect to the optimization iterations.

4. Specific ACR-700 study

4.1 Core void reactivity

In the ACR-700, the core void reactivity is designed to be negative. It has been monitored along the iterations of the optimization process to study its dependence upon the burnup distribution. Results using the MS method for the 7-zone case starting from both initial points are presented on Figure 4. Only full core void reactivity during the objective function minimization step is plotted. The results during the first two steps of the MS method are not represented here, because they actually represents non-feasible configuration thus void reactivity variations are meaningless.

Results shows that the variation of full core void reactivity $\rho_{V,f}$ is very small, for both initial starting points. When the core is voided, the thermal flux increases in the reflector, thus the leakage is increased, leading to a negative void reactivity. In the AECL case, the full core void reactivity decreases a little because peripheral burnups are around $13GWd/t$ at the beginning of the objective function minimization, and they finished around $17GWd/t$. Thus, the flux decreases in the periphery along with the iterations, and consequently when the core is voided, leakage are less increased. For the 'flat' case, it is exactly the opposite, the burnup decreases from approximately $20GWd/t$ to $17GWd/t$ in the periphery. Thus, the full core void reactivity increases a little. Final void reactivity are very similar, because final burnup configurations are almost the same independently of the starting point for the zones case, as we mentionned earlier.

Checkerboard void reactivity has also been monitored. Results are exactly the same except that the void reactivity is around $-9.2mk$ and its variation is around $0.1mk$.

4.2 Maximum channel power limit

The maximum channel power used in the previous calculations was $7300kW$. This limit is however arbitrary and a different value may be used depending on the coolant circuit. In the previous calculations in order to test the validity and the robustness of the optimization methods, a relatively low channel power limit has been used (7300 kW). However, it would be useful to know how the maximum channel limit influence the optimized burnup distribution, and consequently the fuel throughput. Thus, several optimization calculations have been performed for the 7-zone case using the MM method starting from the AECL distribution.

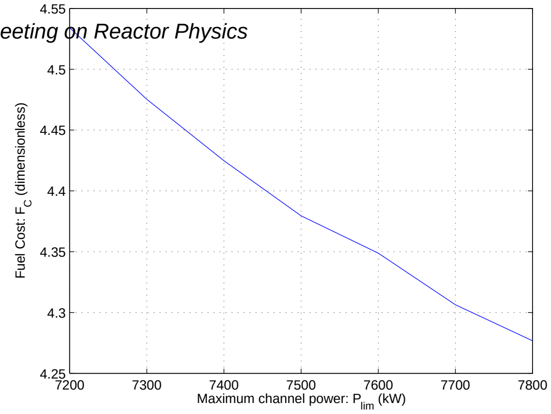


Figure 5: Objective function F_T with respect to the channel power limit P_{lim}.

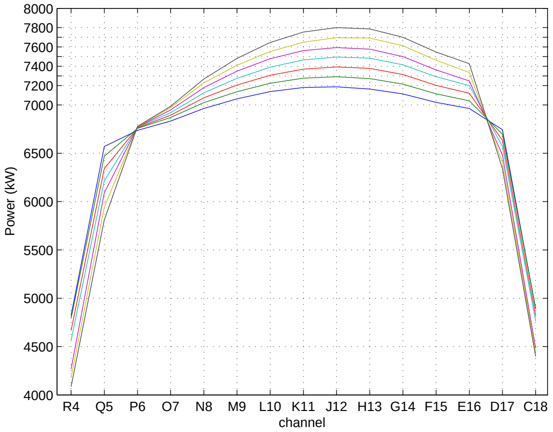


Figure 6: Diagonal profile of channel power with respect to the channel power limit P_{lim}.

The objective function corresponding the optimized exit burnups for different maximum channel power limits is presented on Figure 5. The greater the power limit is, the lower the objective function would become, as expected. We have plotted the channel power profile for the different optimization cases on Figure 6. The power profiles are taken along a bottom-to-top diagonal direction through the reactor from channel R4 to channel C18. (The labels of the corresponding channels are given in Figure 1b). The greater the power limit is, the more the power profile looks like a cosine, in other words, the lower the channel power becomes in the periphery. Thus, the leakage is reduced. Consequently the fuel efficiency is increased, when the power limit is increased.

5. Conclusions

Two alternative gradient approaches, multistep and mixed methods, have been used successfully to solved the ACR-700 fuel management optimization problem. The multistep method may be however too slow to converge on very complex cases. Thus, the mixed method is recommended. Even if it is time consuming, a larger number of burnup zones leads to a significant reduction of the objective function, and a smoother channel power profile. The variation of the core void reactivity (full and checkerboard) is very small in the feasible domain. The power limit constraint actually limits the objective function.

In this study, the fuel enrichment was constant. However, enrichment would have had a direct influence on the objective function and on the core void reactivity. Thus, in a future work, optimization of the ACR-700 fuel management will be carried out taking into account a variation of the fresh fuel enrichment.

Acknowledgment – This work was supported in part by a grant from the Natural Sciences and Engineering Research Council of Canada, Atomic Energy of Canada Limited, and by the Hydro-Québec Chair in Nuclear Engineering.

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