Fluid Dynamics Model in Analyzing Neutron Mass Flow

*Ali Pazirandeh and Arash Haghani
Science and Research Branch, Islamic Azad University

Abstract

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In this paper we have tried to explain a mathematical model based on fluid dynamics laws to treat neutronics problems. In this study the basic laws of fluid flow have been employed to explain neutron flux as a fluid flow. The similarities of neutron behavior in a medium and gas molecule in a diffusing medium made us to assume neutron as a dilute atomic gas. It should bear in mind that in the equation of neutron flow shear stress, viscosity and gravity force have no effect on the motion, therefore, Navier-Stokes equation reduces to:

$$\partial u/\partial t + u.\nabla u = F$$

In the end, simple examples including two group fluxes in a reflected slab core solved by fluid dynamics model and compared with the diffusion theory.

Keywords: Fluid Dynamics, Fluid-particle, Transport Theory, Navier-Stokes equation

1. Introduction

Study of neutron transport and interactions in a multiplying fissile material is a key problem in designing reactor core. This study strongly depends on our information and analysis of neutron behavior in a highly interacting medium. Exact equation of the angular neutron density in a system is derived simply by neutron balancing. Transport theory has high reputation in describing random movement of neutrons in a heterogeneous multiplying medium. Exact transport equation can not be solved analytically. Solving transport equation needs approximations. Successful approximation leads to derivation of diffusion theory. Diffusion theory is widely used in treating neutronics problems. It depends on certain criteria, such as boundary condition(s) and medium absorption properties, which limit its validity.[1]
In this paper we have tried to explain a mathematical model based on fluid dynamic laws to treat neutron problems, and compared results with diffusion equation. We have used some basic ideas which lead to explain neutron flux as a fluid flow. Similarities of neutron behavior in a medium and gas molecules in a diffusing medium, with respect to fluid properties, made us to believe that neutron gas can be simulated as dilute atomic gas. For examining reliability of our model simple examples including two group fluxes in a reflected slab reactor core is solved by our **Neutron Fluid Dynamics Model (NFDM).** For proposing NFDM we had to define new parameters and concepts such as fluid-particle, mean neutron transport velocity and fluid-particle velocity.

* Corresponding author, Tel. +982122006155, Fax. +982188004781, E-mail: paziran@ut.ac.ir

2. Fluid Dynamics Method

Fluid dynamics is based on conservation laws(mass, energy and momentum conservation).
In defining common fluid systems conservation of mass is continuity equation, conservation of energy is *energy equation* which by some approximations reduces to B*ernoulli equation* and conservation of momentum is *Newton's second* law, which by some approximations leads to Navier-*Stokes equation* [2]:

Continuity equation:

$$\int_V \frac{\partial \rho}{\partial t} dV = -\int_S \rho u.dS \qquad (1)$$

Newton's second law:

$$\rho \frac{Du}{Dt} = \sum F \qquad (2)$$

Energy equation:

$$\frac{dQ}{dt} - \frac{dW_K}{dt} = \oiint_{CS} (e)(\rho V.dA) + \frac{\partial}{\partial t} \iiint_{CV} (e)(\rho dv) \qquad (3)$$

3. Neutron Behavior Analysis

Neutron density and flux are important parameters in reactor design and analysis. The *neutron transport equation*, or by some approximations *diffusion equation* is used for calculating these parameters. .*Neutron transport equation* is also used as a conservation of mass law. [1], [3]
We have tried to simulate neutron transport as a fluid flow and define neutron systems and parameters (such as flux) by fluid dynamic laws. For this purpose, conservation of momentum law, i.e. *Newton's second law,* is used.

4. Neutron Fluid Dynamics Model

4.1. Definitions

Now we define some basic parameters important in mass flow.

4.1.1.Neutron Mean Transport Velocity

Neutrons in collision with nuclei of the medium, assuming isotropic property, move in a forward direction. The average forward (transport) mean free path, λ_{tr}, weighted by average cosine of scattering angle, μ is: [3]

$$\lambda_{tr} = (\Sigma_t - \Sigma_s \mu)^{-1} \qquad (4)$$

The average neutron lifetime in the medium is defined as:

$$l_\infty = (v\Sigma_a)^{-1} \qquad (5)$$

Therefore neutron transport velocity, V_{tr}, would be:

$$V_{tr} = \frac{\lambda_{tr}}{l_\infty} \quad (6)$$

4.1.2. Fluid-Particle

Fluid-particle is an imaginary element (control volume) in our neutron fluid that moves forward by velocity u.

4.1.3. Fluid-Particle Velocity

Fluid-particle velocity is equal to the number of neutrons in fluid-particle, n, multiplied by V_{tr}:

$$u = nV_{tr} \quad (7)$$

4.2. The Model

Although in fluid flow each molecule moves in a semi-random direction, but the bulk of fluid flows in a forward direction.
There is the same behavior in neutron transport; therefore we consider a *fluid-particle* that indicates the bulk neutron movement in a forward direction.
Now we can write *Newton's second law, Eq. (2),* for our *fluid-particle* as follows:

$$\frac{\partial u}{\partial t} + u.\nabla u = \frac{\sum F}{\rho} \quad (8)$$

In which

$$\frac{Du}{Dt} = \frac{\partial u}{\partial t} + u.\nabla u \quad (9)$$

Eq. 8 is the general equation of NFDM and in this equation forces have to be specified.
In the steady state condition $\frac{\partial u}{\partial t}$ in Eq. (8) is zero.
Since viscous force, gravitational force and shear stress for *fluid-particles* are negligible we consider only drag force.

5. Some Simple Cases by NFDM

5.1. Infinite plane source in an infinite medium (One group neutron energy)

Eq. (8) for one dimensional geometry becomes:

$$\frac{du}{dx} = \frac{F}{u\rho} \tag{10}$$

And drag force is:

$$F = -bu^2 \tag{11}$$

Inserting F from Eq. (11) in Eq. (10) and integrating give:

$$u = A\exp(-cx) \tag{12}$$

Multiplying Eq. (12) by ρ gives flux:
(ρ is number density of fluid particles)

$$\Phi(x) = A'\exp(-cx) \tag{13}$$

Eq. (13) is the same as the one obtained by *diffusion equation*.

5.2. Point source in an infinite medium (One group neutron energy)

Eq. (8) for one dimensional geometry is:

$$\frac{du}{dr} + \frac{u}{r} = \frac{F}{u\rho} \tag{14}$$

Drag force is:

$$F = -bu^2 \tag{15}$$

Replacing F in Eq. (14) by Eq. (15) and integrating gives:

$$u = \frac{A}{r}\exp(-cr) \tag{16}$$

Multiplying u by ρ gives flux:

$$\Phi(r) = \frac{A'}{r}\exp(-cr) \tag{17}$$

Eq. (17) is the same as that obtained by *diffusion equation*.

5.3. Infinite plane source in an infinite medium (Two group neutron energy)

Eq. (8) for one dimensional, fast group becomes:

$$\frac{du_f}{dx} = \frac{F_1}{u_f\rho} \tag{18}$$

$$F_1 = -b_f u_f^{\ 2} \tag{19}$$

Replacing F_1 and integrating gives:

$$u_f = A_f \exp(-c_f x)$$
(20) $$\Phi_f(x) = A'_f \exp(-c_f x) \tag{21}$$

For thermal neutrons:

$$\frac{du_{th}}{dx} = \frac{F_2}{u_{th}\rho} \tag{22}$$

$$F_2 = -b_{th} u^2{}_{th} + b_f u_f^{\ 2} \tag{23}$$

Replacing F_2 in Eq. (22) by Eq. (23) and integrating gives:

$$u_{th} = \frac{b_f A_f}{b_{th} - b_f} \exp(-c_f x) + C \exp(-b_{th} x) \tag{24}$$

Flux is obtained by multiplying u by ρ :

$$\Phi_{th} = \frac{b_f A'_f}{b_{th} - b_f} \exp(-c_f x) + C' \exp(-b_{th} x) \tag{25}$$

Eq. (25) is the same as that obtained by *diffusion equation*. (See Fig.1)

5.4. Infinite slab, homogeneous multiplying media (One group neutron energy (width 2a))

The form of drag force in multiplying media is different from its form in non-multiplying media. In this model neutron leakage from the system depends on the position of the fluid-particles. Therefore, neutron leakage is a probabilistic event. So we assumed that the drag force is dependent on position, x, and velocity, u .The ingenuity tells us that the force should be:

$$F = -bx^2u^2 \tag{26}$$

Replacing Eq. (26) into Eq. (8) gives:

$$u = A\exp(-c\frac{x^3}{3}) \tag{27}$$

Multiplying Eq.(27) by ρ gives flux:

$$\Phi(x) = A' \exp(-c\frac{x^3}{3}) \quad (28)$$

Eq. (28) is different from the equivalent diffusion equation as shown in Fig. 2.
Two group neutron flux distribution for this case is shown in Fig. 3.

5.5. Infinite slab, homogeneous multiplying media with reflector (Two group neutron energy (width 2a)

Beginning from Eq. (8) and using the same procedure lead to following results:

$$\Phi_f^C(x) = A_f'^C \exp(-b_f^C \frac{x^3}{3}) \quad (29)$$

$$\Phi_{th}^C = \frac{b_f^C A_f'^C}{b_{th}^C - b_f^C} \exp(-c_f^C \frac{x^3}{3}) + C_1' \exp(-b_{th}^C \frac{x^3}{3}) \quad (30)$$

$$\Phi_f^R(x) = A_f'^R \exp(-b_f^R x) \quad (31)$$

$$\Phi_{th}^R = \frac{b_f^R A_f'^R}{b_{th}^R - b_f^R} \exp(-c_f^R x) + C_2' \exp(-b_{th}^R x) \quad (32)$$

The fluxes are shown in Fig. 4.

6. Results and Discussion

In non-multiplying media, i.e. equations 13, 17, 21 and 25, obtained by NFDM are exactly the same as equivalent equations from diffusion theory.
Fig. 1 shows a two group neutron energy fluxes for an infinite plane source in non-multiplying media which are similar to diffusion theory.

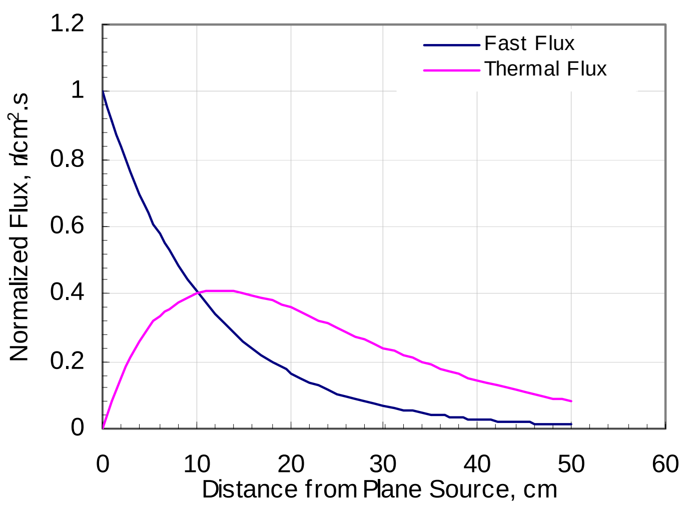

Fig.1- Fast flux distribution in moderating medium by NDFM and build up of

thermal flux as fast neutrons slow down

For multiplying media although NFDM merges on diffusion theory in the center of core, but it deviates from diffusion theory as getting close to the boundaries.
This is a triumph for NFDM because it is closer to the reality, transport theory. It is seen in Fig. 2.

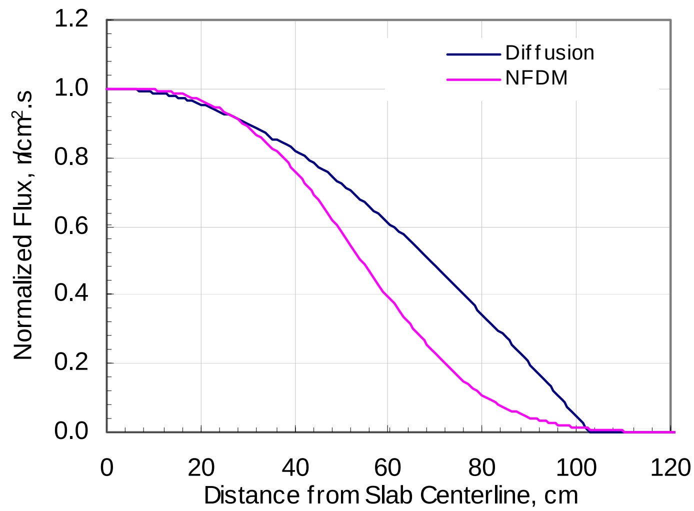

Fig. 2 - Comparison between NFDM and diffusion theory in a slab reactor with 200cm width (one group neutron energy)

Two group neutron flux for an infinite slab reactor obtained by NFDM are shown in Fig.3

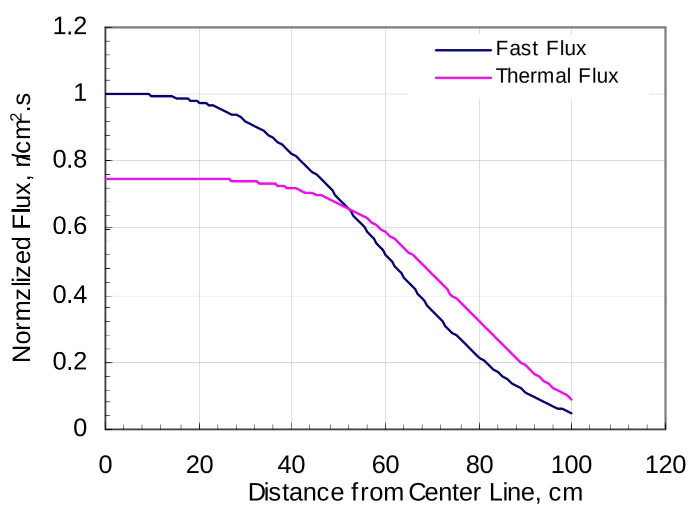

Fig.3 - Fast and thermal neutron fluxes in a slab reactor (width 200 cm) by NFDM

The fluxes for the same case (Fig.3) with reflector are shown in Fig.4.

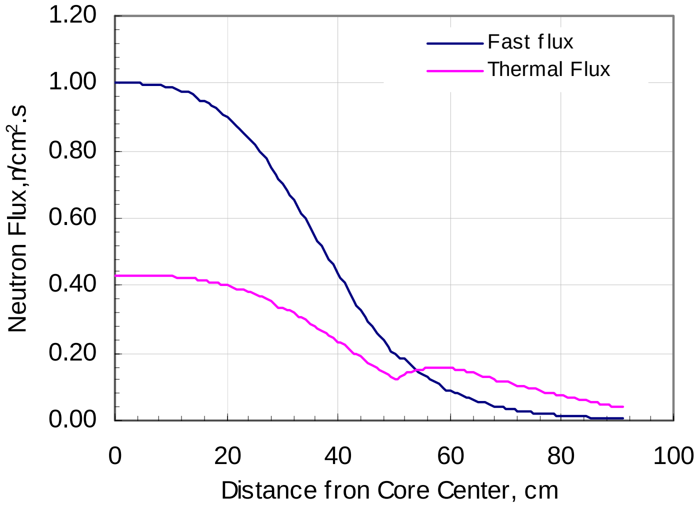

Fig.4- Fast and thermal neutron fluxes in a slab reactor (width 100 cm) with an infinite reflector by NFDM

7. Conclusion

1. The general equation of NFDM (Neutron Fluid Dynamics Model) is a first order differential equation while diffusion equation is a second order equation.
2. Although fluxes in non-multiplying media are the same as diffusion theory, but in multiplying media results are more accurate than diffusion theory, especially near boundaries.

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References:

[1] Duderstadt J.J. and Hamilton L.J, *Nuclear Reactor Analysis*, John Wiley & Sons Ltd, 1976

[2] Hughes W.F., Brighton J.A., *Fluid Dynamics*, third edition, McGraw Hill International Editions Schum's Outline Series, 2000.

[3] Lamarsh, J.R., *Introduction to Nuclear Reactor Theory*, Addison-Wesley Publishing Co., 1966