A Feasibility Study of Low-order Harmonics Expansion Applied to Loading Pattern Search♠

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Abstract

Despite significant progress in core loading pattern search methods over years, there still remains the issue of large computing workload and the need for improving the speed of evaluating loading pattern candidates during the search process. This paper focuses on improving the computing speed for loading pattern evaluation, rather than the method of searching for the patterns. A low order harmonics expansion method for flux distribution representation is proposed for fast LP evaluation application. The novel feature of the method is the separation of the short range local perturbation effect from the long range global tilt effect. The latter effect can be captured by low order harmonics expansion. Demonstration examples are presented to show that even for extremely large perturbations induced by fuel shuffling the proposed method can accurately calculate the flux distribution for the LP with very minimal computation.

KEYWORDS: Loading Pattern Search, Harmonics Expansion

1. Introduction

With the increasing complexity of loading pattern (LP) design and much more attention to the nuclear fuel economy, LP search optimization method has been an active research area. Many different optimization algorithms, based on either stochastic or deterministic search methods, have been extensively studied. Regardless the search method to use, the LP candidates need to be evaluated via solving the neutron diffusion equation during the search process. Although there has been very significant progress in the search methods development over the past years, there still remains the issue of large computing workload and too much time spent in the evaluation of the LP candidates during the LP search process. More recently, there is a trend to directly adopt the engineering design tools in the process of LP optimization, in which case the issue of computing speed becomes even more striking.

This paper focuses on improving the computing speed for loading pattern evaluation, rather than the method of searching for the patterns. A low order harmonics expansion method for flux distribution representation is proposed for fast LP evaluation application. The novel feature of the method is the separation of the short range local perturbation effect from the long range global tilt effect. The global effect can be captured by low order harmonics expansion, and the local effect, although not expandable in few harmonics, can nevertheless be captured by relatively simple perturbation calculation. Demonstration examples are presented to show that even for extremely large perturbations induced by fuel shuffling the proposed method can accurately calculate the flux distribution for the LP with very minimal computation. The speed advantage of the proposed

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method is demonstrated via an exclusive LP search case.

2. THEORY

When two fuel assemblies are switched in LP search, the flux distribution changes to respond to the k-infinity change in these two assembly locations. There are two effects in this flux response to k-infinity change. One is a local effect giving rise to the local flux change confined mainly to the neighborhood of the perturbation site, and the other is a global effect giving rise to a flux tilt redistribution across the core. The local effect is relatively easy to capture with various kinds of approximate perturbation estimation method. It is the global effect that is propagated via neutron diffusion and can only be captured by solving the diffusion equation, which consumes most computing time in LP search. However, the global effect is fairly smooth and systematic. So it is possible to represent the global flux change in terms of an expansion over a small number of base functions, like eigen-functions (harmonic modes) of the reactor *k*-eigenvalue problem. Had the local effect component not been separated from the global one, it could require many harmonic modes to represent the total flux change since all the high order harmonics contribute mostly to the local effect component. (For example, a Delta function requires an infinite number of harmonics of comparable magnitude to expand.) It is natural to use the first few harmonic modes of any reasonable reference core condition as the base functions.

Using the above expansion method, the complete diffusion solver can be reduced to a simple matrix equation solver, where the dimension of the matrix is much smaller than that in conventional diffusion equation solvers. The validity of this representation can cover a large perturbation range, such that the harmonic mode base functions and the functions representing the local flux perturbation effects can be pre-calculated once for all, using well established techniques and tools. This will greatly speed up the diffusion calculation required for the evaluation of various LPs.

2.1 Formulation of Low Order Matrix Solution Problem

Let $\phi^n_{i,j}$ denote the *n*th harmonic mode of a reference core condition at the (i,j) node and $\phi_{i,j}(p)$ stand for the local effect component in flux change at the (i,j) node due to a certain reactivity perturbation (by replacement or shuffling of assemblies) at position p of the reference core. Both $\phi^n_{i,j}$ and $\phi_{i,j}(p)$ are pre-calculated for generic applications.

Consider a new LP different from the reference core condition, and let $\phi'_{i,j}$ be the corresponding neutron flux that we want to obtain. In this proposed method, we first construct a neutron flux distribution $\psi_{i,j}$ as a combination of the fundamental mode $\phi^0_{i,j}$ of the reference core plus the local effect functions corresponding to perturbations at the various locations,

$$\psi_{i,j} = \phi^0_{i,j} + \sum_{p=1}^{P} a_p \phi_{i,j}(p) \tag{1}$$

where P is the total number of perturbed positions in the core. The coefficients a_p can be either determined via using Eq. (3) in below or predetermined directly by assuming the perturbation effect being linear with the perturbation magnitude. In the latter case, the coefficients will be scaled to the ratio of the actual k-infinity change to the k-infinity change assumed in the original calculation of $\phi_{i,j}(p)$. Next we add the global tilt component to represent the total flux distribution

$\phi'_{i,j}$ as,

$$\phi'_{i,j} = c_0\psi_{i,j} + \sum_{n=1}^{N} c_n\phi^n_{i,j} \tag{2}$$

Substituting the above equation into the diffusion equation for the new LP, we can derive a matrix eigen-value problem as follows by adopting the weighted residual technique:

$$[M]\vec{C} = \frac{1}{k}[F]\vec{C} \tag{3}$$

where $\vec{C}$ is the column vector of expansion coefficients and k is the effective multiplication factor. If the coefficients a_p in Eq.(1) are not predetermined already, they will also be part of the vector $\vec{C}$.

Preliminary numerical results in the next section indicate that only the first few harmonic modes are needed to achieve satisfactory accuracy for both k and flux distribution. The dimension of Eq.(3) is much smaller than that of conventional diffusion equation solvers, and hence greatly speeds up the process of LP evaluation.

2.2 Three Different Expansion Methods

In the following demonstration application, three expansion methods based on the above formulation are tested, which are respectively denoted as the Linear Sensitivity Method (LSM), the Local Effect Method (LEM) and the Harmonic Expansion Method (HEM).

- Linear Sensitivity Method (LSM):
  LSM uses Eq.(1) with two local effect functions whose coefficients are determined directly by assuming linear sensitivity of perturbation effect on power shape to the perturbation magnitude.
- Local Effect Method (LEM):
  LEM uses Eq.(2) without the harmonic terms, which is the same as using Eq.(1) except for the coefficients of the two local effect functions being now determined by solving Eq.(3) rather than by assuming the relation of linear sensitivity.
- Harmonic Expansion Method (HEM):
  HEM uses both Eqs, (2) and (3) with three additional lowest order harmonic functions included as well.

3. Examples of Application

For a preliminary feasibility study, we apply the proposed method to a two-loop 121-assembly core problem. A high-leakage out-in LP is chosen as the reference case, the power distribution of which is shown in Fig. 1. Two cases of swapping a pair of assemblies of the same enrichment are considered, one case for a significantly large perturbation and the other for an extremely large perturbation.

- Significantly large perturbation case:
  This case swaps the assemblies identified as 1 and 2 in Fig. 1. Assembly 1 is located next to a core peripheral assembly and assembly 2 is located at the (3,3) location. The burnup of

assembly 2 is 5GWd/tU higher than the burnup of assembly 1.

- Extremely large perturbation case:
  The second case swaps the assemblies identified as 3 and 4 in Fig. 1. Assembly 3 is a fresh feed assembly on the core periphery, and assembly 4 is burnt assembly of 19GWd/tU located next to the core center assembly.

Figure 1: Power Distribution of the Reference Core And Assemblies to be Shuffled

0.6539	0.7868 **(4)**	1.0161	0.7945	0.8771	1.3768 **(1)**	1.1836 **(3)**
	0.9292	0.8702	1.0557	0.8381	1.1752	0.9955
		1.0539 **(2)**	0.8690	1.0724	1.2000	
			0.8745	1.1799	0.9302	
				0.8219		

3.1 Example 1: Significantly Large Perturbation Case

Assemblies 1 and 2 identified in Fig. 1 are shuffled. Fig.2 gives the power distribution after the shuffle. Up to 25% power perturbation relative to the reference case is induced.

Figure 2: Power Distribution Change Induced by Shuffling Assemblies 1 and 2 (Example 1)

0.7377 12.82	0.8862 12.63	1.1239 10.61	0.8303 4.51	0.8152 -7.06	1.0799 -21.56	0.9917 -16.21
	1.0545 13.48	0.9876 13.49	1.1215 6.23	0.8092 -3.45	1.0438 -11.18	0.8641 -13.20
		1.3178 **25.04**	0.9445 8.69	1.0740 0.15	1.1451 -4.58	
Power % change			0.9190 5.09	1.1970 1.45	0.9203 -1.06	
				0.8328 1.33		

The percentage error in power distribution prediction for the shuffled case is given in Fig. 3 for the three expansion methods. Table 1 summarizes the accuracy for the three methods. All the three methods give excellent accuracy for this example case even though the induced perturbation is significantly large (25%).

Figure 3: Power Distribution Prediction % Error For Example 1

0.12 0.07 **0.25**	0.08 0.06 0.22	0.03 0.05 0.16	-0.03 0.01 0.08	-0.39 -0.06 -0.02	-0.39 -0.06 -0.09	**-0.75** -0.24 -0.17
	-0.01 0.06 0.19	-0.20 0.07 0.13	-0.06 0.02 0.06	-0.10 -0.05 -0.03	-0.39 -0.12 -0.10	-0.47 -0.19 -0.14
		0.31 **0.31** 0.19	-0.05 0.07 0.03	0.20 0.00 -0.04	0.18 -0.05 -0.07	
LSM(%) LEM(%) HEM(%)			0.24 0.05 -0.03	0.43 0.04 -0.06	0.46 0.02 -0.07	
				0.55 0.05 -0.07		

Table 1: Accuracy Summary For Example 1

	RMS Error (%)	Maximum Error (%)
LSM	0.32	-0.75
LEM	0.11	0.31
HEM	0.11	0.25

3.2 Example 2: Extremely Large Perturbation Case

Assemblies 3 and 4 identified in Fig. 1 are shuffled. Fig. 4 gives the power distribution after the shuffle. Up to 265% power perturbation relative to the reference case is induced by this extremely large core perturbation. Thus we expect this example to be a very severe test of the proposed approach. Fig. 5 gives the comparison of relative errors of the predicted power distribution by using the three different expansion methods. Table 2 summarizes the accuracy of the three methods for this example.

It can be seen from this comparison that LSM does not work well for such a large perturbation. The maximum error for LSM is -63% at the peripheral location where the fresh assembly is shuffled from. Hence the LSM application in practice will be restricted by the extent of perturbation. LEM greatly improves LSM even though it uses the same expansion functions as LSM. The maximum error for LEM dropped dramatically to -4.5% at a peripheral location close to the diagonal axis. The result of HEM is excellent, with a maximum error of only -1%. This shows that indeed only very few low order harmonics are needed even for the extremely large perturbation.

Figure 4: Power Distribution Change Induced by Shuffling Assemblies 3 and 4 (Example 2)

2.1074	2.8699	1.8810	0.9608	0.7154	0.8039	0.3993
222.28	**264.76**	85.12	20.93	-18.44	-41.61	-66.26
	2.1707	1.4057	1.2011	0.6849	0.7335	0.5045
	133.61	61.54	13.77	-18.28	-37.59	-49.32
		1.3533	0.8699	0.8472	0.8120	
		28.41	0.10	-21.00	-32.33	
Power			0.7485	0.8951	0.6542	
% change			-14.41	-24.14	-29.67	
				0.6042		
				-26.49		

Figure 5: Power Distribution Prediction % Error For Example 2

-20.96	-23.06	-9.26	0.52	4.91	-3.41	**-63.29**
2.16	-1.43	3.69	2.58	-0.47	-2.10	1.59
0.63	-0.79	-0.55	-0.32	1.16	1.23	1.05
	-14.41	-5.41	3.08	7.85	4.35	-9.48
	3.33	3.75	2.12	-0.86	-3.05	-2.75
	-0.26	**-1.05**	-0.10	1.15	0.74	-0.35
		1.41	8.61	14.18	14.28	
		2.88	0.82	-2.05	-4.00	
		-0.86	0.39	1.00	0.28	
LSM(%)			15.21	19.40	20.23	
LEM(%)			-1.26	-3.17	**-4.45**	
HEM(%)			0.74	0.43	-0.30	
				22.00		
				-3.99		
				-0.21		

Table 2: Accuracy Summary For Example 2

	RMS Error (%)	Maximum Error (%)
LSM	17.255	-63.29
LEM	3.137	-4.45
HEM	0.793	-1.05

4. Performance Assessment: An Exclusive Search Case

A more comprehensive assessment of the effectiveness of the proposed method is made by defining a LP search problem that makes an exclusive search of all the available LPs. Fig. 6 shows a 2-loop core of 121 assemblies. Consider the case of fixing all the peripheral assemblies and loading the rest 81 locations with three fuel batches, each of which containing 28, 24 and 29 identical assemblies respectively. There are 112320 available LPs for this 1/8 core symmetric problem.

Figure 6: A 3-Batch 2-Loop Core with Fixed Peripheral Assemblies

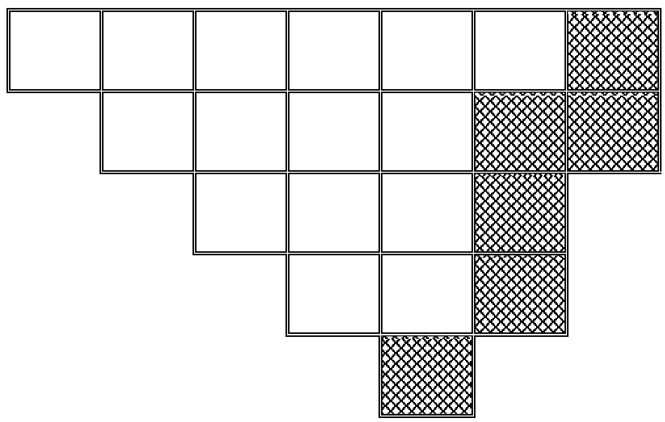
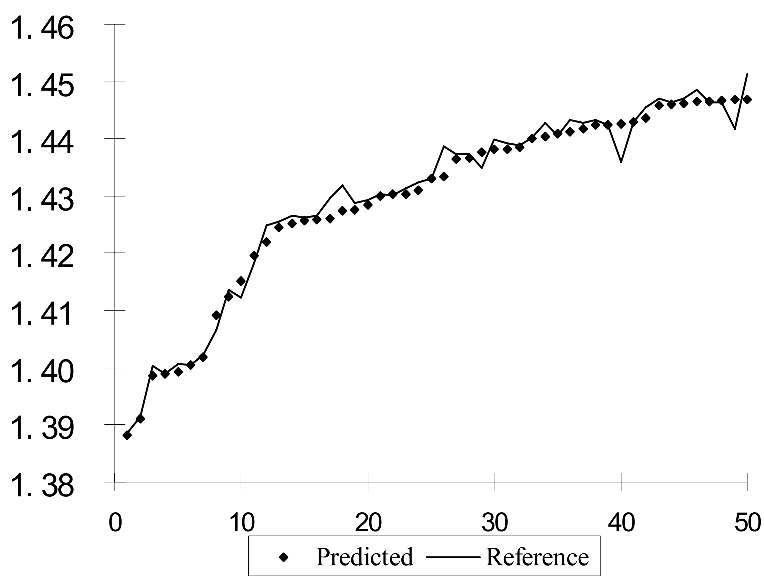


Figure 7: Confirmation of the Best 50 LPs Predicted By the HEM Method
In Terms of Maximum Assembly Power



All the 112320 LPs are evaluated with the HEM method. The best 50 LPs so evaluated are re-analyzed with a finite difference code to compare the HEM predictions versus the "reference" solutions in terms of the maximum assembly power as shown in Fig. 7. The agreement is excellent, confirming the accuracy of the HEM method for LP evaluation application. The evaluation of all the 112320 LPs took 480sec on a Pentium-4, 2.8GHz PC with a single CPU.

4. Conclusion

The results of the preliminary feasibility study indicate that the proposed harmonics expansion method is very promising and worth further investigation. The accuracy assessment study indicates that the validity of the proposed method can cover a very large perturbation range even by using only very few low order harmonics. This has the advantage that for a given fuel inventory the expansion functions can be pre-calculated for generic LP search applications. The experienced speed performance of the proposed method is also very encouraging.