A Stochastic Method for Transient,

Three-Dimensional Neutron Transport

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ABSTRACT

A Stochastic Method, is applied to time-dependent, three-dimensional neutron transport, and the corresponding code has been developed. To verify the validity of this method/code under different status, the variation of neutron quantities/flux along with time is calculated for a steady and supercritical system, respectively. The preliminary results shown in this paper indicate that this method/code is working properly.

Keywords: Neutron Transport, Transient, Stochastic Method

1. INTRODUCTION

As we all known, multigroup, multi-dimensional, diffusion theory has been and is the primary computational model in space-dependent neutron kinetics for applications in transient reactor analysis and reactor safety. However, the validity of diffusion theory based methods is questionable for small high leakage reactors such as space reactors, coolant voiding situations in power reactors, and transient requiring detailed modeling of control rod motion. Alternatively, transport theory based kinetics methods could be used to provide higher accuracy, and to serve as a benchmarking tool for verification of more approximate methods based on diffusion theory. Due to tremendous computing time required to solve the transport equation, transport theory is almost exclusively used for steady-state investigations up to now.

Nevertheless, with the development of the computer (or high performance workstations) and numerical calculation techniques, it has become possible to perform neutron kinetics calculations based on three-dimensional transport theory. In the past several years, the development of the transient calculations based on multi-dimensional transport theory with explicit representation of delayed neutrons has made significant progress. Recent codes like DORTT-TD[1], TDTORT[2,3] or TDMCC[4,5] show that it is feasible to directly solve multi-dimensional transport equation in order to get more accurate neutron flux distribution.

This article is concerned with how to utilize the stochastic method for solving transient neutron transport problems. After the introduction, the method is discussed in brief. Then, numerical results are illustrated. Finally, a summary and conclusion is presented.

2. THEORETICAL BACKGROUND

To solve a transport question, there are two different methods, which are deterministic methods and stochastic methods. The well-known discrete ordinates (Sn) method belongs to the former. Compared with the deterministic methods, the stochastic methods have many obvious advantages, such as the capability of direct simulation to the physics process, lesser limitations of the geometrical condition, the separation between the rate of convergence and the problem dimension, and the parallel computation capability. Certainly, demerits of this approach also exist, for example, relatively long computation time. Thereby, it is vital for the direct simulation approach how to deal with the conflict between the calculation precision and computation time.

To be the same with other Monte Carlo codes, desired results can be obtained by direct simulation of neutrons. However, for a transient problem, it's insufficient to simulate neutron behavior only. The precursors of delayed neutrons should be also dealed with. The remainder of this section will present the process of direct simulation of neutron and precursor behaviors and the tallies, respectively. Before it, random numbers are discussed.

2.1 Random numbers

Like any other Monte Carlo program, this code uses a sequence of random numbers to sample from probability distributions. The linear congruential method[7] is used. A sequence of random numbers is generated by

$$x_{i+1} = \alpha x_i + c \mod M$$
$$\xi_{i+1} = x_{i+1} / M$$

where, α is the random number multiplier, x_0 is the initial random seed, c is an additive constant.

Considering the correlation of the above random numbers, some improvements are adopted. At first, a short sequence of random number is generated by the linear congruential method. Then, the random number required for calculation is selected from the sequence at random. Once one random number has been selected, a new random number generated by the method replaces it.

2.2 Simulation of neutron behaviors

This is composed of neutron tracks and neutron interactions with nuclides. For neutron tracks, the following well-known expression is used to calculate the distance l, which is neutron track length before the first collision:

$$l = -\frac{1}{\Sigma_t} ln(\xi)$$

where Σ_t is the macroscopic total cross section of the medium, and ξ is the random number on [0,1).

When a neutron collides with a nucleus, the following sequence occurs:

1. the collision nuclide is identified;
2. the free gas thermal treatment is adopted for low-energy neutrons(<1eV);

3. the collision type is identified;
4. if scattering is selected, the new energy and direction of the outgoing track are determined;
5. if fission is selected, the new neutrons and precursors are birthed

Except for the step 2, others are relative simple and have been discussed in many books on Monte Carlo Methods, such as the reference 6. So, it is not repeated here. However, it is necessary to describe specially the free gas thermal treatment.

For low-energy neutrons, a collision between a neutron and an atom is affected by the thermal motion of the atom. The free gas thermal treatment assumes that the medium is a free gas. It consists of adjusting the elastic cross section and taking into account the velocity of the target nucleus when the kinematics of a collision is being calculated.

The first aspect of the free gas thermal treatment is to adjust the elastic cross section by raising it by the factor

$$F = (1 + 1/2a^2)erf(a) + exp(-a^2)/(a\sqrt{\pi})$$

Where, $a^2 = AE/kT$, A = atomic weight, E = neutro nenergy,and T = temperature .

The total cross section is also increased by the amount of the increase in the elastic cross section.

The second aspect of the free gas thermal treatment takes into account the velocity of the target nucleus. The distribution to target velocities can be denoted by the Maxwellian probability density function p(V):

$$\mathrm{p(v)} = \frac{4}{\pi^{1/2}}\beta^3 \mathrm{v}^2 e^{-\beta^2 \mathrm{v}^2}$$

with β defined as : $\beta = (\frac{AM_n}{2kT})^{1/2}$

According to the introduction in MCNP manual, the following algorithm is used to sample the target velocity.[8]

1. With probability $\alpha = 1/(1 + (\sqrt{\pi}\beta v_n/2))$, the target velocity V is sampled from the distribution $P_1(V) = 2\beta^4 V^3 e^{-\beta^2 V^2}$. The transformation $V = \sqrt{y}/\beta$ reduces this distribution to the sampling distribution for y: $P(y) = ye^{-y}$.

2. With probability $1-\alpha$, the target velocity is sampled from the distribution $P_2(V) = (4\beta^3/\sqrt{\pi})V^2 e^{-\beta^2 V^2}$. Substituting $V = y/\beta$ reduces this distribution to the sampling distribution for y: $P(y) = (4/\sqrt{\pi})y^2 e^{-y^2}$.

3. The cosine of the angle between the neutron velocity and the target velocity is sampled uniformly on the interval $-1 \le \mu_t \le +1$.

4. The rejection function $R(V, \mu_t)$ is computed using

$$R(V,\mu_t)=\frac{\sqrt{v_n^2+V^2-2Vv_n\mu_t}}{v_n+V}\le 1$$

With probability $R(V,\mu_t)$, the sampling is accepted; otherwise, the sampling is rejected and the procedure is repeated.

And scattering is isotropic in the center of mass frame. After obtaining the target velocity and the direction, the scattered neutron energy and travel direction is determined from two-body kinematics.

2.3 Simulation of precursor behaviors

According to the half-lives, precursors are typically divided into six groups. Because the thermal motion of precursors can be ignored, its simulation becomes easier. There are two processes for it. One is to generate precursors based on the corresponding delayed neutron fraction when fission occurs. The other is to produce the delayed neutron along with the precursor decays, which follows the exponential decay rule. Once a delayed neutron is produced, the approach to track neutrons is utilized to simulate it.

2.4 Tallies

At the first time interval (t_0,t_1), initial neutrons and precursors are always provided for the code. Otherwise, the code will generate the initial sources following a certain rule. Once the initial particles have exit, the code begin to simulate their behaviors. When the time exceeds the time interval, the information of this particle is stored. After completing the simulations of all the initial particles, the neutron quantity/flux is obtained by counting the stored neutrons. And the stored particles, including neurons and precursors, are used as the initial particle sources for the next time internal.

3. NUMERICAL RESULTS

3.1 Setting up the testing problem

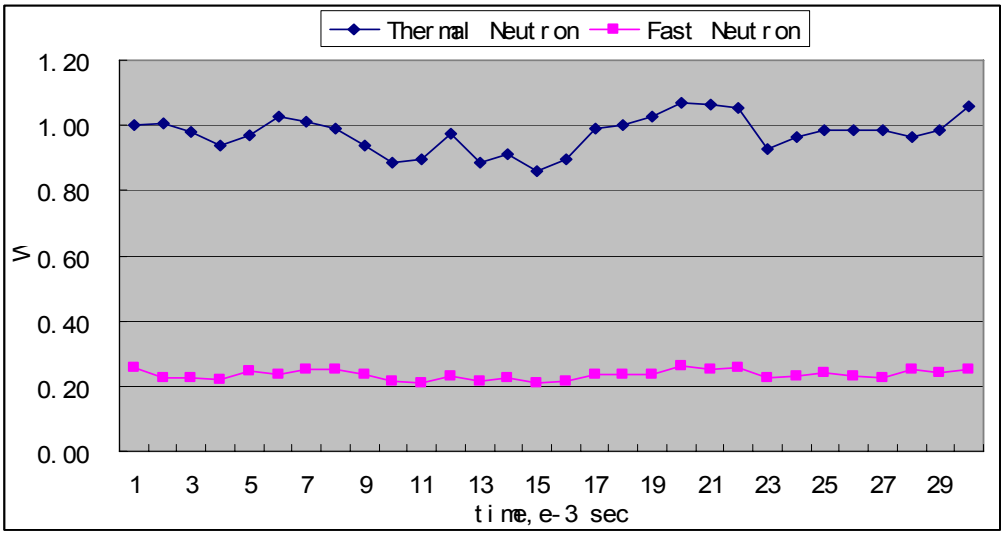
It is a 3-D transient transport problem with two neutron energy groups and six delayed neutron precursor groups. The geometry is composed of two areas. The center is reactor core with 1.0m×1.0m×1.0m, composed of ^{235}U and ^{12}C. Except for the z-axis direction, the core is surrounded by ^{12}C reflector of 0.15 m thickness. In the core, the nuclear density of C is $8.025\times10^{28}/m^3$, and U is $8.025\times10^{25}/m^3$. In the reflector, the nuclear density is $6.55\times10^{28}/m^3$.

3.2 Results of the testing problem

Under the above condition, the system is at steady state (keff=0.99932). The dynamic calculation consists of 2 steps.

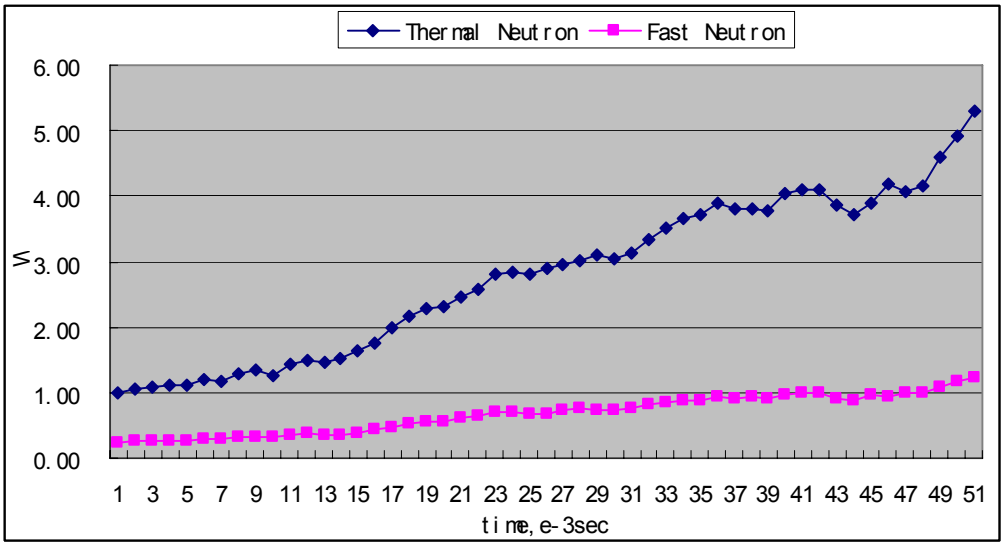
Step 1 corresponds to the above steady conditions. For this purpose 20 timesteps of 0.001sec duration were computed. Considering the computational time, there are only 9325 initial thermal neutrons and 2004 initial fast neutron. And the results are presented in figure 1. The ordinates are the system thermal and fast neutron quantities normalized to one initial thermal neutron.

Figure 1: The system neutron quantities variation with time under steady state



The Step 2 corresponds to the step change at the initial state of the material nuclear density in the core region. The nuclear densities of ^{235}U and ^{12}C are increased to $8.03 \times 10^{25}/m^3$ and $8.03 \times 10^{28}/m^3$, respectively. 50 timesteps of 0.001sec duration each were computed. Results are presented in Figure 2 .

Figure 2: The system neutron quantities variation with time under steady state



4. CONCLUSIONS

The preliminary calculation results show it is practical to deal with the kinetics problem by the stochastic method in the contemporary computing condition. Further work will concentrate on improving the code, such as adopting the parallel algorithm for higher computational efficiency, the more precise physics model (i.e. S(α, β)model) to simulate the neutron thermalization. Also,

for the practical application requirement, we must consider the combination with thermo-hydraulic codes.

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