PHYSOR-2006, ANS Topical Meeting on Reactor Physics
Organized and hosted by the Canadian Nuclear Society. Vancouver, BC, Canada. 2006 September 10-14

Application of Shannon filter to dynamical behavior of BWR neutronic signals

V. Garcia i Llorens*[1] and G. Verdú[1]
[1] *Institut de Seguretat Industrial Radiofísica i Mediambiental, Camí de Vera s/n*
Tech. Univ. of València, Spain

Abstract

The analysis of neutron signals from BWR will be performed in this paper using Shannon filtering methodology as a dynamical tool. Signal analysis is nowadays, and has been since the beginning of this technology, the way used to keep under control and inside safety parameters BWR in operational conditions. Some problems come when those techniques require large series of sampled points, so large computational time to give back an answer. This is the weakest point since we need to follow the core behavior in as much continuous way as possible.

KEYWORDS: Shannon method, neutron signal analysis, digital filter techniques

1. Introduction

Stability of BWR is and has been since the beginning of the use of this technology one of the most important topics of study. Joined to signal analysis, both in time and frequency domain, other techniques as exclusion regions has been used to ensure the operational safety.

Using the assumption that the signal is stationary, through an autoregressive model, the linear stability parameters have been obtained in other works [10].

As published by some other authors [9], the BWR are non-linear systems which when working in steady state point can be threated as a linear systems, and linear stability parameters as the DR can be used with enough reliability to characterize their stability regime [11], but only in such situations. In a previous wok [6] we also proposed the use of adaptative methods to calculate those stability parameters and their time evolution. If we want to use any of this techniques on-line they should be very fast and robust.

The DR is still the most used parameter to characterize the stability behavior of a BWR, instead some authors claimed to rethink about it. A recent review of signal analysis techniques and DR calculation was performed in Forsmark 1 and 2 Benchmark [12]. The large variability of the DR determination was one of the conclusions of the Benchmark.

Now, based on Shannon functions [2], and with the knowledge that from a signal analysis methodology we expect fastness, robustness, adaptativity and reliability, we will introduce an elegant method to clean and filter neutron signals.

*Corresponding author, E-mail: vigarllo@isirym.upv.es

Shannon sampling theorem is a very well known technique of signal processing. In this work we will investigate for which other types of signals this method holds, since it is based on bandlimited signals. But bandlimited signals should be unlimited on time duration. By the other hand, the ”sinc” function used to reconstruct a signal from its samples has infinite support [4]. And this is what we are looking for: reconstruction of signals of compact support and finite duration. Here the extension of Shannon Sampling Theorem made by Papoulis [8] fits perfectly to develope this new Generalized Discrete Shannon Functions. This type of functions is the natural bridge between the filter techniques family and the Fourier Transform method, and are the basis for a new methodology to signal conditioning, which clean and filter the signals in a selected band of frequencies. As far as our knowledge arrives Shannon functions have been applied in wavelet techniques, but never on stability analysis on nuclear field.

The paper is organized as follows, the introduction is presented in section 1. The definitions and developing of the generalized discrete Shannon functions are reserved to section 2. In section 3 we apply the methodology to a couple of real signals. Finally, in section 4 the main conclusions of the work are presented.

2. Generalized discrete Shannon functions

Consider a signal *f(t)* sampled each ΔT seconds, $f(t_j)$, $t_j = j \cdot \Delta T$, $j = 0, ..., N$, and N is the number of sampled points. In Eq. 1 we present the Papoulis modification of the Shannon Sampling Theorem [8].

$$\widetilde{f}(t) = \sum_j f(t_j) \frac{\sin\left[\omega_0 \left(t - j\Delta T\right)\right]}{\omega_2(t - j\Delta T)} \tag{1}$$

where w_2 is the Nyquist frequency, $\omega_2 = \frac{\pi}{\Delta T}(rad)$, $\widetilde{f}(t)$ is the filtered signal in the $[0, w_0]$ band of frequencies, and the frequencies keep the following relation $0 \leq \omega_0 \leq \omega_2$.

Applying the Discrete Fourier Transform, we can arrive to Eq. 2

$$\widetilde{F}(\omega) = \sum_j f(t_j) e^{-i\omega j \Delta T} \Delta T \chi[-\omega_{0,}\omega_0], \tag{2}$$

where $\sum\limits_j f(t_j) e^{-i\omega j\Delta T} \Delta T$ is exactly the Discrete Fourier Transform (DFT) of the sampled signal, and the $\chi[-\omega_{0,}\omega_0]$ is the characteristic function with value 1 inside the interval, and 0 outside.

Discretizing Eq. 1 we arrive to a sampling equation which suits for finite duration signals,

$$\widetilde{f}(t_i) = \sum_j f(t_j) \frac{\sin \omega_0 (i - j)\Delta T}{\pi(i - j)\Delta T} \Delta T \tag{3}$$

where, $t_i = i\Delta T, t_j = j\Delta T$ are the sampled points, $i = 0, ..., N, j = 0, ..., N.$

From Eq. 3 we can select a sampling functions, the so called generalized scaling discrete Shannon Functions, which we can choose as a filter function.

$$\varphi^l_{j,i} = \frac{\sin \omega_l (i - j)\Delta T}{\pi(i - j)\Delta T} \frac{1}{\sqrt{2\Omega_l}} \tag{4}$$

where $\omega_l = 2^l \omega_0, \omega_l = 2\pi\Omega_l, l \in Z$, and $\sqrt{2\Omega_l}$, is the normalization factor.

Functions presented in expression 4 have good properties for signal analysis, but they can be improved after some arrangements as shown in [3].

Reformulating Eq. 1 through the modification of the Shannon sampling theorem made by Papoulis [8] we arrive to

$$\widehat{f}(t)=\sum_{j} f(t_j)\frac{\cos[\frac{3}{2}\omega_0(t-j\Delta T)]\sin\left[\frac{1}{2}\omega_0\left(t-j\Delta T\right)\right]}{\frac{1}{2}\omega_2(t-j\Delta T)} \tag{5}$$

which can be transformed into Eq. 6, using trigonometric relations,

$$\widehat{f}(t)=\sum_{j} f(t_j)\frac{\sin\left[2\omega_0\left(t-j\Delta T\right)\right]-\sin\left[\omega_0\left(t-j\Delta T\right)\right]}{\omega_2(t-j\Delta T)} \tag{6}$$

With the same theory shown before is easy to prove that function $\widehat{f}(t)$ is a function banded on the interval $[-2\omega_0,-\omega_0]\cup[\omega_0,2\omega_0]$ from signal $f(t_j)$.

Also function presented in Eq. 6 can be discretized and improved in order to be able to be used in a narrower bands of frequencies [3]. The generalized discrete Shannon Functions are presented in Eq. 7, which with an appropriate selection of l,k,r parameters and w_0 main frequency act over $B_{l,k,r}=[-(2^l+(r+1)2^{l-k})\omega_{0,}-(2^l+r2^{l-k})\omega_0]\cup\ [(2^l+r2^{l-k})\omega_{0,}(2^l+(r+1)2^{l-k})\omega_0]$ band of frequency.

$$\psi_{j,i}^{l,k,r}=\frac{\cos[2^{l-k}(2^k+\frac{2r+1}{2})\omega_0(i-j)\Delta T]\sin[2^{l-k}\omega_0(i-j)\Delta T]}{\frac{1}{2}\pi(i-j)\Delta T}\frac{1}{\sqrt{2^{1-k}\Omega_l}} \tag{7}$$

With functions of Eq. 7 the sampling equation will have the shape presented in Eq. 8.

$$\widehat{f_{l,k,r}}(t_i)=\sum f(t_j)\frac{\cos[2^{l-k}(2^k+\frac{2r+1}{2})\omega_0(i-j)\Delta T]\sin[2^{l-k}\omega_0(i-j)\Delta T]}{\frac{1}{2}\pi(i-j)\Delta T}\Delta T, i=0,1,..,N \tag{8}$$

Finally, once the functions $\widehat{f_{l,k,r}}(t_i)$, have been calculated using Eq. (8), we are able to obtain directly the pseudospectrum [3] of the filtered signal, applying the Discrete Fourier Transform obtaining Eq. (9).

$$G^{l,k,r}(\omega)=\sum \widehat{c_j^{l,k,r}}e^{-i\omega j}, j=0,1,..N; \widehat{c_j^{l,k,r}}=\widehat{f_{l,k,r}}(t_j)\sqrt{2^{1-k}\Omega_l} \tag{9}$$

3. Numerical results

The application of the Shannon methodology to analyze the evolution of the core has shown itself to be a very powerful tool to this affair. The amount of sampled points needed to describe the state of the core is very small, so with it we can achieve a very satisfactory computational times. Through filtering neutron signals with Shannon methodology we can obtain the decay ratio (DR) of the analyzed series of points and also its spectrum in the frequency domain, with a stressed good behavior over the border effects.

Some analysis of dynamical behavior of the core through decomposition of signals into blocks have been done in this work. The applied methodology has been the same for all signals. We selected a constant length of time series to analyze, and then shift it some points depending on the selected overlapping. From each cleaned time series the pseudospectrum and the Decay Ratio (DR) is calculated in order to follow the signal behavior.

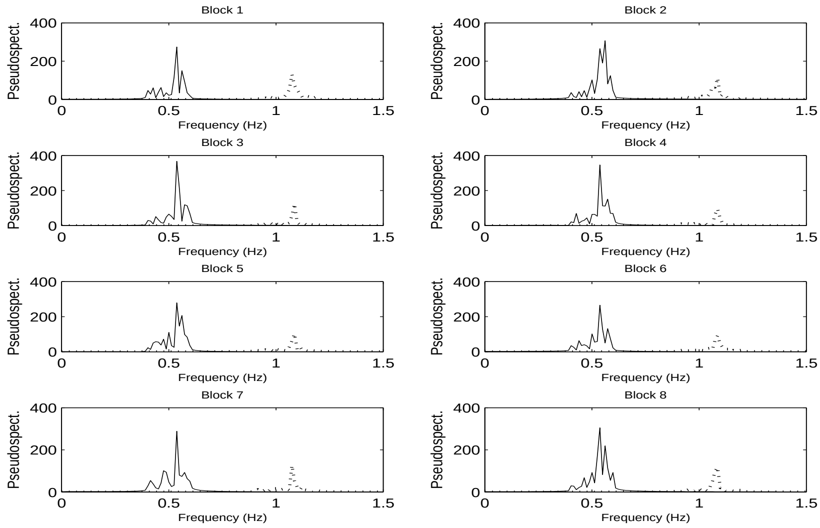

Figure 1: Pseudospectrum evolution of signal lprm.g00 after been cleaned in $[0.4, 0.6]$ Hz and $[0.9, 1.2]$ Hz frequency band with Shannon method

Two different real neutron signals are analyzed: $lprm.g00$ from Ringhals Benchmark and $case\_4\_aprm$ from Forsmark Benchmark.

3.1 $lprm.g00$

Here we present the obtained evolution of signal $lprm.g00$ from Ringhals Benchmark [5]. This signal contains a non-linear behavior of the core, characterized by the presence of a component of the signal in the frequency domain in the surrounding of $1Hz$. In Fig. 1 we present the pseudospectrum of each analyzed series of points of the signal. Each block of points has 1024 points, and they have been overlapped 512 point each time.

As can be observed the contribution of frequency components coming from outside of the band pass nearly does not exist thanks to the application of the filter. Also the results of our Shannon methodology has been compared with other used and well known methodologies, revealing that this new method can lead us to more confident results. The same analyzed series of points applying Butterworth filter of 5^{th} order are presented in Fig. 2.

While through the application of our proposed Shannon filter the contribution of other parts of the frequency is fully removed, using Butterworth filter the obtained result is not so satisfactory. The small components from the undesired part of the frequency domain do not fully disappears, and what is more easy to observe, the band in which we are interested has more problems to cross the filter. As a result can be seen how the same peaks in Fig. 2 have been cut more than in Fig. 1, so modifying their original shape and amplitude in a greater size than when Shannon methodology is applied.

In Table 1 we present some results of DR for each block after applying Shannon methodology. There we can expect an important oscillation in both $[0.4, 0.6]$ and $[0.9, 1.2]$ Hz frequency bands, as can be seen in Fig. 1. In the study of the evolution of this signal we can see how in the spectrum from the Shannon cleaned signal each component is clearly separated one from the other, giving as a result that the DR from both bands of frequency high values of DR are obtained, indicating a strong non-linear signal behavior.

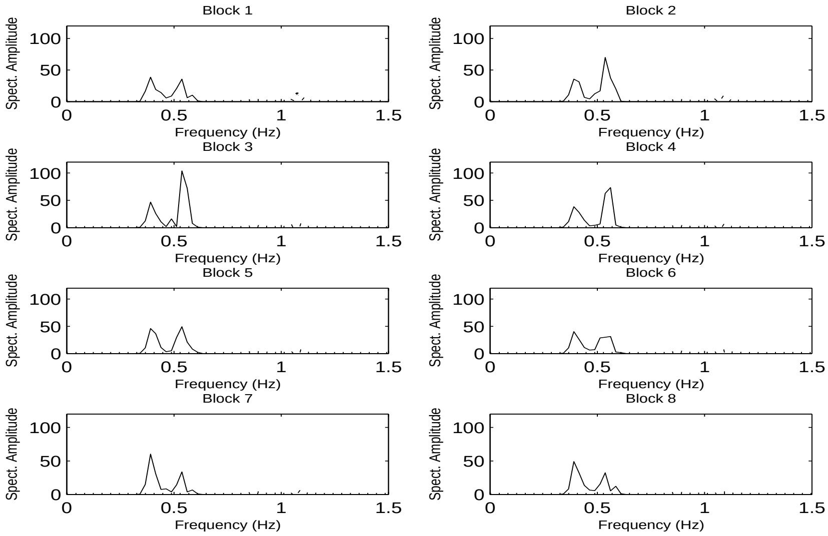

Figure 2: Spectrum evolution of signal lprm.g00 after been cleaned in $[0.4, 0.6]$ Hz and $[0.9, 1.2]$ Hz frequency band with Butterworth method

To carry out the comparison of the analysis of this signal, we present Table 2 where DR calculated for each block of lprm.g00 signal after been cleaned with Butterworth method is shown. This is a non-linear signal, and it has an important component in the surrounding of 1 Hz frequency band as explained before. But it looks also clear how applying Butterworth method part of the behavior of the signal is lost, and despite still the DR of the 1 Hz component is bigger than the one of the 0.5 Hz component, the absolute value of the DR is very much lower than after applying the Shannon method. So we can conclude in this point that the Shannon method has less influence over the component of the signal which we do not want to affect than Butterworth method.

3.2 $case\_4\_aprm$

As a second example we perform an analysis of the signal $case\_4\_aprm$ for the Forsmark Benchmark. This signal was presented in the benchmark as a signal which contained a mixture between a global and a regional oscillation. The presented results by the different participants where not too dispersed in the DR (around 0.8). What we try to do here is an analysis more in detail, following the evolution of the signal and also trying to extract their different components applying the new proposed Shannon methodology.

In this case the signal has been analyzed through four blocks of 2048 points each one, with an overlapping between them of 1024 points. In a first roughly analysis we could detect a couple of peaks of the signal in the frequency domain when analyzing the full signal series, but both in the $0.5Hz$ frequency band. Our goal now is to split both peaks of frequency that conforms the signal, and calculate their respectively DR evolution in order to compare them with the DR evolution of the original signal.

In Fig. 3 and Fig. 4 we present how the splitting has been achieved for each of the four blocks. Through selection of parameter values $l = 3, k = 2$ and $r = 5$ with main frequency $w_0 = 0.025$ we could extract the component which belongs to $[0.45, 0.5]Hz$ frequency band, as shown in Fig. 3. By the other hand through selection of parameter values $l = 3, k = 2$

Table 1: Comparison of DR from $0.5Hz$ component and from $1Hz$ component for lprm.g00 signal filtered with Shannon method

Block	**DR of $0.5Hz$ comp.**	**DR of $1Hz$ comp.**
1	0.897	0.999
2	0.902	1.002
3	0.922	0.984
4	0.920	0.989
5	0.939	0.992
6	0.917	0.976
7	0.919	0.974
8	0.850	0.940

Table 2: Comparison of DR from $0.5Hz$ component and from $1Hz$ component for lprm.g00 signal filtered with Butterworth filter

Block	**DR of $0.5Hz$ comp.**	**DR of $1Hz$ comp.**
1	0.239	0.515
2	0.396	0.504
3	0.398	0.526
4	0.427	0.549
5	0.399	0.521
6	0.610	0.500
7	0.269	0.535
8	0.249	0.494

and $r = 5$ and the same main frequency we could extract the component which belongs to $[0.5, 0.55]Hz$ frequency band, as shown in Fig. 4.

To finalize the results obtained in this analysis we present in table 3 the obtained DR in the original signal, the DR in the $0.45Hz$ frequency component and the DR in the $0.5Hz$ frequency component. There we can see how each component can be analyzed independently and the DR of the original signal can be perturbed very much when they are analyzed all together.

4. Conclusions

In this work the Shannon sampling theory has been presented as a tool for cleaning and analyzing times series of neutronic signals. As an example of its usefulness we have applied the method to a couple of signals which were studied in the past in international benchmarks.

The analysis performed here lead us to think that many other questions that there were not taken into account in those benchmarks are still without answer, and tools like the Shannon sampling methodology presented here can be applied to give more insight to them. With the Shannon methodology we can obtain more information about how many main components in the frequency domain conform the signal, and we can separate them in order to analyze them

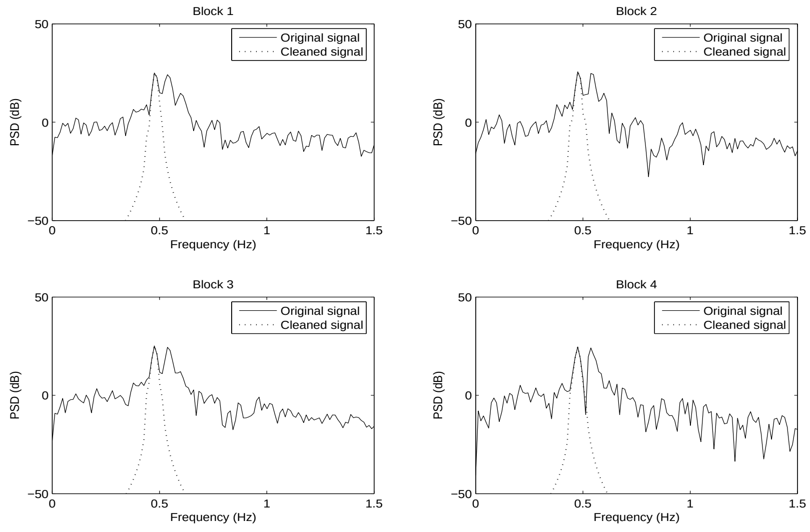

Figure 3: Spectrum evolution of signal $case\_4\_aprm$ after been cleaned in $[0.45, 0.5]$ Hz and PSD of original signal

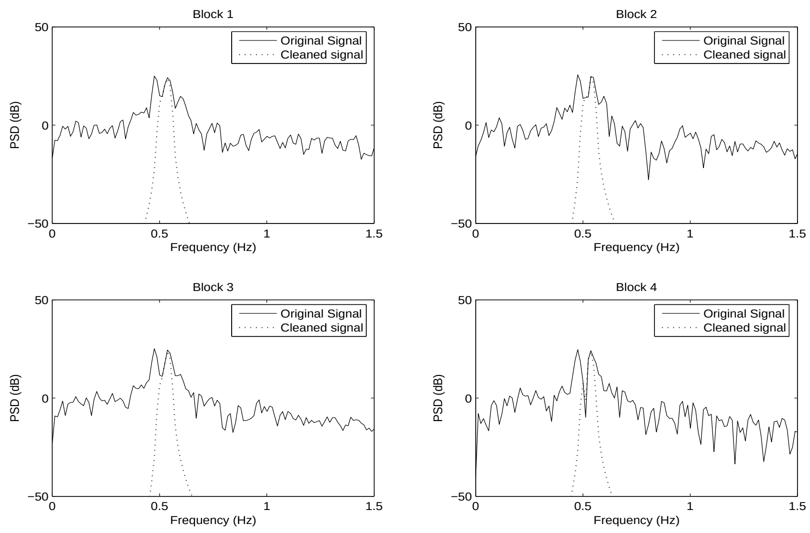

Figure 4: Spectrum evolution of signal $case\_4\_aprm$ after been cleaned in $[0.5, 0.55]$ Hz and PSD of original signal

Table 3: Comparison of DR from original signal, $0.45Hz$ component and from $0.5Hz$ component for $case\_4\_aprm$ signal

Block	**DR of orig. sig.**	**DR of $0.45Hz$ comp.**	**DR of $0.5Hz$ comp.**
1	0.705	1.001	0.982
2	0.695	0.998	0.988
3	0.659	0.999	0.986
4	0.659	0.975	0.958

separately, since the analysis of all them together can lead us to erroneous conclusions.

Finally remark that this tool can be used with a quite short time series, so the evolution of the system can be followed with detail, allowing to detect when a new component appears and how it acts over the whole signal.

References

1) "FORSMARK 1 and 2 BWR STABILITY BENCHMARK - Final Report, *NEA/NSC/DOC(99)9*, (1999)

2) X. Li,L. Qu, G. Wen, C. Li, "Application of wavelet packet analysis for fault detection in electro-mechanical systems based on torsional vibration measurement, *Mechanical System and Signal Processing*, **17**,(6), 1219-1235, (2003).

3) V. G. i Llorens, G. Verdú, "Neutronic Signal Conditioning Using Shannon Functions, *International Topical Meeting on Mathematics and Computation*, Avignon, (2005).

4) Abdul J. Jerri, "The Shannon Sampling Theorem-Its Various Extensions and Applications: A Tutorial Review, *Proceedings of the IEEE*, **65**, No. 11, (1977).

5) T. Lefvert, "RINGHALS1 STABILITY BENCHMARK - Final Report, *NEA/NSC/DOC(96)22*, (1996)

6) Navarro-Esbrí J. Ginestar D., Verdú G., "BWR stability monitoring using adaptive methods, *Annals of Nuclear Energy*, **30**, 755-773, (2003).

7) Ogata K., "Discrete-time control systems, *Prentice Hall. Upper Saddle River. NJ*, (1987).

8) A. Papoulis, "Systems and Transformations with Applications in Optics., *New York: McGraw-Hill*, (1968).

9) Suzudo T., "Application of a nonlinear dynamical descriptor to BWR stability analysis.,*Progress in Nuclear Energy*, **43**, No. 1-4, 217-223. (2003).

10) Van Der Hagen T.M.J.J., Paszit I.,Thomson O., Melkerson B., *Nucl. Technol.* **107**,193. (1994).

11) Van Der Hagen T.M.J.J., Zboray R., de Kruijf W.J.M.,"Questioning the use of the decay ratio in BWR stability monitoring. *Ann. Nucl. Energy*, **27**, 727-732. (2000).

12) Verdú G., Ginestar D., Muñoz-Cobo, J. L., Navarro-Esbrí J., Palomo M.J., Lansaker P., Conde J.M., Recio M., Sartori E., "Forsmark 1 and 2 Stability Benchmark. Time Series Analysis Methods for Oscillations during BWR Operation, *Final Report NEA/NSC/DOC(2001)2.*, (2001).