# Feedback Reactivity Calculations Using Exact Expression from $LAMBDA - REAC$ Code

V. Garcia i Llorens[*1] , C. Lange[2], D. Hennig[3], R. Miró[1] and G. Verdú[1]
[1]*Institut de Seguretat Industrial Radiofísica i Mediambiental, Camí de Vera s/n Tech. Univ. of València, Spain*
[2]*Institut für Energietechnik, TU Dresden*
[3]*Present address: 13055 Berlin, Heckelberger Ring 4*
*dieter@dhennig-berlin.de (PhD thesis supervisor at TU Dresden)*

***Abstract***

In the framework of BWR stability analysis based on reduced order models (ROM) with (spatial) mode kinetic models one of the key task's is the calculation of the mode-coupling feedback reactivities $\rho_{mn}(t)$ (m,n=0,1,... number of the spatial modes). This type of dynamical reactivity is a result of the expansion of the neutron group fluxes (or power) in a series of suitable spatial modes (so called $\lambda$-modes or $\omega$-modes. In the current paper the feedback reactivity calculation of the mode kinetic model of an extended and modified ROM will be shortly discussed.

***KEYWORDS: feedback reactivity, void fraction perturbation, Doppler perturbation, Reduced Order Models***

## 1. Motivation and objectives

For BWR stability analysis, reduced order models should be considered as complementary tools to complex system codes [a] (not alternatives).

Non-linear BWR stability analysis with the aid of complex system codes is currently common practice in many laboratories. A particular demand on the system codes used for stability analysis is the integration of a 3D neutron kinetics model for the core, thereby permitting analysis of regional or higher mode stability behavior (the ones known as out-of-phase oscillations).

Because of the large computational effort required, system codes cannot in practice be employed for a detailed investigation of the complete manifold of solutions of the nonlinear equations describing the stability behavior [1], [8]. In this context, so-called reduced-order models (ROM's) become necessary. Such models contain a minimum number of system equations describing the physical phenomena of interest with adequate sophistication, but the geometrical complexity is reduced (e.g. with a few-channel model). Examples of reduced-order models employing different approximations are given in [2], [1].

*Corresponding author, E-mail: vigarllo@isirym.upv.es

[a] Complex system codes are codes which includes physical models of all nuclear power plant components which are significant for a particular transient analysis

Currently the reduced order model developed previously at PSI (in collaboration with University of Illinois) [2] will be extended and modified in the framework of 2 PhD theses at TU Valencia and TU Dresden. One of the main tasks is the critical discussion of the weak points of the calculation of the mode-coupling feedback reactivities in the previous model. In the current paper we demonstrate the first results achieved by a new feedback reactivity methodology developed by the corresponding author.

## 2. Short discussion of previous methods used for the $\rho_{mn}$ calculation

The feedback reactivities are defined in the mode kinetic equations as the influence that each expanded mode has over itself and also over the other modes. It can be observed in the equations 1 and 2, which describe the neutron flux and the delayed neutron precursors evolution.

$$\frac{d}{dt}n_n(t) = \frac{1}{\Lambda_n}\left[\left(\rho_m^{st} - \beta\right) n_n(t) + \sum_{m=0}^{1} \rho_{m,n}^{F} \cdot n_m(t) - \sum_{m=0}^{1} \rho_{m,n}^{D} \cdot n_m(t)\right] + \lambda \cdot C_n(t) \quad (1)$$

$$\frac{d}{dt}C_n(t) = \frac{1}{\Lambda_n}\left[\beta \cdot n_n(t) + \sum_{m=0}^{1} \rho_{m,n}^{D} \cdot n_m(t)\right] - \lambda \cdot C_n(t) \quad (2)$$

where $n_m(t)$ is the neutron flux mode expansion coefficients, $C_n(t)$ is the delayed neutron precursors mode expansion coefficients, $\Lambda_n$ is the generation neutron time, $\rho_m^{st}$ is the static reactivity of the mode *m*, $\beta$ is the fraction of delayed neutrons, $\lambda$ is the decaying constant of the neutron delayed precursors, $\rho_{m,n}^{D}$ is the delayed reactivity from mode *m* into mode *n* and $\rho_{m,n}^{F}$ is the feedback reactivity from mode *m* into mode *n*. This last reactivity is our parameter of interest which we will call $\rho_{mn}$. We would like to remark that $\rho_{m,n}^{D}$ has been neglected because it has usual values about two orders of magnitude smaller than $\rho_{m,n}^{F}$.

Inside the ROM $\rho_{mn}$ should be defined as $\rho_{mn}(\delta\alpha(t), \delta T_f(t))$, where $\delta\alpha(t)$ and $\delta T_f(t)$ comes from the thermal-hydraulic model and from the fuel element dynamic model respectively. Hence we need approximative expressions $\rho_{mn}(\delta\alpha(t), \delta T_f(t))$. In principle we have for it two opportunities: polynomial approximation, linear [2] or quadratic [1] in void, for instance, or using interpolation tables procedures, which is the methodology that we are presenting in this work.

Polynomial approximation was used in the previous models [1], [2], [4] to estimate the polynomial coefficients. In the framework of this type of approximations we could not correctly reproduce all mode coupling reactivities [2]. In the current paper we demonstrate the new interpolation table method developed at the TU Valencia.

## 3. Novel methodology

The proposed methodology in this work has a different concept than the previous [2], [1], [4]. What we propose is a direct calculation of the feedback reactivity parameter through an interpolation of the exact values from a table previously generated.

The starting point to develop our methodology is to obtain the steady state core configuration from which we will work, and which will be perturbed to generate a transient. The first step is to obtain the steady state set of cross sections and void fraction and fuel temperature averaged over the full core. Further anytime that we reefer to void fraction or fuel temperature perturbation we will be talking about averaged values over the full core.

Then using the RAMONA 5-2.5 [7] code we could introduce controlled perturbations of void fraction and fuel temperature homogeneously in all the core, which will be translated into the cross sections as a quadratic dependence on the void perturbation, and a square root of the fuel temperature perturbation. Then, after a steady state run we obtain a set of perturbed cross-sections equivalent to a perturbation due to the amount of void fraction or fuel temperature deviation respect the steady state value which we introduced. This was made with special input options of RAMONA5 code.

This perturbation procedure has been done for a large amount of combinations of values of void and fuel temperature perturbations. More concretely we made void perturbations $\delta\alpha \in [-0.15, 0.15]$ by steps of $0.01$ and fuel temperature perturbations $\delta T_f \in [-70, 70]$ by steps of $10$ degrees.

As a result of each perturbation made we got a new set of cross sections which represents such a perturbed core. Now using the perturbed cross sections together with the steady state set of cross sections, and with the LAMBDA-REAC code [5], we can solve equation 3.

$$\rho_{mn}(t) = \frac{\langle \phi_m^+, (\delta M - \delta L)\, \phi_n \rangle}{\langle \phi_m^+, M_0 \phi_n \rangle} = \rho_{mn}^V(t) + \rho_{mn}^D(t) \tag{3}$$

where the expressions for each term are

$$\rho_{mn}^V(t) = \frac{\left\langle \phi_m^+, \left( \frac{\partial M}{\partial \alpha} - \frac{\partial L}{\partial \alpha} \right) \delta\alpha \phi_n \right\rangle}{\langle \phi_m^+, M_0 \phi_n \rangle} \tag{4}$$

$$\rho_{mn}^D(t) = \frac{\left\langle \phi_m^+, \left( \frac{\partial M}{\partial T_f} - \frac{\partial L}{\partial T_f} \right) \delta T_f \phi_n \right\rangle}{\langle \phi_m^+, M_0 \phi_n \rangle}, \tag{5}$$

and $\delta M$ and $\delta L$ are the perturbation in the production and in the leakage operators of neutron flux, respectively, since $M_0$ is the unperturbed production operator [3].

As a final result we have developed a transformation, from void fraction and fuel temperature perturbations into values of feedback reactivity parameter $\rho_{mn}(\delta\alpha(t), \delta T_f(t))$.

Then, to obtain the value of the feedback reactivity parameter we should have the perturbation of the void fraction and fuel temperature in the core given by RAMONA. With those values, we will go to our map of feedback reactivities calculated for some exact amounts of those perturbations and it will return back to us the feedback reactivity parameter which corresponds to such void fraction or fuel temperature perturbation.

## 4. Validation and physical discussion

Before begin with our method we should check that the main tool we were going to use (LAMBDA-REAC) was in concordance with the code RAMONA. To do it we had to check that the reactivity calculated by RAMONA (it is only able to calculate the fundamental reactivity) had the same value that the fundamental reactivity calculated by LAMBDA-REAC. The result was satisfactory, as shown in Fig. 1.

The first step was to perform an analysis of the behavior of the reactivity parameter ($\rho_{mn}$) when the core suffers a perturbation in each node of some amount of void fraction and fuel temperature. The obtained result is shown in Fig. 2 for the case of $\rho_{00}$.

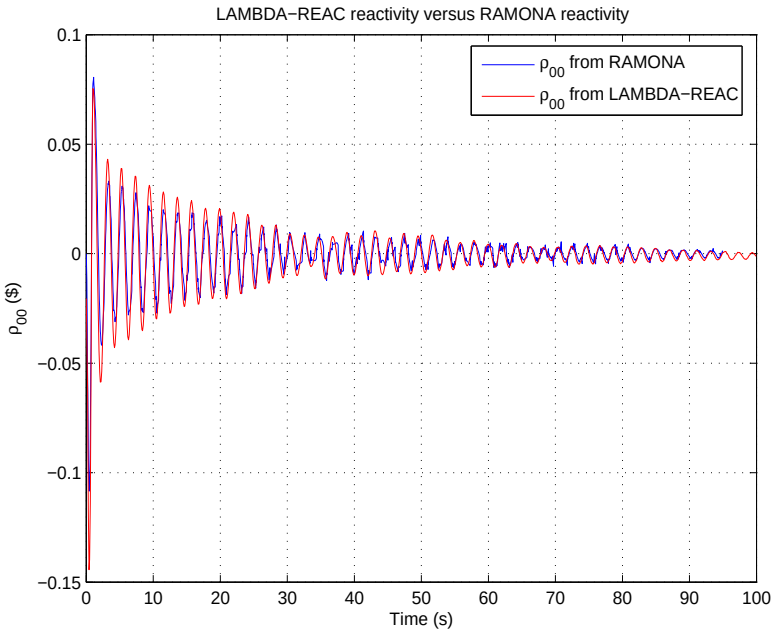


**Figure 1:** Fundamental feedback reactivity comparison between LAMBDA-REAC reactivity and RAMONA reactivity

In Fig. 2 can be observed how under homogeneous perturbations of void fraction and fuel temperature over the whole core the $\rho_{00}$ parameter returns a linear answer. This result is in accordance with results obtained in [2], where was also obtained that the answer of $\rho_{mn}$ parameter has linear dependence respect to the void fraction and also respect to the fuel temperature perturbation.

In order to evaluate the coupled reactivity parameters we should split the core in almost two regions, depending on the configuration of the first harmonic mode where it has the symmetric line. This was obtained also with LAMBDA-REAC code [5], which allows the user to obtain as many Lambda modes as desired. In Fig. 3 we present the flux distribution of the first harmonic, where can be easily seen both regions of the core respect this division. As done in Fig. 2 we will present in Fig. 4 the map over which we interpolate to obtain the $\rho_{10}^2$ value in the second region of the core. As can be seen, the behavior of the parameter is qualitatively similar to the behavior of the fundamental parameter $\rho_{00}$.

With these maps of reactivities we can try to reproduce the evolution of $\rho_{mn}$ parameters in order to evaluate the reliability of the proposed methodology. To obtain the evolution of those parameters we only need to know the evolution of the void fraction and fuel temperature perturbations which were obtained from running the transient of study with RAMONA. In Fig. 5 we present such evolution, which will be used as an input to calculate the reactivity.

Making use of the values presented in Fig. 5 we obtain the $\rho_{00}$ evolution through the transient shown in Fig. 6. Just to remark that the reference values of the figures have been obtained from LAMBDA-REAC code [5]. Also another point to clarity is that is known from the previous investigations that a factor increasing the feedback amplitude was introduced in all ROMs. The necessity for the factor results from the discrepancy between the obtained amplitude when the new methodology is applied, and the reference results. The physical reasons are not yet

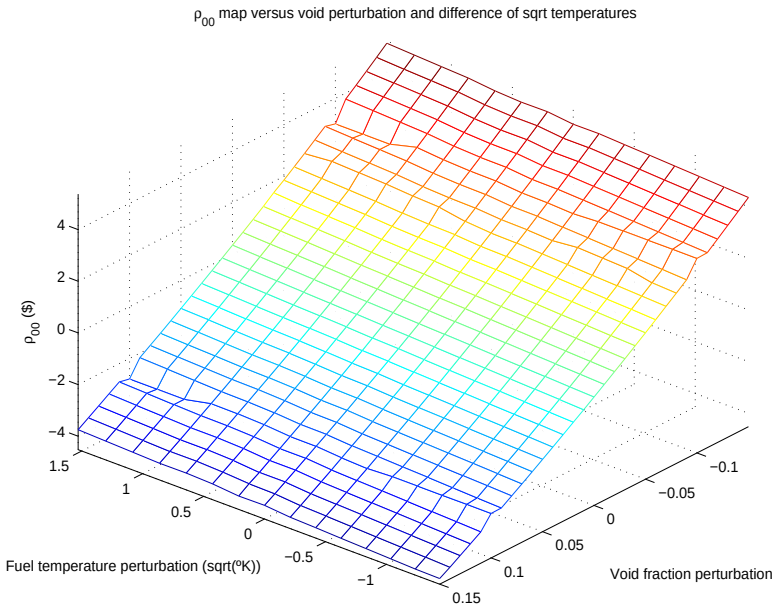


**Figure 2:** Feedback reactivity map parameter versus void fraction and fuel temperature perturbations

clear. But it seems that the physical interpretation of this factor in [8] as a core configuration indicator is not to accept because we used the same core configuration for both methods (reference method with LAMBDA-REAC and table interpolation method). The application of this factor for our case was to multiply the value of the obtained reactivity parameter from table interpolation by $2.5$.

In Fig. 6 can be easily seen how to try to reproduce the $\rho_{00}$ parameter evolution returns results in high degree of agreement with the reference values. Just to remark that the first 2 seconds the result is really different from the reference because in those moments the control rods are inserted to develop the oscillation, so we can neglect those moments and consider only the rest of the transient.

Then we try also to reproduce the evolution of the coupled modes. As an example we present in Fig. 7 the evolution of the $\rho_{10}^2$ obtained with the methodology previously explained.

As can be seen in Fig. 7 the agreement between the reference parameter value and the calculated parameter value is really good. So we can conclude that in spite of the linear behavior presented in this work, which has been criticized some times as not to be in accordance with the real behavior, the calculus of the reactivity parameter lead us to a values very close to the reference values.

## 5. Conclusions

Using the methodology explained in this work we are able to neglect e.g. all weighting factors which are necessary in the previous methods, because the current construction of a map with exact values allows to obtain the desired value with direct interpolation. And, as has been shown, with this methodology the results of calculation the coupling $\rho_{mn}$ parameters are also in good agreement with the reference values, so will allow to study the activation of out-of-phase

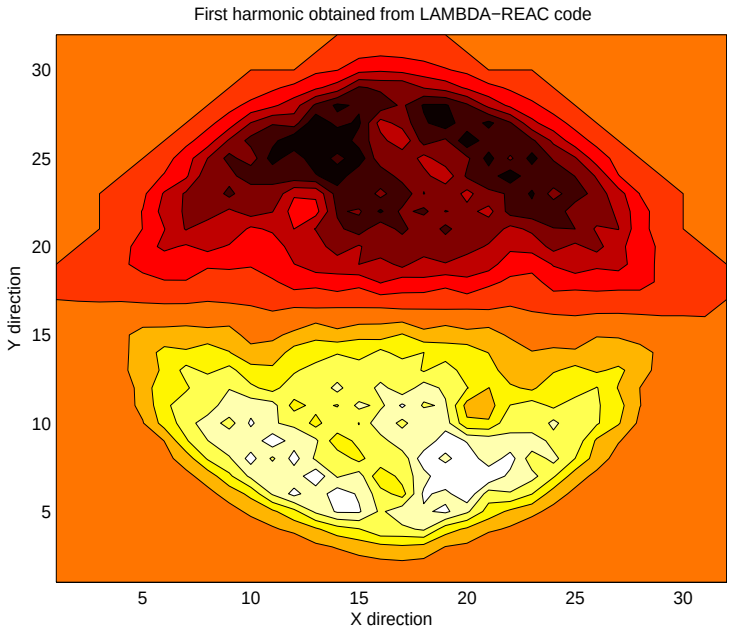


**Figure 3:** First harmonic of the lambda modes expansion calculated with LAMBDA-REAC code

oscillations from an in-phase oscillation. The initial goals have been achieved.

Finally this methodology has been developed in a way that allows itself to be further implemented inside the full ROM, since it has a enough simply structure to be used as a sub-model inside a larger model, and realize in the future studies about the bifurcation and non-linear behavior of a BWR.

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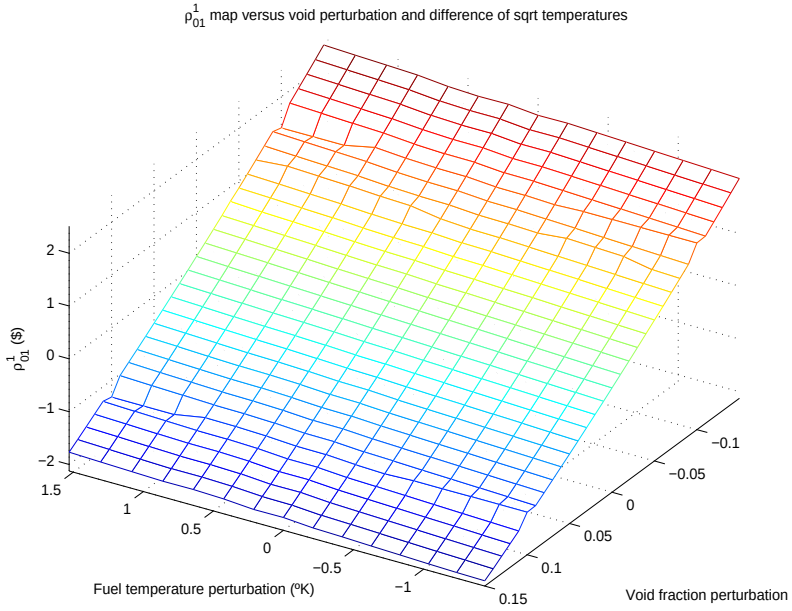


**Figure 4:** Feedback reactivity map parameter versus void fraction and fuel temperature perturbations

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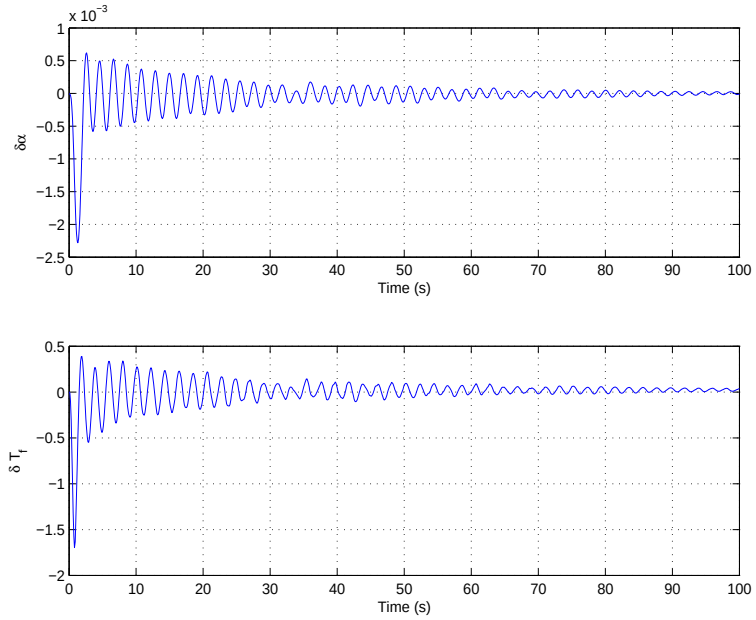


**Figure 5:** Parameter evolution in the studied transient

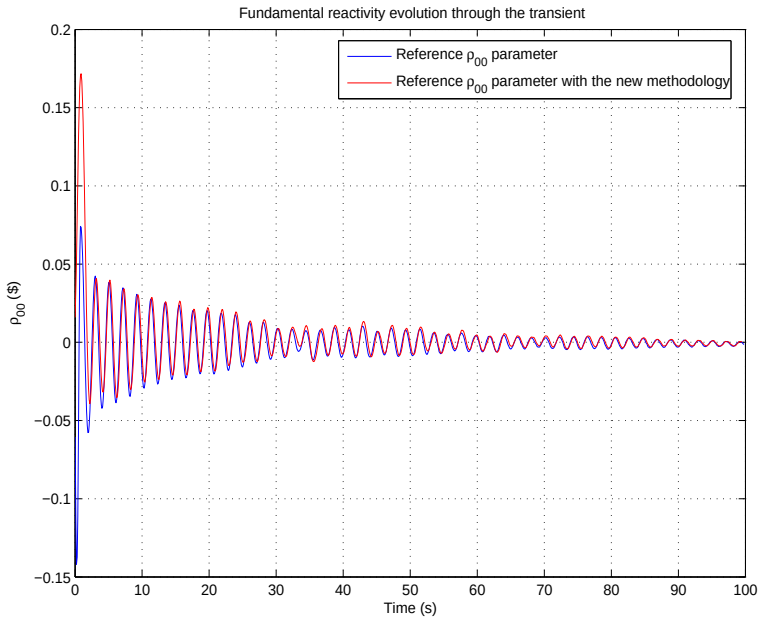


**Figure 6:** Reference $\rho_{00}$ versus $\rho_{00}$ calculated with the new methodology

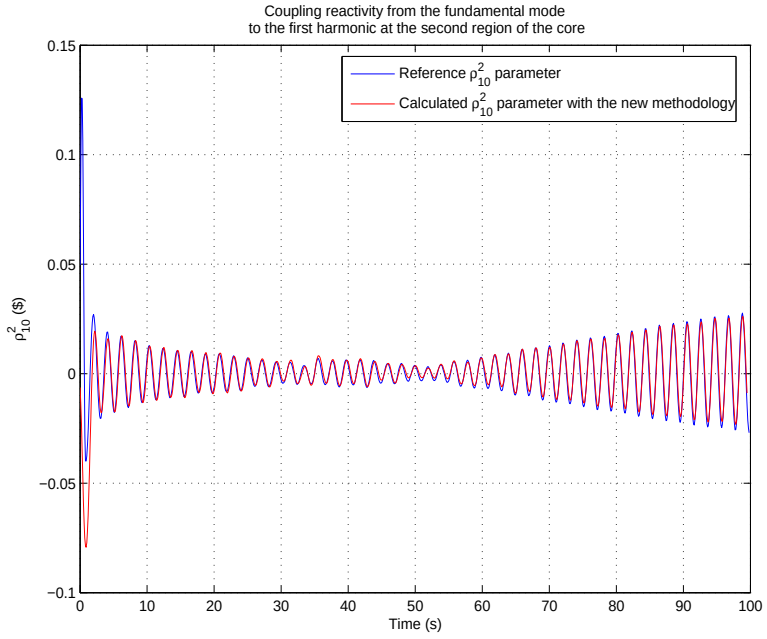


**Figure 7:** Reference $\rho_{10}^2$ versus $\rho_{10}^2$ calculated with the new methodology