Kinetic Parameters for Source Driven Systems

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Abstract

The definition of the characteristic kinetic parameters of a subcritical source-driven system constitutes an interesting problem in reactor physics with important consequences for practical applications. Consistent and physically meaningful values of the parameters allow to obtain accurate results from kinetic simulation tools and to correctly interpret kinetic experiments. For subcritical systems a preliminary problem arises for the adoption of a suitable weighting function to be used in the projection procedure to derive a point model.

The present work illustrates a consistent factorization-projection procedure which leads to the definition of the kinetic parameters in a straightforward manner. The reactivity term is introduced coherently with the generalized perturbation theory applied to the source multiplication factor k_S, which is thus given a physical role in the kinetic model. The effective prompt lifetime is introduced on the assumption that a neutron generation can be initiated by both the fission process and the source emission. Results are presented for simplified configurations to fully comprehend the physical features and for a more complicated highly decoupled system treated in transport theory.

KEYWORDS: Reactor dynamics, Source-driven systems, Kinetic parameters

1. Introduction

The derivation of proper kinetic models for source-driven systems is an important task in the development of the technology of subcritical reactors. The classic procedure to derive the kinetic equations requires a factorization of the unknown neutron flux followed by a projection on a suitable weighting function [1]. This procedure is consistent and physically meaningful for critical reference reactors; however, it can be somewhat arbitrarily extended to subcritical systems. Some problems connected to the most adequate weighting function to be used in the projection procedure have been recently outlined [2, 3]. On the other hand, several parameters have been introduced in order to characterize subcritical system and to evidence the physical features connected to the presence of the source that drives the neutron population [4]. Some of these parameters, although quite useful to characterize the physical and engineering features of the system, do not retain a completely consistent meaning.

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A paper by Gandini [5] tackles the problem on a self-consistent basis analysing in a comprehensive way the problems arising in the kinetics of subcritical systems through a heuristic generalized perturbation theory approach. In that paper definitions for the kinetic parameters are given, introducing in particular suitable reactivity terms.

The present paper is motivated by the same objective as Gandini's paper. It has appeared in many applications that the use of standard models with conventional definitions of the parameters usually employed for nearly critical systems may become questionable when analysing subcritical reactors. Also, tailored models could have been envisaged for specific applications and have proved adequate for experimental data interpretation, although by some arbitrary definitions of the parameters. The present work illustrates a consistent factorization-projection procedure which leads to the definition of kinetic parameters in a straightforward manner. The approach differs somewhat from Gandini's especially in the definition of the effective neutron lifetime and introduces a source reactivity into the model through the source multiplication factor k_S, to which the generalized perturbation technique is naturally applied.

2. Theory

The general formulation of the time-dependent problem for a source-driven system is considered in its most general form [6]:

$$\begin{cases} \dfrac{1}{v}\dfrac{\partial \phi}{\partial t} = \hat{L}\phi + \hat{M}_p\phi + \lambda C + S, \\ \\ \dfrac{\partial C}{\partial t} = \hat{M}_d\phi - \lambda C, \end{cases} \tag{1}$$

where the operators herewith appearing may be specified according to the physical model adopted. The assumption of one delayed neutron family poses no restriction and can be easily removed. A reference steady-state system is defined and the corresponding neutron distribution is determined by the solution of the following equation:

$$\hat{L}_0\phi_S + \hat{M}_0\phi_S + S_0 = 0, \tag{2}$$

where the total multiplication operator is introduced as: $\hat{M}_0 = \hat{M}_{p,0} + \hat{M}_{d,0}$. With the usual Henry procedure [1], the solution ϕ to Eq. (1) is factorized as a product of shape and amplitude functions, as $\phi = \phi_S P$, having indicated with P the time-dependent only amplitude function.

The shape function ϕ_S is slowly varying in time. If assumed as a constant, a generalization of the standard point kinetic model is obtained. If updated on a scale slower than for the amplitude, the consistent quasi-static method [7] is derived as a general extension of the classic quasi-static method of reactor theory [8]. Recently a variational approach has also been pursued [9] with the objective of giving a consistent mathematical foundation to the whole procedure applied to source-driven systems.

The factorized formula is then introduced into Eq. (1) and a projection is taken on a weight function. A particularly physically significant result is obtained if the weighting function is assumed as the solution of the following source-driven reference system adjoint equation:

$$\hat{L}_0^+\phi_S^+ + \hat{M}_0^+\phi_S^+ + S_0^+ = 0, \tag{3}$$

where $S_0^+ = \nu\Sigma_{f,0}$. The choice of this adjoint source allows a significant interpretation of the adjoint flux in terms of importance, e.g. see [5,7]. The paper submitted for publication referred above [9] addresses the problem in terms of a variational functional framework, therefore in a totally unambiguous manner. After the projection is carried out, the first equation of system (1) can be easily given the following form:

$$\begin{aligned}\left\langle \phi_S^+ \right| \frac{1}{v}\,\phi_S\rangle\,\frac{dP}{dt} = \Big[\left\langle \phi_S^+ \right| \left(\delta\hat{M}+\delta\hat{L}\right)\phi_S\rangle - \left\langle \phi_S^+ \right| \hat{M}_d\,\phi_S\rangle - \left\langle \phi_S^+ \right| S_0\rangle\Big]\,P \\ +\lambda\left\langle \phi_S^+ \right| C\rangle + \left\langle \phi_S^+ \right| S\rangle\,,\end{aligned} \tag{4}$$

where the perturbation operators are explicitly introduced.

Generally this equation and the corresponding projected equations for the precursors can be solved as a system, once the integral terms herewith appearing are computed. In the following the integral terms are manipulated in order to obtain a more physically significant formulation. A normalization procedure is then in order. This step can be carried out in a somewhat arbitrary manner. Differently from previous proposals, in this work it is suggested to divide all terms by the total importance of the first generation neutrons introduced into the system by both fission events and source emissions:

$$I = \left\langle \phi_S^+ \right| \hat{M}_0\,\phi_S\rangle + \left\langle \phi_S^+ \right| S_0\rangle\,. \tag{5}$$

It is therefore assumed that source and fission neutrons are physically undistinguishable for the multiplying system. This choice has a straight consequence on the definition of the kinetic parameters, with particular regard to the effective neutron lifetime. All the terms of equation (4) are divided by this integral. The objective is to write the model as:

$$\begin{cases}\Lambda_S\dfrac{dP}{dt} = \left[\rho_S-\beta_S\right]P + \lambda\dfrac{\left\langle \phi_S^+ \right| C\rangle}{I} + \dfrac{\left\langle \phi_S^+ \right| S\rangle}{I}, \\ \dfrac{d}{dt}\left(\dfrac{\left\langle \phi_S^+ \right| C\rangle}{I}\right) = \beta_S P - \lambda\dfrac{\left\langle \phi_S^+ \right| C\rangle}{I}.\end{cases} \tag{6}$$

In Eq. (6) above, a total ”‘source-driven”’ reactivity is introduced as in the following sum:

$$\rho_S = \tilde{\rho_S} + \rho_{S,0}, \tag{7}$$

where the perturbation-induced source reactivity and the initial subcriticality level are defined as:

$$\begin{aligned}\tilde{\rho_S} &= \frac{\left\langle \phi_S^+ \right| \left(\delta\hat{M}+\delta\hat{L}\right)\phi_S\rangle}{I}, \\ \rho_{S,0} &= -\frac{\left\langle \phi_S^+ \right| S_0\rangle}{I}.\end{aligned} \tag{8}$$

The first reactivity term can be related to the source multiplication factor that is introduced by the following formula [10]:

$$k_S = \frac{\left\langle \hat{M}\phi_S \right\rangle}{\left\langle \hat{M}\phi_S + S \right\rangle}. \tag{9}$$

In general, a first order perturbation approach allows to evaluate the relative perturbation of the k_S parameter. To construct the perturbation of $\left\langle \hat{M}\phi_S \right\rangle$, the generalized perturbation theory [11] is required. The full expression is following:

$$\frac{\delta k_S}{k_S} = \frac{\langle S_0 \rangle}{\left\langle \hat{M}_0 \phi_S \right\rangle + \langle S_0 \rangle} \left[\frac{\langle \phi_S^+ | \left(\delta \hat{M} + \delta \hat{L} \right) \phi_S \rangle}{\left\langle \hat{M}_0 \phi_S \right\rangle} - \frac{\langle \delta S \rangle}{\langle S_0 \rangle} \right]. \tag{10}$$

The above equation can be further simplified in the assumption of a leading role for the perturbation of the numerator of Eq. (9) with respect to the denominator terms. This assumption amounts to disregard the contribution of the source and to write explicitly:

$$\frac{\delta k_S}{k_S} = \frac{\langle \phi_S^+ | \left(\delta \hat{M} + \delta \hat{L} \right) \phi_S \rangle}{\left\langle \hat{M}_0 \phi_S \right\rangle}. \tag{11}$$

As a consequence, the source-driven reactivity term is written as:

$$\tilde{\rho_S} = \frac{\delta k_S}{k_S} \frac{\left\langle \hat{M}_0 \phi_S \right\rangle}{I}, \tag{12}$$

as introduced in Eq. (8).

All reactivity terms are obviously different from the ones obtained by the usual static reactivity definitions; also these terms are determined by the source-driven flux shape and by the corresponding source-driven importance function. It is easily verified that the standard values are approached asymptotically at criticality.

The effective delayed neutron fraction is also introduced for the source-driven system, namely:

$$\beta_S = \frac{\langle \phi_S^+ | \hat{M}_d \, \phi_S \rangle}{I}, \tag{13}$$

while the effective generation time is defined as:

$$\Lambda_S = \frac{\langle \phi_S^+ | \frac{1}{v} \, \phi_S \rangle}{I}. \tag{14}$$

A few observations can be summarized:

- Equation (12) gives a physical role to k_S;
- The derivation of the model is consistent with the quasi-static philosophy applied to source-driven systems;
- On approaching criticality, also the kinetic parameters Λ_S and β_S approach the usual parameters defined for critical systems;
- On approaching criticality, ρ_S approaches the standard reactivity as evaluated with standard perturbation theory.

All these definitions are physically motivated as in a source-driven subcritical system the originating process for neutrons is the combination of both source and fission emissions.

3. Results

The results presented in the following concern a simplified one-dimensional reactor, in order to get a general idea of the effects of the redefinition of the kinetic parameters with respect to the standard technique. The neutron balance adopts the one-group diffusion model; the equations are solved for a one-dimensional slab reactor.

Table 1 reports the kinetic parameters and the corresponding time constants obtained through the solution of the inhour equation [1]. The observation of the results confirms the considerations set forth in the previous section. In particular, when increasing the subcriticality level, one notices that the effective delayed neutron fraction significantly decreases, which is consistent with the fact that the dominance of the source is enhanced, with a consequent reduction of the role of delayed neutrons. This feature is further confirmed by observing the behavior of the time constants. In fact, the time constant associated to the prompt response, ω_2, decreases significantly, implying a quicker response.

Table 1: Reformulated and standard kinetic parameters for a system with different levels of subcriticality and δ-type $+10\%$ perturbation of the absorption cross section. System dimension: $H = 30L$.

k_{eff}	0.9	0.95	0.98	0.999
Λ_S [10^{-5}s]	4.98	4.96	4.95	4.95
Λ [10^{-5}s]	5.50	5.21	5.05	4.95
β_S [pcm]	543	572	588	599
β [pcm]	600	600	600	600
$\rho_{S,0}$ [pcm]	-9451	-4712	-1919	-100
ρ_0 [pcm]	-11111	-5263	-2041	-100
perturbation distance from the source: $9L$				
ρ_S [pcm]	-26	-41	-56	-68
ρ [pcm]	-76	-72	-70	-69
$\omega_{1,S}$ [s^{-1}]	-0.0947	-0.0893	-0.0770	-0.0218
ω_1 [s^{-1}]	-0.0949	-0.0899	-0.0779	-0.0219
$\omega_{2,S}$ [s^{-1}]	-2013.9	-1073.5	-517.8	-155.2
ω_2 [s^{-1}]	-2145.0	-1140.1	-537.2	-155.4
perturbation distance from the source: $3L$				
ρ_S [pcm]	-191	-190	-186	-180
ρ [pcm]	-200	-189	-184	-180
$\omega_{1,S}$ [s^{-1}]	-0.0947	-0.0895	-0.0781	-0.0318
ω_1 [s^{-1}]	-0.0950	-0.0901	-0.0787	-0.0318
$\omega_{2,S}$ [s^{-1}]	-2047.0	-1103.5	-544.0	-177.8
ω_2 [s^{-1}]	-2167.5	-1162.6	-559.7	-177.8

In Table 2 a large system is considered. As can be seen, due to the fact that from such a large source-driven system almost no neutrons escape, the parameter Λ_S is almost insensitive on the subcriticality level. Fluxes and adjoints are drawn in Fig. 2 in the reference source-driven configuration.

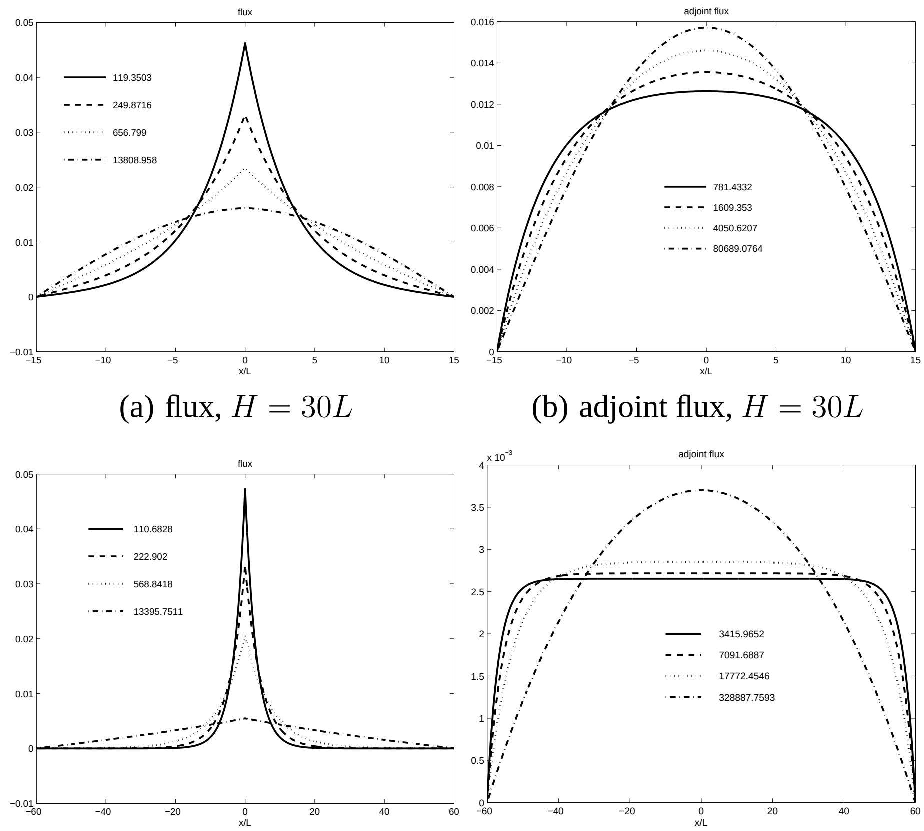
A more complicated highly decoupled system is now considered by a three-group two-dimensional transport model [12]. The system is highly heterogeneous and is characterized

Table 2: Reformulated and standard kinetic parameters for a system with different levels of subcriticality and δ-type $+10\%$ perturbation of the absorption cross section. System dimension: $H = 120L$.

k_{eff}	0.9	0.95	0.98	0.999
Λ_S [10^{-5}s]	5.00	5.00	5.00	5.00
Λ [10^{-5}s]	5.55	5.26	5.10	5.00
β_S [pcm]	540	570	588	599
β [pcm]	600	600	600	600
$\rho_{S,0}$ [pcm]	-9938	-4935	-1937	-96
ρ_0 [pcm]	-11111	-5263	-2041	-100
perturbation distance from the source: $15L$				
ρ_S [pcm]	-4	-12	-26	-42
ρ [pcm]	-48	-45	-44	-43
$\omega_{1,S}$ [s^{-1}]	-0.0948	-0.0897	-0.0769	-0.0188
ω_1 [s^{-1}]	-0.0949	-0.0898	-0.0776	-0.0192
$\omega_{2,S}$ [s^{-1}]	-2096.6	-1103.5	-510.2	-147.8
ω_2 [s^{-1}]	-2118.0	-1123.4	-526.6	-148.6
perturbation distance from the source: $3L$				
ρ_S [pcm]	-185	-172	-138	-60
ρ [pcm]	-55	-53	-51	-50
$\omega_{1,S}$ [s^{-1}]	-0.0949	-0.0899	-0.0779	-0.0207
ω_1 [s^{-1}]	-0.0949	-0.0899	-0.0777	-0.0200
$\omega_{2,S}$ [s^{-1}]	-2132.7	-1135.5	-532.7	-151.4
ω_2 [s^{-1}]	-2119.4	-1124.8	-528.0	-150.0

by an inner core with a hard spectrum and an outer core surrounded by a reflector with a much softer spectrum. Systems of this type have interesting physical features that make them rather attractive. The perturbation is introduced locally at two positions having the same abscissa, where absorption is perturbed in all groups by an amount that implies a 20% change in the total cross section. Flux distributions for two different levels of subcriticality are reported in Figs. 2 and 3.

The results for the kinetic parameters and corresponding roots of the inhour equation are reported in Tables 3 and 4. The most significant difference with respect to standard critical evaluations appears for the reactivity term, which induces different values of the inhour time constants.



(a) flux, $H = 30L$ (b) adjoint flux, $H = 30L$

(c) flux, $H = 120L$ (d) adjoint flux, $H = 120L$

Figure 1: Fluxes and adjoint functions, renormalized in order to evidence spatial effects. The values of their integral are reported.

4. Conclusions

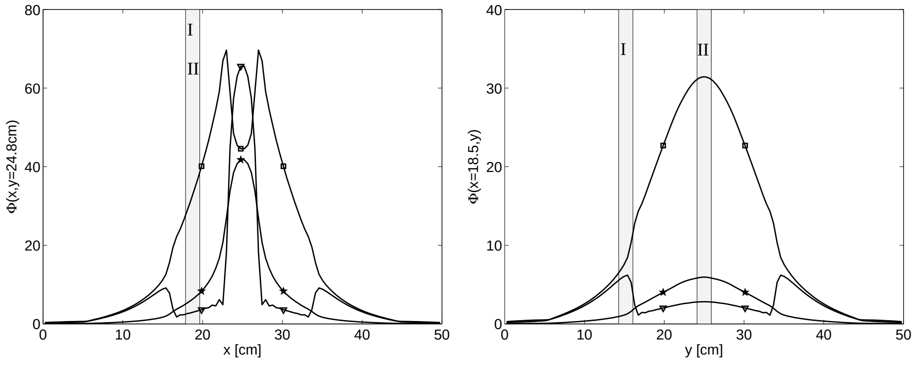
The work presented in this paper illustrates an approach specifically designed to derive consistent kinetic parameters for subcritical source-driven systems. The approach should lead to sets of parameters physically meaningful for the kinetic analysis of subcritical systems and for the interpretation of experiments. Some work is on-going in the application of these techniques to the interpretation of kinetic experiments.The results presented for simplified and for

Table 3: Kinetic parameters for a decoupled system with $k_{eff} = 0.95$.

–		perturbation I		perturbation II	
Λ_S [10^{-6}s]	1.14	ρ_S [pcm]	-157	ρ_S [pcm]	-34
Λ [10^{-6}s]	1.10	ρ [pcm]	-150	ρ [pcm]	-43
β_S [pcm]	642	$\omega_{1,S}$ [s^{-1}]	-0.0691	$\omega_{1,S}$ [s^{-1}]	-0.0672
β [pcm]	651	ω_1 [s^{-1}]	-0.0893	ω_1 [s^{-1}]	-0.0891
$\rho_{S,0}$ [pcm]	-1282	$\omega_{2,S}$ [10^3s^{-1}]	-18.266	$\omega_{2,S}$ [10^3s^{-1}]	-17.189
ρ_0 [pcm]	-5263	ω_2 [10^3s^{-1}]	-55.365	ω_2 [10^3s^{-1}]	-54.394

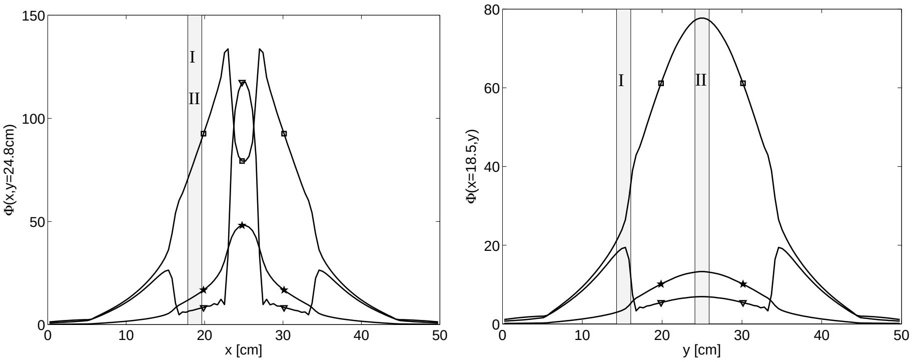
Table 4: Kinetic parameters for a decoupled system with $k_{eff} = 0.98$.

–		perturbation I		perturbation II	
Λ_S [10^{-6}s]	1.11	ρ_S [pcm]	-153	ρ_S [pcm]	-38
Λ [10^{-6}s]	1.10	ρ [pcm]	-149	ρ [pcm]	-43
β_S [pcm]	647	$\omega_{1,S}$ [s^{-1}]	-0.0515	$\omega_{1,S}$ [s^{-1}]	-0.0469
β [pcm]	651	ω_1 [s^{-1}]	-0.0771	ω_1 [s^{-1}]	-0.0762
$\rho_{S,0}$ [pcm]	-533	$\omega_{2,S}$ [10^3s^{-1}]	-12.008	$\omega_{2,S}$ [10^3s^{-1}]	-10.971
ρ_0 [pcm]	-2041	ω_2 [10^3s^{-1}]	-25.940	ω_2 [10^3s^{-1}]	-24.969



(a) (b)

Figure 2: Flux distribution along x (a) and y (b) for a system with $k_{eff} = 0.95$; perturbed regions shaded in grey. Stars: fast group; squares: intermediate group; triangles: low-energy group.



(a) (b)

Figure 3: Flux distribution for a system characterized by $k_{eff} = 0.98$. Curves are identified as in Fig. 2.

highly decoupled systems studied in transport theory evidence large effects with respect to the standard definition of the parameters that shall significantly affect temporal predictions.

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