A Remedy to Prevent the P1SA Scheme Degradation in Multidimensional S_n Calculations in the Presence of Material Discontinuities

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Abstract

In the paper a way to prevent the P1 synthetic acceleration (P1SA) scheme degradation in solving small absorption highly heterogeneous (SAHH) multidimensional problems that ensures fast pointwise convergence of the P1SA scheme is discussed. Numerical experiment has shown that the lack of the difference scheme monotonicity is the reason of the consistent P1SA scheme degradation in solving SAHH problems. So, improvement of the difference scheme monotonicity also improves convergence of the consistent P1SA scheme in solving SAHH problems. In the paper we discuss remedies those improve the difference scheme monotonicity without essential degradation in accuracy. We also present results which demonstrate that a suitable choice of the fix-up function in the adaptive weighted diamond difference (AWDD) scheme essentially extends the class of SAHH problems, which can be efficiently accelerated by the consistent P1SA scheme.

KEYWORDS: P1SA scheme, small absorption highly heterogeneous multidimensional problems, monotonicity, AWDD scheme

1. Introduction

There are two types of transport problems in solving which the diffusion or P1 synthetic acceleration (DSA [1] or P1SA [2, 3]) scheme for acceleration of inner iterations convergence (also known in Russia as the KP1 scheme [4]), consistent with differencing scheme used, degenerates or becomes unstable. The first type problems are the problems with highly anisotropic scattering [4, 5], such as electron transport problems. The second one is the small absorption highly heterogeneous (SAHH) multidimensional problems [6]. Numerical experiment has shown that performance of the consistent P1SA scheme is differencing scheme dependent in solving SAHH problems in 2D and 3D geometries [3]. Being applied to the diamond difference (DD) scheme in solving SAHH problems, the P1SA scheme degenerates, it becomes sensitive to the accuracy of the P1 system solving, etc. But the P1SA scheme, being applied to the Step scheme, works quite efficiently for all problems solved.

To avoid the DSA scheme degeneration in solving SAHH problems the use of the GRMES algorithm, preconditioned by the DSA scheme, was proposed [6]. Our experience in the use of the GRMES algorithm for improving of the P1SA scheme performance in solving SAHH problems has shown that being applied to the DD scheme, preconditioned with the P1SA scheme, it essentially decreases the number of inner iterations if the scalar flux convergence is checked in the L_2 norm. But if the pointwise (C norm) convergence criterion is used, the gain in

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the use of the GRMES algorithm is minimal (if any). Being applied to the Step scheme, preconditioned with the P1SA scheme, the GRMES algorithm does not decreases the number of inner iterations in comparison with the P1SA scheme even if the scalar flux convergence is checked in the L_2 norm. But in this case fast convergence of inner iterations takes place for the P1SA scheme both in L_2 and C norm.

In the paper we discuss an alternative remedy to prevent the P1SA scheme degradation in solving SAHH problems that ensures fast pointwise convergence of the consistent P1SA scheme.

Similar to the Step scheme results in solving SAHH problems are received also for the weighted DD (WDD) scheme with the weighting coefficients, chosen from monotonicity conditions of the scheme. So, it seems that the lack of the difference scheme monotonicity is the reason of the consistent P1SA scheme degradation in solving SAHH problems. Unfortunately, monotonicity conditions are quite strong and being applied to all problem cells produce an overweighted scheme that results in accuracy degradation. The use of the adaptive WDD (AWDD) scheme that ensures positivity and in some extent improves monotonicity properties of the DD scheme does not resolve the problem, but we found that by appropriate choice of the “fix-up” function, used by the AWDD scheme, it is possible to extend class of SAHH problems that can be accelerated by the P1SA scheme.

The needed improvement in monotonicity of the scheme can be received if the choice of the WDD scheme weighting coefficients is performed from the necessary and sufficient scheme monotonicity conditions (monotonicity fix-up) for “critical” extrapolations, where an essential variance in the effective total cross-section/source ratio takes place and rough approximation errors are originated. Numerical experiment has shown that some improvement can be achieved if the monotonicity fix-up is applied for extrapolations, which are located in enter of zones with strongly increased total cross-sections.

The efficiency of the developed algorithms is tested both for model problem, defined in [6], and the similar one.

2. Approximation of Transport Equation and Monotonicity Conditions

We discuss the problem in an example of transport equation for 3D r,ϑ,z geometry with two angular variables φ and $\mu = Cos\theta$ to specify direction:

$$\eta\frac{\partial\psi}{\partial\vartheta} + \mu r\frac{\partial\psi}{\partial z} + \xi\frac{\partial}{\partial r}(r\psi) - \frac{\partial}{\partial\varphi}(\eta\psi) + \sigma r\psi(r,\vartheta,z;\mu,\varphi) = rS(r,\vartheta,z;\mu,\varphi)\,, \tag{2.1}$$

where $\xi = \sqrt{1-\mu^2}Cos\varphi$, $\eta = \sqrt{1-\mu^2}Sin\varphi$. Here the normal ranges are: $-1 \le \xi,\mu,\eta \le 1$, $0 \le \theta \le \pi$, $0 \le \varphi \le 2\pi$, so, the discrete direction $\vec{\Omega}$, $\vec{\Omega} = (\xi,\eta,\mu)$, spans a sphere. The spatial variables r, ϑ and z change in the limits: $0 \le r_{\text{int}} \le r \le r_{ext}$, $0 \le \vartheta_0 \le \vartheta \le \vartheta_{end} \le 2\pi$ and $z_{bot} \le z \le z_{top} = H$. The balance equation is obtained by integrating of Eq. (2.1) over a cell $(r_{i-1/2}, r_{i+1/2})\times(\vartheta_{j-1/2},\vartheta_{j+1/2})\times(z_{k-1/2},z_{k+1/2})\times(\varphi_{l,m+1/2},\varphi_{l,m-1/2})\times(\mu_{l-1/2},\mu_{l+1/2})$, $i=1,..,I$, $j=1,..,J$, $k=1,..,K$:

$$|\eta|\,\Delta r\Delta z(\psi_\vartheta^+ - \psi_\vartheta^-) + |\mu|\,v\Delta\vartheta(\psi_z^+ - \psi_z^-) + \Delta z\Delta\vartheta\left[|\xi|(A^+\psi_r^+ - A^-\psi_r^-) + \frac{C}{w}(\alpha_{m+1/2}\psi_{m+1/2} - \alpha_{m-1/2}\psi_{m-1/2})\right] +$$
$$+\sigma_t^q V\psi^q = VS^q\,, \tag{2.2}$$

where

$$\alpha_{l,m+1/2}-\alpha_{l,m-1/2}=-w_{l,m}\xi_{l,m}, \quad \alpha_{l,1/2}=\alpha_{l,M_{l+1/2}}=0, \quad m=1,..,M_l, \quad l=1,..,L,$$

$$A_{i\pm1/2}=r_{i\pm1/2}, \; v_i=\frac{1}{2}(r_{i+1/2}^2-r_{i-1/2}^2), \; C_i=\Delta r_i=r_{i+1/2}-r_{i-1/2}, \; V_{i,j,k}=v_i\Delta\vartheta_j\Delta z_k, \; \Delta\vartheta_j=\vartheta_{j+1/2}-\vartheta_{j-1/2},$$

$$\psi_\vartheta^\pm=\begin{cases}\psi_{i,j\pm1/2,k,l,m}, \eta_{l,m}>0\\ \psi_{i,j\mp1/2,k,l,m}, \eta_{l,m}<0\end{cases}, \; \psi_z^\pm=\begin{cases}\psi_{i,j,k\pm1/2,l,m}, \mu_l>0\\ \psi_{i,j,k\mp1/2,l,m}, \mu_l<0\end{cases}, \; \psi_r^\pm=\begin{cases}\psi_{i\pm1/2,j,k,l,m}, \xi_{l,m}>0\\ \psi_{i\mp1/2,j,k,l,m}, \xi_{l,m}<0\end{cases}, \; A^\pm=\begin{cases}A_{i\pm1/2}, \xi_{l,m}>0\\ A_{i\mp1/2}, \xi_{l,m}<0\end{cases}$$

The WDD scheme supplementary equations are:

$$\psi_r^+=(1+P_r)\psi-P_r\psi_r^-, \quad \psi_\vartheta^+=(1+P_\vartheta)\psi-P_\vartheta\psi_\vartheta^-, \quad \psi_z^+=(1+P_z)\psi-P_z\psi_z^-,$$

$$\psi_{m+1/2}=(1+P_\xi)\psi-P_\xi\psi_{m-1/2}, \; 0\le P_r,P_\vartheta,P_z,P_\xi\le 1. \tag{2.3}$$

It is possible to rewrite the system (2.2), (2.3) in the quasi one-dimensional form in any variable, for which the WDD equation is used:

$$|\xi|[(A^+\psi_r^+-A^-\psi_r^-)+(A^--A^+)\psi]+\sigma_r v\psi=vS_r, \qquad \psi_r^+=(1+P_r)\psi-P_r\psi_r^-,$$

$$|\eta|(\psi_\vartheta^+-\psi_\vartheta^-)+\sigma_\vartheta\Delta\vartheta\psi=\Delta\vartheta S_\vartheta, \qquad \psi_\vartheta^+=(1+P_\vartheta)\psi-P_\vartheta\psi_\vartheta^-,$$

$$|\mu|(\psi_z^+-\psi_z^-)+\sigma_z\Delta z\psi=\Delta z S_z, \qquad \psi_z^+=(1+P_z)\psi-P_z\psi_z^-,$$

$$\frac{C}{w}\Big[\left(\alpha_{m+1/2}\psi_{m+1/2}-\alpha_{m-1/2}\psi_{m-1/2}\right)+\left(\alpha_{m-1/2}-\alpha_{m+1/2}\right)\psi\Big]+\sigma_\xi v\psi=vS_\xi, \; \psi_{m+1/2}=(1+P_\xi)\psi-P_\xi\psi_{m-1/2},$$

where σ_r, σ_ϑ, σ_z, σ_ξ and S_r, S_ϑ, S_z, S_ξ are the effective total cross-sections and sources for extrapolation in r, ϑ, z and φ variables, respectively, defined in [3].

Necessary and sufficient condition of the WDD scheme monotonicity can be written in the following form [10]:

$$\left(\psi_r^--\frac{S_r}{\sigma_r}\right)\left(\psi_r^+-\frac{S_r}{\sigma_r}\right)\ge 0, \qquad \left(\psi_\vartheta^--\frac{S_\vartheta}{\sigma_\vartheta}\right)\left(\psi_\vartheta^+-\frac{S_\vartheta}{\sigma_\vartheta}\right)\ge 0,$$

$$\left(\psi_z^--\frac{S_z}{\sigma_z}\right)\left(\psi_z^+-\frac{S_z}{\sigma_z}\right)\ge 0, \qquad \left(\psi_{m-1/2}-\frac{S_\xi}{\sigma_\xi}\right)\left(\psi_{m+1/2}-\frac{S_\xi}{\sigma_\xi}\right)\ge 0. \tag{2.4}$$

For the WDD scheme the following equalities take place:

$$\left(\psi_r^+-\frac{S_r}{\sigma_r}\right)=\frac{|\xi|\left(A^-+A^+P_r\right)-P_r\sigma_r v}{|\xi|\left(A^-+A^+P_r\right)+\sigma_r v}\left(\psi_r^--\frac{S_r}{\sigma_r}\right), \quad \left(\psi_\vartheta^+-\frac{S_\vartheta}{\sigma_\vartheta}\right)=\frac{|\eta|(1+P_\vartheta)-P_\vartheta\sigma_\vartheta\Delta\vartheta}{|\eta|(1+P_\vartheta)+\sigma_\vartheta\Delta\vartheta}\left(\psi_\vartheta^--\frac{S_\vartheta}{\sigma_\vartheta}\right),$$

$$\left(\psi_z^+-\frac{S_z}{\sigma_z}\right)=\frac{|\mu|(1+P_z)-P_z\sigma_z\Delta z}{|\mu|(1+P_z)+\sigma_z\Delta z}\left(\psi_z^--\frac{S_z}{\sigma_z}\right),$$

$$\left(\psi_{m+1/2}-\frac{S_\xi}{\sigma_\xi}\right)=\frac{C/w\left(\alpha_{m-1/2}+\alpha_{m+1/2}P_\xi\right)-P_\xi\sigma_\xi v}{C/w\left(\alpha_{m-1/2}+\alpha_{m+1/2}P_\xi\right)+\sigma_\xi v}\left(\psi_{m-1/2}-\frac{S_\xi}{\sigma_\xi}\right). \tag{2.5}$$

So, we can deduce that the following choice of weighting coefficients ensures monotonicity of the scheme:

$$P_r=\begin{cases}1, & |\xi|\left(A^-+A^+\right)-\sigma_r v\ge 0\\ |\xi|A^-/\left(\sigma_r v-|\xi|A^+\right), & |\xi|\left(A^-+A^+\right)-\sigma_r v<0\end{cases},$$

$$P_\vartheta = \begin{cases} 1, & 2|\eta| - \sigma_\vartheta \Delta\vartheta \ge 0 \\ |\eta| / (\sigma_\vartheta \Delta\vartheta - |\eta|), & 2|\eta| - \sigma_\vartheta \Delta\vartheta < 0 \end{cases}, \quad P_z = \begin{cases} 1, & 2|\mu| - \sigma_z \Delta z \ge 0 \\ |\mu| / (\sigma_z \Delta z - |\mu|), & 2|\mu| - \sigma_z \Delta z < 0 \end{cases},$$

$$P_\xi = \begin{cases} 1, & \left(C/w\right)\left(\alpha_{m-1/2} + \alpha_{m+1/2}\right) - \sigma_\xi \nu \ge 0 \\ \left(C/w\right)\alpha_{m-1/2} \Big/ \left[\sigma_\xi \nu - \left(C/w\right)\alpha_{m+1/2}\right], & \left(C/w\right)\left(\alpha_{m-1/2} + \alpha_{m+1/2}\right) - \sigma_\xi \nu < 0 \end{cases}. \tag{2.6}$$

We note that quantities σ_r, σ_ϑ, σ_z, σ_ξ are weighting coefficients dependent. For example, $\sigma_r = \sigma_r\left(P_\vartheta, P_z, P_\xi\right)$. So, the iterative refinement of weighting coefficients is, generally, required. But numerical experiment has shown that the zeroth iteration with sequential refinement of effective cross-sections usually gives an acceptable result.

The use of Eq. (2.6) to calculate the weighting coefficients gives a monotonous solution. Unfortunately, in this case the degradation in accuracy takes place. But the similar flux monotonicity can be received if the monotonicity fix-up is performed only for "critical" extrapolations, where an essential variance in the effective total cross-section/source ratio takes place. This algorithm, known as the MDS_n scheme [10], requires an identification algorithm for critical extrapolations. To identify the 'critical' extrapolations information about the effective total cross-section/source ratio for adjacent already calculated cells is, generally, required.

The simplest variant of the identification algorithm is to refer to critical the extrapolations those are located in enter of zones with strongly increased (10 or more times) total cross-sections. But in this case not all critical extrapolations are included, as some ones can be located at other flux singularities.

Below we call the scheme with weighting coefficients calculated by formulas (2.6) for all cells as the MDS_n full scheme

3. The Fix-up Function Choice in the AWDD Scheme

The use of the adaptive WDD (AWDD) scheme [7, 10, 8, 9] that ensures positivity and in some extent improves monotonicity properties of the DD scheme does not resolve the problem of the P1SA scheme degradation in solving SAHH problems, but numerical experiment has shown that by appropriate choice of the "fix-up" function, used by the AWDD scheme, it is possible to extend class of SAHH problems that can be accelerated by the P1SA scheme.

In the AWDD scheme the weight P_r, for example, is calculated on the base of preliminary estimation of flux gradient in radial variable by the DD scheme with $P_r = 1$:

$$P_r = \begin{cases} 1, & U_r \le U_0^r \\ P_r\left(U_r, \delta_r\right), & U_r > U_0^r \end{cases}, \qquad \delta_r = A^+ / (A^- + A^+), \; U_r = b_r |u_r|, \qquad u_r = \frac{\psi^- - \psi_{DD}}{\psi_{DD}}, \tag{3.1}$$

where b_r and U_0^r are parameters: $b_r \ge 1$, $0 < U_0^r < 1$; $P_r(U)$ is a fix-up function that is chosen from requirement of the fix-up "softness":

$$P_r\left(U = U_0^r\right) = 1, \qquad P_r'\left(U = U_0^r\right) = 0, \tag{3.2}$$

positivity of extrapolation and ability to improve monotonicity properties of the scheme:

$$0 \le P\left(U, \delta\right) \le \frac{1-\delta}{U} \quad \text{for } U > 1 - \delta. \tag{3.3}$$

The following choice of the fix-up function satisfies these requirements:

$$P(U,\delta)=\frac{(1-\delta)U+\beta}{U^2+\gamma U+\alpha},\ \gamma=1-\delta-2U_0,\ \beta=(1-\delta)\gamma,\ \alpha=\beta+U_0^2, \tag{3.4}$$

Usually, two partial cases of this formula are used:

$$P(U,\delta)=\frac{1-\delta}{U}, \qquad U_0=1-\delta, \tag{3.5}$$

$$P(U,\delta)=\frac{1-\delta}{U+(U_0)^2/U}, \qquad U_0=\frac{1-\delta}{2}. \tag{3.6}$$

Function (3.5) is the simplest choice, but in this case the second equality in Eq. (3.2) is not satisfied. Function (3.6) ensures more "soft" fix-up as both equalities in Eq. (3.2) are satisfied in this case. Further decreasing of parameter U_0 in Eq. (3.4) both increases softness of fix-up functions, improves monotonicity of the scheme and extends the class of SAHH problems that can be accelerated by the P1SA scheme. For z and ϑ variables $\delta=1/2$ and fix-up function is simplified.

4. Results

The performance of the P1SA scheme in solving highly heterogeneous problems with small absorption was tested by solving isotropically scattered r,ϑ,z geometry test problem 1, defined in paper [6]. The problem consists of a heterogeneous cylinder, 25 cm in radius and 50 cm in height. The circular section of the problem is depicted in Figure 1a. The "thin" duct region is filled by material 2. It is surrounded by a "thick" region, filled by material 1, and bends around the central disk of material 1. There is a unit isotropic source incident on the left face of the duct. An isotropic source of strength 10^{-6} particles/cm^3 is distributed throughout the problem. $0<\vartheta<\pi/2$ sector of symmetry is used in calculation. Really, the problem is two-dimensional and can be solved in r,z geometry. The similar problem can be defined for x,y,z geometry case. In this case the test problem consists of a heterogeneous parallelepiped with rectangular (in axial $0<z<15$ and $35<z<50$ sections) duct region. Only a quarter of the region with reflected $x=0$ and $y=0$ boundaries can be solved in this case.

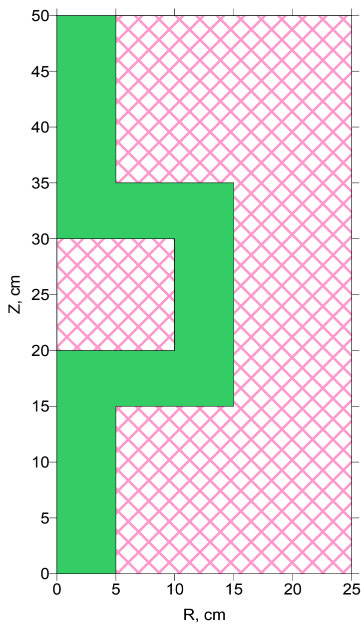
Test problem 2, depicted in Figure 1b, is received from problem 1 by replacing the half of the region with $z>25$ by reflecting boundary at $z=25$. Test problem 2 can be solved in r,z geometry. The similar problem can be also defined for x,z geometry case. Test problem 2 for x,z geometry consists of an infinite in y direction parallelepiped with rectangular duct region along z axis and reflecting boundaries at $x=0$ and $z=25$.

In Tables 1-4 number of inner iterations required to achieve pointwise convergence criterion 10^{-5} in solving problem 2 in S_8 approximation for some values of material 1 ($\sigma_{t,1}$) and material 2 ($\sigma_{t,2}$) total cross-sections, as well as the scattering ratio $c=\sigma_s/\sigma_t$, are given both for r,z and x,z geometry case for spatial mesh 50×50 for the Step, MDS$_n$ full, DD, MDS$_n$, AWDD and AWDD+MDS$_n$ schemes. By the AWDD+MDS$_n$ scheme is marked a hybrid scheme where the AWDD scheme fix-up is joined with monotonicity fix-up for critical extrapolations, used by the MDS$_n$ scheme. In Table 5 results for solving problem 2 with a fine spatial mesh 200×200 are given. The accuracy of solving the P1 system for accelerating corrections ε_{P1} was chosen 0.03 in

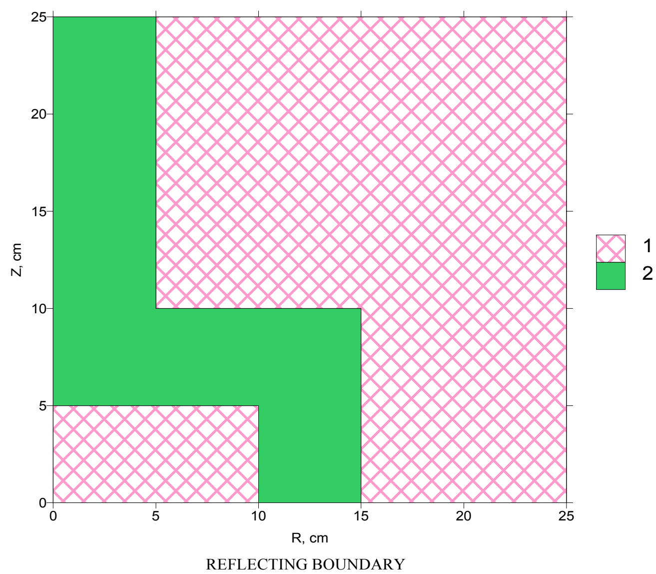
Tables 1-3 and 5, and 0.5 in Table 4.

In Tables 6-8 number of inner iterations required to achieve pointwise convergence criterion 10^{-5} in solving problem 1, depicted in Figure 1a, in S_6 approximation for some values of material 1 ($\sigma_{t,1}$) and material 2 ($\sigma_{t,2}$) total cross-sections, as well as the scattering ratio $c=\sigma_s/\sigma_t$, are given both for r,z, r,ϑ,z and x,y,z geometries for spatial meshes 50×100, 50×3×100 and 50×50×100, respectively, for the Step, MDS$_n$ full, DD, MDS$_n$, AWDD and AWDD+MDS$_n$ schemes.

Figure 1: Circular sections: a) the r,ϑ,z geometry problem 1, b) r,z geometry problem 2.



a)



b)

Table 1: Number of inner iterations required for solving Problem 2 for r,z and x,z geometry case for the Step, MDS$_n$ full and DD schemes.

		$\sigma_{t,2}$					
		10^2			10^1		
$\sigma_{t,1}$	c	Step	MDS$_n$ full	DD	Step	MDS$_n$ full	DD
10^{-3}	0.9999 0.99	23 (23)[1] 13 (13)	(23) (13)	112 (135) 149 (129)	23 (25) 19 (21)	(28) (23)	45 (39) 34 (40)
10^{-2}	0.9999 0.99	21 (22) 12 (13)	(24) (13)	99(118) 133 (129)	22 (22) 18 (18)	(25) (20)	36 (38) 31 (34)
10^0	0.9999 0.99	12 (12) 9 (9)	(13) (10)	76 (66) 106 (86)	12 (11) 12 (11)	(12) (13)	20 (16) 22 (22)

[1] Results for x,z geometry case in Tables 1-5 are given in round brackets.

Table 2: Number of inner iterations required for solving Problem 2 for r,z and x,z geometry case for the AWDD scheme with $U_0 = (1-\delta)/4$ and $b = 1.3$, the MDS_n and AWDD+MDS_n schemes.

		$\sigma_{t,2}$					
		10^2			10^1		
$\sigma_{t,1}$	c	AWDD	MDS_n	AWDD +MDS_n	AWDD	MDS_n	AWDD +MDS_n
10^{-3}	0.9999 0.99	a (a) 20 (20)	(39) (20)	a[2] (a) 17 (16)	31 (28) 31 (29)	(34) (37)	29 (29) 26 (27)
10^{-2}	0.9999 0.99	a (a) 19 (19)	(44) (18)	a (a) 16 (17)	28 (29) 27 (23)	(39) (27)	27 (28) 23 (21)
10^{0}	0.9999 0.99	a (a) 18 (18)	(24) (14)	a (a) 16 (16)	18 (17) 20 (20)	(15) (15)	17 (16) 19 (18)

Table 3: Number of inner iterations required for solving Problem 2 for r,z and x,z geometry case for the AWDD scheme in dependence of the fix-up function choice.

		$\sigma_{t,2}$					
		10^2			10^1		
$\sigma_{t,1}$	c	AWDD $U_0 = 1-\delta$ $b = 2.0$	AWDD $U_0 = \frac{1-\delta}{2}$ $b = 1.6$	AWDD $U_0 = \frac{1-\delta}{4}$ $b = 1.3$	AWDD $U_0 = 1-\delta$ $b = 2.0$	AWDD $U_0 = \frac{1-\delta}{2}$ $b = 1.6$	AWDD $U_0 = \frac{1-\delta}{4}$ $b = 1.3$
10^{-3}	0.9999 0.99	a (a) 20 (27)	(a) 20 (21)	a (a) 20 (20)	33 (30) 28 (30)	31 (30) 26 (28)	31 (28) 31 (29)
10^{-2}	0.9999 0.99	a (a) 44 (a)	(a) 19 (19)	a (a) 19 (19)	26 (28) 28 (27)	28 (28) 27 (25)	28 (29) 27 (23)
10^{0}	0.9999 0.99	a (a) a (14)	(a) 22 (17)	a (a) 18 (18)	21 (20) 27 (28)	19 (18) 23 (23)	18 (17) 20 (20)

5. Conclusion

Numerical experiment has shown that if the monotonous Step or MDS_n full schemes for approximation of transport equation are used then the consistent P1SA scheme works quite efficiently for all problems solved. MDS_n full scheme is the optimal linear monotonous scheme from the WDD schemes family as its weights are close to 1 at most as possible. Though it has the first order of accuracy and requires a fine spatial mesh, its accuracy is essentially higher than the accuracy of the Step scheme. So, taking into account a good compatibility of the MDS_n full scheme with the P1SA scheme, maybe, its use will be acceptable for parallel calculations.

The tested variant of the MDS_n scheme with a primary algorithm for identification of critical

[2] By symbol ‘a’ the cases where no convergence was achieved are marked.

extrapolations prevents degradation of the consistent P1SA scheme in solving SAHH problems, but it requires an additional fix-up to ensure positivity of the scheme. It seems that performance of the MDS_n scheme can be essentially improved by the use more elaborate algorithm for identification of critical extrapolations.

The adaptive fix-up used by the AWDD scheme improves monotonicity of the scheme and in this way prevents degradation of the P1SA scheme. But for SAHH problems with $c \to 1$ there is some critical level that depends on softness of the fix-up function used, starting with which the nonlinearity, introduced by the adaptive fix-up, becomes a source of the P1SA scheme instability (convergence stops after achieving some accuracy that is less than required). To avoid this type degradation either the spatial mesh should be refined (see Table 5) or parameter U_0 in Eq. (3.4) should be decreased (Table 4). With increasing of the problem dimension requirements for the fix-up function softness also increase (Table 8).

Table 4: Number of inner iterations required for solving Problem 2 for x,z geometry case with ε_{P1}=0.5 for the AWDD and AWDD+MDS_n schemes for fix-up function (3.4) with $U_0 = (1-\delta)/10$ and $b=1$.

		$\sigma_{t,2}$			
		10^2		10^1	
		$U_0 = (1-\delta)/10$, $b=1$			
$\sigma_{t,1}$	c	AWDD	AWDD +MDS_n	AWDD	AWDD +MDS_n
10^{-3}	0.9999 0.99	(52) (23)	(42) (17)	(41) (31)	(36) (25)
10^{-2}	0.9999 0.99	(48) (21)	(38) (17)	(33) (20)	(32) (21)
10^{0}	0.9999 0.99	(35) (20)	(35) (16)	(20) (18)	(17) (16)

Table 5: Number of inner iterations required for solving Problem 2 for r,z and x,z geometry case for fine spatial mesh 200×200 with ε_{P1}=0.03 for the AWDD scheme with $U_0 = (1-\delta)/4$ and $b=1.3$, MDS_n and AWDD+MDS_n schemes.

		$\sigma_{t,2}$		
		10^2		
$\sigma_{t,1}$	c	AWDD	MDS_n	AWDD +MDS_n
10^{-3}	0.9999 0.99	29 (38) 28 (26)	(48) (30)	33 (34) 23 (24)
10^{-2}	0.9999 0.99	31 (35) 27 (26)	(47) (29)	32 (34) 23 (22)
10^{0}	0.9999 0.99	24 (23) 25 (25)	(28) (24)	21 (20) 22 (22)

The use of the hybrid AWDD+MDS_n scheme, as usual, leads to some decreasing the number of inner iterations.

Received results have broadened the class of SAHH problems those can be accelerated by the P1SA scheme with flux poinwise convergence. But the problem of degradation of the P1SA scheme in solving SAHH problems requires additional investigation.

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Table 6: Number of inner iterations required for solving Problem 1 for r,z, r,ϑ,z and x,y,z geometry case with ε_{P1}=0.03 for the Step, MDS_n full and DD schemes.

		$\sigma_{t,2}$					
		10^2			10^1		
$\sigma_{t,1}$	c	Step	MDS_n full	DD	Step	MDS_n full	DD
10^{-3}	0.9999	25 [25] (33)[3]	(36)	145	26 [26] (34)	(37)	64
	0.99	15 (17)	(17)	169 (184)	21 (25)	(26)	49 (63)
10^{-2}	0.9999	21 [21] (26)	(27)	141	23 [23] (26)	(27)	41
	0.99	14 (14)	(15)	170 (153)	18 (20)	(22)	40 (53)
10^{0}	0.9999	7 [9] (14)	(15)	76	14 [15] (15)	(16)	27
	0.99	10 (11)	(11)	64 (133)	12 (12)	(14)	31 (36)

Table 7: Number of inner iterations required for solving Problem 1 for r,z, r,ϑ,z and x,y,z geometry case by the AWDD and AWDD+MDS_n schemes. P1 system for accelerating corrections was solved with ε_{P1}=0.5 for 2D r,z and ε_{P1}=0.3 for 3D geometries.

		$\sigma_{t,2}$					
		10^2			10^1		
		$U_0=(1-\delta)/2$, $b=1.6$	$U_0=(1-\delta)/4$, $b=1.3$		$U_0=(1-\delta)/2$, $b=1.6$	$U_0=(1-\delta)/4$, $b=1.3$	
$\sigma_{t,1}$	c	AWDD	AWDD	AWDD +MDS_n	AWDD	AWDD	AWDD +MDS_n
10^{-3}	0.9999				48	48	
	0.99	31[27](a)	31[25](26)	23 (31)	37[42](a)	37[42](40)	32 (34)
10^{-2}	0.9999				36	37(a)	
	0.99	29[26](a)	29[24](31)	22 (26)	31[36](a)	30[36](35)	25 (29)
10^{0}	0.9999	a[a]	a[40]		22[27]	21[24]	(a)
	0.99	22[20](a)	21[21](22)	19 (21)	26[27](26)	22[24](23)	21 (22)

[3] Results for r,ϑ,z and x,y,z geometry case in Tables 6-8 are given in square and round brackets, respectively.

Table 8 Number of inner iterations required for solving Problem 1 for r, z, r, ϑ, z and x, y, z geometry case with ε_{P1}=0.03 by the AWDD and AWDD+MDS$_n$ schemes.

		$\sigma_{t,2}$			
		10^2		10^1	
		$U_0 = (1-\delta)/20$, $b = 1$			
$\sigma_{t,1}$	c	AWDD	AWDD+MDS$_n$	AWDD	AWDD+MDS$_n$
10^{-3}	0.9999 0.99	88 [a] 25 [28](26)	45 21 (21)	44 [72] 29 [41] (37)	38 24 (29)
10^{-2}	0.9999 0.99	a [79] 25 [24] (24)	59 22 (21)	33 [35] (44) 28 [32] (32)	30 25 (29)
10^0	0.9999 0.99	50 [a] (a) 23 [21] (23)	a (a) 20 (22)	19 [21] (61) 20 [25] (20)	18 (24) 18 (19)

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