Generalized Perturbation Theory Based on the Method of Cyclic Characteristics

M. Assawaroongruengchot and G. Marleau*
Institut de génie nucléaire, Département de génie physique,
École Polytechnique de Montréal, 2900 Boul. Édouard-Montpetit,
Montréal, Québec, H3T 1J4, Canada

Abstract

A GPT algorithm for estimation of eigenvalues and reaction-rate ratios is developed for the neutron transport problems in 2D fuel assemblies with isotropic scattering. In our study the GPT formulation is based on the integral transport equations. The mathematical relationship between the generalized flux importance and generalized source importance functions is applied to transform the generalized flux importance transport equations into the integro-differential forms. The resulting adjoint and generalized adjoint transport equations are then solved using the method of cyclic characteristics (MOCC). Because of the presence of negative adjoint sources, a biasing/decontamination scheme is applied to make the generalized adjoint functions positive in such a way that it can be used for the multigroup rebalance technique. To demonstrate the efficiency of the algorithms, perturbative calculations are performed on a 17×17 PWR lattice.

KEYWORDS: Generalized perturbation theory, method of characteristics, cyclic tracking, neutron transport theory

1. Introduction

Generalized perturbation theory (GPT) is a technique used to estimate the change in performance functionals, such as reaction-rate ratios, eigenvalues etc., affected by small changes in reactor core compositions. It can be applied to problems such as computations of power density distribution, sensitivity and uncertainty analyses etc. [1]. The GPT method has been mostly applied to problems using the neutron diffusion equation [1,2]. For the neutron transport problems, a few applications can be found such as a variational nodal transport perturbation theory [3] based on the response matrix equations and GPT applications based on the CP equation [4]. Here we present an algorithm for GPT estimations of eigenvalues and reaction-rate ratios based on integral transport equations and the MOCC transport solution method. The relationship between generalized adjoint and generalized flux importance functions is applied to transform the generalized flux importance transport equations into an integro-differential form.

Because of the presence of negative adjoint sources in the generalized adjoint problem, the direct use of these sources in the adjoint MOCC algorithm shall result in the negative generalized adjoints and errors in the rebalance factors. In order to remedy this problem, a flux

*Corresponding author e-mail: guy.marleau@polymtl.ca

biasing scheme is applied to make the generalized adjoints positive in such a way that the rebalance technique can be applied. After the convergence is reached, the decontamination scheme extracts the orthogonal component from the biased generalized adjoints.

The verification of GPT algorithms is performed for a 17×17 PWR lattice problem. Comparisons of the generalized adjoint functions obtained by the MOCC and CP methods are presented as well as the numerical results of GPT estimates and sensitivity coefficients.

2. GPT Formulations

2.1 Transport Equations

Let's first define the multigroup discrete ordinate forward integral transport equations [5] in direction $\vec{\Omega}_l$ for a multiplicative k_{eff} problem with isotropic scattering as follows,

$$(\mathcal{L}^g - \lambda \mathcal{F}^g)\Phi^g(\vec{r}, \vec{\Omega}_l) = 0 \tag{1}$$

where

$$\mathcal{L}^g\Phi^g = \Phi^g - \mathcal{T}^{g,-1} \sum_{g'=1}^{G} \Sigma_s^{g \leftarrow g'} \phi^{g'} \qquad \mathcal{F}^g\Phi^g = \mathcal{T}^{g,-1} \sum_{g'=1}^{G} \chi^g \nu \Sigma_f^{g'} \phi^{g'}$$

$$\mathcal{T}^{g,-1} = (\vec{\Omega}_l \cdot \vec{\nabla} + \Sigma^g(\vec{r}))^{-1} \qquad \phi^g(\vec{r}) = \sum_{l=1}^{N^\Omega} W_l^\Omega \Phi^g(\vec{r}, \vec{\Omega}_l)$$

Here the explicit quadrature we will select is such that a set of cyclic tracking lines can be generated for a 2D Cartesian problem [6]. One can associate with the integral transport equation (1) a flux importance transport equation using the conservation relationship,

$$\left\langle \Phi^{\dagger g}, (\mathcal{L}^g - \lambda \mathcal{F}^g)\Phi^g \right\rangle = \left\langle \Phi^g, (\mathcal{L}^{\dagger g} - \lambda \mathcal{F}^{\dagger g})\Phi^{\dagger g} \right\rangle = 0 \tag{2}$$

The flux importance transport equations then take the form,

$$(\mathcal{L}^{\dagger g} - \lambda \mathcal{F}^{\dagger g})\Phi^{\dagger g}(\vec{r}, \vec{\Omega}_l) = 0 \tag{3}$$

where

$$\mathcal{L}^{\dagger g}\Phi^{\dagger g} = \Phi^{\dagger g} - \sum_{g'=1}^{G} \Sigma_s^{g' \leftarrow g} \sum_{l'=1}^{N^\Omega} W_{l'}^\Omega \mathcal{T}^{*g',-1} \Phi^{\dagger g'} \qquad \mathcal{T}^{*g,-1} = (-\vec{\Omega}_l \cdot \vec{\nabla} + \Sigma^g(\vec{r}))^{-1}$$

$$\mathcal{F}^{\dagger g}\Phi^{\dagger g} = \sum_{g'=1}^{G} \chi^{g'} \nu \Sigma_f^g \sum_{l'=1}^{N^\Omega} W_{l'}^\Omega \mathcal{T}^{*g',-1} \Phi^{\dagger g'} \qquad \phi^{\dagger g}(\vec{r}) = \sum_{l=1}^{N^\Omega} W_l^\Omega \Phi^{\dagger g}(\vec{r}, \vec{\Omega}_l)$$

$\Phi^{\dagger g}(\vec{r}, \vec{\Omega}_l)$ and $\phi^{\dagger g}(\vec{r})$ are the angular and scalar flux importance functions respectively. The angular and scalar flux importance functions are respectively the adjoint angular and scalar functions of the integral neutron transport equations [1]. The angular flux importance is related to the adjoint angular function by,

$$\Phi^{*g}(\vec{r}, \vec{\Omega}_l) = \mathcal{T}^{*g,-1} \Phi^{\dagger g}(\vec{r}, \vec{\Omega}_l) \tag{4}$$

where $\Phi^{*g}(\vec{r},\vec{\Omega}_l)$ is the adjoint angular function (or angular source importance). Operating on both sides of (3) with $\mathcal{T}^{*g,-1}$ and using (4), we can arrange the flux importance transport equations (3) to the integro-differential adjoint transport equations. To determine the scalar flux importances using the adjoint scalar functions, we apply (4) to (3) and integrate the resulting equations over angular domain to obtain,

$$\phi^{\dagger g}(\vec{r}) = \sum_{g'=1}^{G} [\Sigma_s^{g'\leftarrow g}(\vec{r}) + \chi^{g'}(\vec{r})\nu\Sigma_f^g(\vec{r})]\phi^{*g'}(\vec{r}) \tag{5}$$

where $\phi^{*g}(\vec{r})$ is the adjoint scalar function (or scalar source importance). One may note that the angular and scalar source importances are the adjoint angular and scalar functions of the integro-differential transport equations. The scalar flux importance function (5) is equivalent to the total adjoint source of the integro-differential adjoint transport problem with isotropic scattering.

2.2 GPT Method

The classical first order approximation for the perturbed eigenvalue $\Delta\lambda$ based on the integral transport equation is in the form [5],

$$\Delta\lambda = \frac{\langle \Phi^{\dagger g}, (\Delta\mathcal{L}^g - \lambda\Delta\mathcal{F}^g)\Phi^g\rangle}{\langle \Phi^{\dagger g}, \mathcal{F}^g\Phi^g\rangle} \tag{6}$$

The main difference between the eigenvalue estimate in (6) and that based on the integro-differential transport equations [4] is that the flux importance functions are applied to eliminate the first order forward flux variations in (6) while the adjoint functions are used for the integro-differential equations. The perturbed system operators $(\Delta\mathcal{L}^g - \lambda\Delta\mathcal{F}^g)\Phi^g$ depend on the first order variations in the cross-sections and the inverse transport operators (i.e. $\Delta(\mathcal{T}^{g,-1})$) take into account the changes of neutron transfer characteristics between the discretized regions via the tracking lines. On the other hand the perturbed system operators based on the integro-differential transport equations depend only on the first order variations in cross-sections [1]. This is because the integro-differential transport equations represent the infinitesimal systems in differential volume [5]. For the GPT estimation of reaction-rate ratio $R_1 = \langle\Sigma_\kappa^g, \Phi^g\rangle / \langle\Sigma_\zeta^g, \Phi^g\rangle$, the first order approximation for ΔR_1 based on (1) can then be obtained,

$$\Delta R_1 = \frac{(\langle\Delta\Sigma_\kappa^g, \Phi^g\rangle - R_1\langle\Delta\Sigma_\zeta^g, \Phi^g\rangle)}{\langle\Sigma_\zeta^g, \Phi^g\rangle} + \langle T_\beta^{\dagger g}, -(\Delta\mathcal{L}^g - \lambda\Delta\mathcal{F}^g)\Phi^g\rangle \tag{7}$$

with the use of the generalized flux importance transport equations to eliminate the first order flux variations,

$$(\mathcal{L}^{\dagger g} - \lambda\mathcal{F}^{\dagger g})T^{\dagger g} = S^{\dagger g} \tag{8}$$

where $S^{\dagger g} = (\Sigma_\kappa^g - R_1\Sigma_\zeta^g)\langle\Sigma_\zeta^g, \Phi^g\rangle^{-1}$, and $T^{\dagger g}(\vec{r},\vec{\Omega}_l)$ is the generalized angular flux importance function.

Similar to the mathematical relationship between flux importance and source importance functions (4), the relationship between the generalized flux importance and the generalized adjoint function is

$$T^{*g}(\vec{r},\vec{\Omega}_l) = \mathcal{T}^{*g,-1}T^{\dagger g}(\vec{r},\vec{\Omega}_l) \tag{9}$$

where T^{*g} is the generalized adjoint angular function. Operating on both sides of (8) with $\mathcal{T}^{*g,-1}$ and using (9), we can arrange the resulting equations into the integro-differential form which can then be solved using the MOCC method. The general solutions of the integro-differential generalized adjoint transport equations are of the form,

$$T^{*g}_{\beta} = T^{*g} + \beta\Phi^{*g} \tag{10}$$

We select $\beta = -\langle T^{*g}, B^g\Phi^g\rangle / \langle \Phi^{*g}, B^g\Phi^g\rangle$ in order to satisfy $(\langle T^{*g}_{\beta}, B^g\Phi^g\rangle = 0)$. We can indirectly compute the generalized flux importances $t^{\dagger g}_{\beta}(\vec{r})$ from the generalized adjoints $t^{*g}_{\beta}(\vec{r})$ by applying (9) and (10) to (8), and integrate the resulting equations over angular domain using the angular quadrature technique,

$$t^{\dagger g}_{\beta}(\vec{r}) = \sum_{g'=1}^{G} [\Sigma_s^{g' \leftarrow g}(\vec{r}) + \chi^{g'}(\vec{r})\nu\Sigma_f^g(\vec{r})] t^{*g'}_{\beta}(\vec{r}) + S^{\dagger g} \tag{11}$$

One can see that the generalized scalar flux importance is equivalent to the total generalized adjoint source (adjoint fission, adjoint scattering and external generalized adjoint sources) of the integro-differential generalized adjoint transport problem with isotropic scattering. The sensitivity coefficient of R_1 with respect to N_q is,

$$S^q_{R_1} = \frac{N_q}{R_1 \Delta N_q} \left[\frac{(\langle \Delta\Sigma^g_{\kappa}, \Phi^g\rangle - R_1 \langle \Delta\Sigma^g_{\zeta}, \Phi^g\rangle)}{\langle \Sigma^g_{\zeta}, \Phi^g\rangle} + \langle T^{\dagger g}_{\beta}, -(\Delta\mathcal{L}^g - \lambda\Delta\mathcal{F}^g)\Phi^g\rangle \right] \tag{12}$$

2.3 Mathematical Relationship Between Generalized Adjoint Functions by MOCC and CP Methods

The adjoint scalar function obtained by the MOCC method is shown equivalent to the scalar source importance function by the CP method [7]. For the generalized adjoint transport problem, the characteristics formulations are similar to the formulations for the adjoint problem except the external generalized adjoint sources S^{*g} are included. The MOCC generalized adjoint function can be written in the form of CP equation as follows,

$$[t^*] = [P_{VV}] \left(\left[[\Sigma_s]^T + \lambda \left[[\chi] \cdot [\nu\Sigma_f] \right]^T \right] [t^*] + [S^*] \right) \tag{13}$$

where $[P_{VV}]$ is the collision probability matrix and $[S^*] = [S^{\dagger}]$. The MOCC-based generalized adjoint CP equation (13) is the same as the generalized source importance CP equation [4]. Thus the MOCC generalized adjoint function is equivalent to the CP generalized source importance. From the relationship between $T^{*g}(\vec{r}, \vec{\Omega}_l)$ and $T^{\dagger g}(\vec{r}, \vec{\Omega}_l)$ in (9), the integrations of (9) over the angular domain and region j shall result in,

$$[t^*] = [P_{VV}][t^{\dagger}] \tag{14}$$

where the generalized flux importances are assumed isotropic and constant within each region. Applying (14) to (13), the generalized flux importance equation are obtained,

$$[t^{\dagger}] = \left[[\Sigma_s]^T + \lambda \left[[\chi] \cdot [\nu\Sigma_f] \right]^T \right] [P_{VV}][t^{\dagger}] + [S^{\dagger}] \tag{15}$$

One can again see that the generalized flux importances are the total generalized adjoint sources of the integro-differential generalized adjoint transport problem.

2.4 Evaluation of Perturbed Transport Functionals

The first order approximation for $\left\langle \Phi^{\dagger g}, (\Delta\mathcal{L}^g - \lambda\Delta\mathcal{F}^g)\Phi^g \right\rangle$ is,

$$\left\langle \Phi^{\dagger g}, (\Delta\mathcal{L}^g - \lambda\Delta\mathcal{F}^g)\Phi^g \right\rangle \approx -\left\langle [\phi^\dagger], ([P_{VV}(\Sigma+\Delta\Sigma)] - [P_{VV}(\Sigma)])\,[Q] \right\rangle - \left\langle [\phi^*], ([\Delta\Sigma_s][\phi] + \lambda[\chi][\Delta(\nu\Sigma_f)][\phi]) \right\rangle \tag{16}$$

where

$$\Delta\mathcal{L}^g\Phi^g \approx -\Delta(\mathcal{T}^{g,-1})\sum_{g'=1}^{G}\Sigma_s^{g\leftarrow g'}\phi^{g'} - \mathcal{T}^{g,-1}\sum_{g'=1}^{G}\Delta\Sigma_s^{g\leftarrow g'}\phi^{g'} \tag{17}$$

$$\Delta\mathcal{F}^g\Phi^g \approx \Delta(\mathcal{T}^{g,-1})\sum_{g'=1}^{G}\chi^g\nu\Sigma_f^{g'}\phi^{g'} + \mathcal{T}^{g,-1}\sum_{g'=1}^{G}\chi^g\Delta(\nu\Sigma_f^{g'})\phi^{g'} \tag{18}$$

$$\Delta(\mathcal{T}^{g,-1}) = \mathcal{T}_p^{g,-1} - \mathcal{T}^{g,-1} \tag{19}$$

$$[Q] = [[\Sigma_s] + \lambda[\chi][\nu\Sigma_f]]\,[\phi] \tag{20}$$

The flux importance and adjoint functions are assumed isotropic and constant within regions. The term $(([P_{VV}(\Sigma+\Delta\Sigma)] - [P_{VV}(\Sigma)])[Q])$ will be approximated using the isotropic approximation (IA) and linearization approximation (LA) methods. For the IA method, we add the term $\Delta\Sigma\Phi^g$ on both sides of integro-differential forward transport equation [5],

$$\vec{\Omega}_l\cdot\vec{\nabla}\Phi^g + \Sigma^g\Phi^g + \Delta\Sigma^g\Phi^g = \sum_{g'=1}^{G}[\Sigma_s^{g\leftarrow g'} + \lambda\chi^g\nu\Sigma_f^{g'}]\phi^{g'} + \Delta\Sigma^g\Phi^g \tag{21}$$

We know that the flux solutions of the transport equation (21) is the same as the solutions of the unmodified transport equation. By assuming that $\Delta\Sigma\Phi^g$ on RHS of the modified transport equation (21) is isotropic, we can write the solutions of (21) and the unmodified transport equations in the CP form [6],

$$\phi_j^g = \sum_{i=1}^{N_J} p_{j\to i}^g\cdot q_i^g = \sum_{i=1}^{N_J}\tilde{p}_{j\to i}^g\cdot(q_i^g + \Delta\Sigma\phi_i^g) \tag{22}$$

where $\tilde{p}_{j\to i}^g \equiv p_{j\to i}^g(\Sigma+\Delta\Sigma)$. The equation (22) in the matrix form is,

$$([P_{VV}(\Sigma+\Delta\Sigma)] - [P_{VV}(\Sigma)])[Q] \approx -[P_{VV}(\Sigma)][\Delta\Sigma][\phi] \tag{23}$$

Applying (23) into (16), one finds that the approximated perturbed integral transport functional (16) using IA method is equivalent to the approximated perturbed integro-differential transport functional $\left\langle \Phi^{*g}, (\Delta A^g - \lambda\Delta B^g)\Phi^g \right\rangle$. For the LA method, when $\Delta\Sigma\Phi^g$ on RHS of (21) is assumed anisotropic, (22) becomes,

$$\sum_{i=1}^{N_J}(\tilde{p}_{j\to i}^g - p_{j\to i}^g)q_i^g = -\sum_{i=1}^{N_J}\left[\frac{1}{V_j}\int_{V_j}d^3r\int_{V_i}d^3r'\frac{e^{-\tau^g((\Sigma+\Delta\Sigma),\vec{r'},\vec{r})}}{\|\vec{r'}-\vec{r}\|^2}\delta_i(\vec{r'})\Delta\Sigma^g\Phi^g(\vec{r'},\vec{\Omega}_l)\right] = -\sum_{i=1}^{N_J}\left[\Xi_{j\to i}^g\Delta\Sigma_i^g\right] \approx -\sum_{m=1}^{N_{MIX}}\left[\hat{\Xi}_{m,j}^g\Delta\Sigma_m^g\right] \tag{24}$$

where $\Delta\Sigma^g(\vec{r'}) = \Delta\Sigma_i^g$ for $\vec{r'}$ in a region i and N_{MIX} is the number of mixtures. Because the direct computation for the $\Xi_{j\to i}^g$ may be complicated, we pre-evaluate the linearization factor $\hat{\Xi}_{m,j}^g$ instead as follows,

$$\hat{\Xi}_{m,j}^g = -\frac{1}{d\Sigma_m^g}\sum_{i=1}^{N_J}(p_{j\to i}^g(\Sigma^g + d\Sigma_m^g) - p_{j\to i}^g(\Sigma^g))q_i^g \tag{25}$$

Here we use $d\Sigma_m^g = 10^{-4}\Sigma_m^g$. To determine these linearization factors, we use the MOCC algorithm to perform total $G \times N_{MIX}$ scanning over tracking lines (to evaluate $[P_{VV}(\Sigma + d\Sigma)][Q]$ for each group and mixture). The same procedures are applied to evaluate the perturbed transport functional $\left\langle T^{\dagger g}, (\Delta\mathcal{L}^g - \lambda\Delta\mathcal{F}^g)\Phi^g \right\rangle$ where $T^{*g}(\vec{r})$ and $T^{\dagger g}(\vec{r})$ are assumed isotropic and constant within each region.

2.5 Biasing and Decontamination Schemes

Because $S^{*g}(\vec{r})$ may be negative in some regions, these negative sources will result in the negative generalized adjoint solutions leading to instabilities in the multigroup adjoint rebalance scheme [7]. This is because in the adjoint balance equations the adjoint functions and spatial-integrated adjoint balance sources must be positive. In order to avoid this problem, the generalized adjoints are made positive using a biasing scheme defined as follows,

$$[t_{bias}^*] = [t^*] + \alpha_{bias}[\phi^*] \tag{26}$$

$$\alpha_{bias} = \max_{i,g}\left(\frac{f_b q_i^{*g} - S_i^{*g}}{q_i^{*g}}\right) \tag{27}$$

$$q_i^{*g} = \sum_{g'=1}^{G}(\Sigma_{s,i}^{g'\leftarrow g} + \frac{1}{k_{eff}}\chi_i^{g'}\nu\Sigma_{f,i}^g)\phi_i^{*g'} \tag{28}$$

where α_{bias} is the global biasing factor with f_b a positive source biasing factor. The adjoint functions are used because they are the homogeneous solutions of the generalized adjoint transport equations. After the multigroup iteration is completed, the orthogonal generalized adjoints $t_\perp^{*g}$ are extracted using a decontamination scheme,

$$[t_\perp^*] = [t_{bias}^*] - \alpha_{decon}[\phi^*] \tag{29}$$

where $\alpha_{decon} = ([t_{bias}^*]^T[\phi^*]/[\phi^*]^T[\phi^*])$ in order to satisfy $[t_\perp^*]^T[\phi^*] = 0$. The decontamination scheme is to remove the homogeneous solutions from the total inhomogeneous solutions. Here we propose four types of biasing/decontamination schemes as follows,

1. A 2 step biasing/decontamination scheme: the power iterations are divided into 2 steps. In the 1st step the generalized adjoints are biased once in the first power loop. These biased generalized adjoints are then used in the power and multigroup iterations. The decontamination scheme is turned off in this step. When the biased generalized adjoints converge, the orthogonal generalized adjoints are extracted. In the 2nd step the unbiased orthogonal generalized adjoints are directly used in the iterations. The rebalance and decontamination schemes are turned off. When the 2nd step iterations converge, the last decontamination scheme extracts the orthogonal generalized adjoints and evaluate t_β^{*g}.

2. A biasing/decontamination scheme is performed in each power iteration loop. The generalized adjoints are biased at the beginning of each power loop. When multigroup iterations converge, the orthogonal generalized adjoints are extracted and used in the next power loop. The convergence of multigroup iterations relies on the biased generalized adjoints while convergence of power iterations relies on the orthogonal generalized adjoints.

3. A 2 step biasing/decontamination scheme similar to 1 except that, in the second step, the decontamination is performed in each power iteration and the rebalance is turned off.

4. A 2 step biasing/decontamination scheme similar to 1 except that, in the first step, the biasing and decontamination are performed in each power iteration (as described in Scheme 2).

3. Numerical Results

The problem we will study is a 2D 1/8 symmetry 17×17 PWR lattice with reflective boundaries discretized into 135 regions according to the exact geometries with 69 group cross-sections taken from the WIMS-AECL library [8]. The azimuthal angular integration is performed using a 19 point Gaussian quadrature [6] while the polar integration consists in a 10 point Gauss-Legendre quadrature. The tracking density selected is 20 lines/cm. Convergence criterion is 10^{-6}. We compare our results with the generalized source importances obtained by the CP method [9] using the same cyclic tracking procedure. The testing computer is a PENTIUM 4-3.2GHz CPU. The relative error between the MOCC generalized adjoints and CP generalized source importances is defined as $\epsilon_f = \|[t^*_{MOC}] - [t^*_{CP}]\|/\|[t^*_{CP}]\|$. The functionals of reaction-rate ratio used in the numerical simulations are the one-group homogenized cross sections $\Sigma^H_x = \left\langle \Sigma^g_x, \phi^g \right\rangle / \left\langle \phi^g \right\rangle$, where Σ^H_x can be the homogenized total (Σ^H), transport correction (Σ^H_{tc}), fission neutron production ($\nu\Sigma^H_f$), fission (Σ^H_f), absorption (Σ^H_a) or scattering (Σ^H_s) cross sections.

3.1 Generalized Adjoint Solutions

We select $f_b = 10^4$, for the biasing/decontamination scheme 1, 3 and 4, and $f_b = 500$ for scheme 2 because these values give the lowest number of one-group solutions (TNOS) with minimum relative error ϵ_f. The results for computing time using the MOCC and CP methods are shown in Table 1. The comparisons of the generalized adjoints using various biasing and decontamination schemes are shown in Table 2. The total computing time by the MOCC method is longer than the total time by the CP method. This is because the pre-evaluation of the CP matrices is more efficient than using MOCC method (to perform the spatial and angular integrations over adjoint sources) to compute the 6 types of generalized adjoints in our study. Amongst all the schemes, scheme 2 results in the lowest computing time. This is because the decontamination is performed at each power iteration to extract the orthogonal component thereby removing the undesired adjoint components. In the remaining schemes, the generalized adjoints are freely iterated (no multigroup rebalance) with the decontaminated components once the contaminated flux converges and a large value for the TNOS is then obtained .

3.2 GPT Estimates

The GPT estimates of variations in cell averaged cross sections, due to perturbation in the temperature of fuel, clad and coolant by 1, 10, 25 and 50%, are shown in Table 3 and 4.

Table 1: CPU time in minutes used in the MOCC and CP methods.

Types of GPT Functional	CP Method	MOCC			
	Scheme 1	Scheme 1	Scheme 2	Scheme 3	Scheme 4
Σ^H	0.28	336.0	352.5	336.0	354.1
Σ^H_{tc}	0.32	282.2	352.7	282.2	283.1
$\nu\Sigma^H_f$	0.35	262.6	204.4	262.6	262.0
Σ^H_f	0.36	374.6	186.5	368.6	298.0
Σ^H_a	0.63	382.5	189.3	382.5	336.7
Σ^H_s	0.34	329.9	350.7	329.9	332.3
CP Matrix Computing Time	1359.0	—	—	—	—
Total Computing Time	1361.3	1967.9	1636.1	1961.9	1866.2

As one can see, the absolute relative errors ($\epsilon_r = |(R_p - R_{GPT})/R_u|$) of GPT estimates are within 2.3×10^{-4} for perturbation in temperature as large as 10%. The GPT estimates using IA method are comparable to the results using LA method. This is because in the PWR problem the angular fluxes are relatively isotropic due to the fact that the fuel rods are uniformly positioned over the spatial domain. The higher order angular terms in ($\Delta\Sigma\Phi$) then produce low contribution to the GPT estimates. The sensitivity coefficients of homogenized

Table 2: The total numbers of one-group solutions (TNOS).

Types of GPT Functional	TNOS			
	Scheme 1 ($f_b = 10^4$)	Scheme 2 ($f_b = 500$)	Scheme 3 ($f_b = 10^4$)	Scheme 4 ($f_b = 10^4$)
Σ^H	3748	3932	3748	3950
Σ^H_{tc}	3148	3934	3148	3158
$\nu\Sigma^H_f$	2929	2280	2929	2922
Σ^H_f	4179	2080	4112	3324
Σ^H_a	4267	2112	4267	3756
Σ^H_s	3680	3912	3680	3707
Maximum ϵ_f	1.5E-06	3.8E-06	1.50E-06	1.40E-06

cross-sections to various lattice parameters are shown in Table 5. The reference sensitivities ($S^q_{R_1,ref} = (N_q/(\Delta N_q R_1))(R_{1,+ptb\%} - R_{1,-ptb\%})$) are determined using direct calculation with $\pm0.1\%$ perturbation in boron density ($N_{\mathrm{B_{nat}}}$), coolant density ($N_{\mathrm{H_2O}}$) and fuel temperature (T_f), and $\pm0.01\%$ perturbation in U^{235} enrichment. The sensitivities of homogenized total cross sections Σ^H using GPT method are evaluated with +1.0% perturbation in $N_{\mathrm{B_{nat}}}$, $N_{\mathrm{H_2O}}$ and T_f, and +0.10% perturbation in U^{235} enrichment. Generally the sensitivities evaluated using the LA method result in lower relative error ($\epsilon_s = |(S_{ref} - S_{GPT})/S_{ref}|$) than those using the IA method. The main exception to this behavior is for S^{T_f} where the LA method gives large error due to the nonlinear effect in Doppler broadening. However if we re-evaluate the S^{T_f} using $\pm0.1\%$ perturbation, we obtain a reference value for S^{T_f} of -3.876865$\times10^{-3}$ and

$\epsilon_s = 1.2 \times 10^{-6}$ now becomes much smaller than for the IA method. In the LA method, the linearization factors are evaluated using small perturbations ($d\Sigma_m^g = 10^{-4} \times \Sigma_m^g$), therefore the sensitivity results give lower error than the results using the IA method in the cases studied here.

Table 3: MOCC GPT estimates for perturbation Δ in fuel, clad and coolant temperatures using IA method.

	k_{eff}		Σ^H		$\nu\Sigma_f^H$		Σ_s^H	
Δ	GPT Estimate	ϵ_r	GPT Estimate	ϵ_r	GPT Estimate	ϵ_r	GPT Estimate	ϵ_r
0%	1.262363	-	6.61101E-1	-	2.78388E-2	-	6.39048E-1	-
1%	1.261930	7.5E-7	6.61132E-1	8.1E-7	2.78185E-2	4.7E-7	6.39088E-1	8.4E-7
10%	1.258089	1.9E-5	6.61350E-1	5.4E-5	2.76328E-2	2.1E-4	6.39385E-1	6.2E-5
25%	1.251947	1.6E-4	6.61949E-1	3.6E-4	2.73201E-2	1.3E-3	6.40124E-1	4.1E-4
50%	1.242350	6.7E-4	6.63065E-1	1.3E-3	2.68056E-2	4.9E-3	6.41475E-1	1.5E-3

Table 4: MOCC GPT estimates for perturbation Δ in fuel, clad and coolant temperatures using LA method.

	k_{eff}		Σ^H		$\nu\Sigma_f^H$		Σ_s^H	
Δ	GPT Estimate	ϵ_r	GPT Estimate	ϵ_r	GPT Estimate	ϵ_r	GPT Estimate	ϵ_r
1%	1.261929	0.0	6.61133E-1	8.1E-7	2.78185E-2	2.2E-6	6.39089E-1	8.4E-7
10%	1.258074	3.1E-5	6.61361E-1	7.0E-5	2.76322E-2	2.3E-4	6.39396E-1	7.8E-5
25%	1.251928	1.8E-4	6.61975E-1	4.0E-4	2.73190E-2	1.4E-3	6.40150E-1	4.5E-4
50%	1.242330	6.8E-4	6.63117E-1	1.4E-3	2.68038E-2	5.0E-3	6.41528E-1	1.5E-3

4. Conclusions

We study the GPT estimations of eigenvalue and reaction-rate ratio based on integral transport equations. The flux importance and generalized flux importance transport equations are defined in order to eliminate the first order flux variations. The biasing/decontamination schemes in each power loop results in the lowest computing time among the 4 schemes. This is because the orthogonal generalized adjoints are used in each power iteration since the decontamination scheme removes the undesired components proportional to the adjoint functions. To evaluate the perturbed integral transport functionals in the GPT estimates, the isotropic approximation (IA) and linearization approximation (LA) methods are applied to approximate the perturbed integral transport functional. Using the IA method, the GPT estimate based on the integral transport equations is equivalent to the GPT estimate based on the integro-differential transport equation. The GPT estimate using IA method is comparable to the GPT estimate using the LA

Table 5: Sensitivities of one-group homogenized total cross-sections to lattice parameters.

Sensitivity for Σ^H	Direct Calculation	IA method		LA method	
		Sensitivity	ϵ_s	Sensitivity	ϵ_s
S^{T_f}	-3.87687E-3	-3.86779E-3	2.3E-3	-3.84071E-3	9.3E-3
$S^{N_{\mathrm{B_{nat}}}}$	-1.00529E-2	-1.00348E-2	1.8E-3	-1.00348E-2	1.8E-3
$S^{N_{\mathrm{H_2O}}}$	8.42317E-1	8.40955E-1	1.6E-3	8.42109E-1	2.5E-4
$S^{\mathrm{U}^{235}}$	-3.55951	-3.578886	5.4E-3	-3.559142	1.0E-4

method because in the PWR lattice the angular fluxes are relatively isotropic due to the uniform fuel-rod distribution in the assembly. Therefore the high order angular terms in $(\Delta\Sigma^g\Phi^g)$ produce low effect in the GPT estimation using the LA method.

Acknowledgments

This work is supported in part by the Natural Science and Engineering Research Council of Canada.

References

1) M. Williams, "CRC handbook of nuclear reactors calculations: volume 3", Y. Ronen, Ed., CRC Press, Boca Raton, Fl. (1986).

2) W.J. Stacey, "Variational estimates of reactivity worths and reaction-rate ratios in critical nuclear reactors," Nuc. Sci. Eng., **48**, 444 (1972).

3) K.F. Laurin-Kovitz, E. Lewis, "Variational nodal transport perturbation theory," Nuc. Sci. Eng., **123**, 369 (1996).

4) T. Courau, G. Marleau, "Perturbation theory for lattice cell calculations," Nuc. Sci. Eng., **143**, 19 (2003).

5) G. Bell, S. Glasstone, "Nuclear reactor theory," R. E. Krieger Pub. Co., Huntington, N.Y. (1979).

6) G. Marleau, "DRAGON theory manual: part I- Collision probability calculation," Report IGE-236, École Polytechnique de Montréal, Institut de génie nucléaire (2001).

7) M. Assawaroongruengchot, G. Marleau, "Multigroup adjoint transport solution using method of cyclic characteristics," accepted for publication in Nuc. Sci. Eng. (2006).

8) G. Marleau, A. Hébert, R. Roy, "A user guide for DRAGON," Report IGE-174 Rev. 5, École Polytechnique de Montréal, Institut de génie nucléaire (2000).

9) T. Courau, G. Marleau, "Adjoint and generalized adjoint flux calculations using the collision probability technique," Nuc. Sci. Eng., **141**, 46 (2002).