Generalized Bias Factor Method for Accurate Prediction of Neutronics Characteristics

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Abstract

A generalized bias factor method is proposed to improve the prediction accuracy of neutronics characteristics of a target core. The generalized bias factor method uses conventional bias factors calculated for several critical assemblies. The weighting factors for individual bias factors are determined to minimize the variance of neutronic characteristics predicted for the target core. Numerical calculations are performed to investigate the uncertainty reductions of neutronics characteristics for a tight-lattice core. Though the uncertainty is not remarkably reduced for k_{eff} , that for the reaction rate ratio of ^{238}U capture / ^{239}Pu fission is remarkably reduced : For example, the uncertainty reduction of the reaction rate ratio in the upper core is 0.871 for the present method, and 0.657 for the conventional bias factor method.

KEYWORDS: bias factor, neutronics characteristics, variance, uncertainty of neutronics characteristics, sensitivity coefficient, cross section error

1. Introduction

The reduction of prediction uncertainty of neutronics parameters is essential in designing innovative reactors with complicated configuration. To reduce the prediction uncertainty, one should use a reliable calculational method such as Monte-Carlo method and a reliable data such as JEF-2.2 [1], JENDL-3.3 [2], ENDF/B-V, VI. [3,4]

However, even if one uses such a reliable method and data, there still remains some discrepancy between calculations and experiments. One of the example is observed for k_{eff} as indicated by Okumura.[5] Therefore, for detailed design calculations of innovative reactors, it is required to further reduce the uncertainty.

To reduce the uncertainty, there are two techniques. One is the bias factor method [6], and the other is the cross section adjustment method. [7] Here, we concentrate on the bias factor method.

In the bias factor method, the bias factor, that is the ratio of measured neutronics characteristics and the calculated neutronics characteristics, is obtained using the data of critical experiments. This factor is multiplied to calculated neutronics characteristics for a target reactor core to improve the accuracy. This method is simple, and easy to apply to design calculations.

The bias factor includes the calculation error and the experimental error of critical assembly. The calculation error is caused by the cross section error and the method error.

The contribution from the cross section error is expressed by the product of sensitivity coefficients and the cross section error. On the other hand, the error of the target core is expressed by the method error and the cross section error multiplied by the sensitivity coefficients of the target core.

Therefore, when one uses the same cross section set and the same calculation method in calculating the neutronics characteristics of the critical assembly and the target core, the errors will be cancelled each other for the cases when the sensitivity coefficients are very similar for the two cores. Thus, the conditions necessary for the usefulness of the bias factor method are as follows:

1. the sensitivity coefficients are very similar for the critical assembly and the target core.
2. the experimental error is small for the critical assembly.
3. the same cross section set and the same calculation method should be used for the two cores.

However, the bias factor method has a drawback that only one bias factor is used for the design calculation. Therefore, one has to make a critical experiment to be very close to the target reactor core. However, there is some restriction in performing the critical experiment such as the plutonium and uranium inventories, the plutonium isotopic composition and some special material. Also, there is some restriction of the core size due to the limited space of the experiment. These restrictions introduce some difference between the critical assembly and the target core, and this difference may lead to the deterioration of the conventional bias factor method.

The purpose of this paper is to generalize the bias factor method to use several experimental data, and to improve the prediction accuracy. The TSUNAMI code can calculate the correlation of individual experiments. [8] However, there is no suggestion how to use the individual experimental data to the design calculation of the target reactor core. In the paper, we derive how to use the individual data for the target core.

The following chapter describes the theory. The numerical example will be shown in Chap.3. Chapter 4 draws some conclusions.

2. Theory

Let us first consider the conventional bias factor method. The bias factor f is defined by the ratio of experimental and calculated neutronics characteristics of the critical assembly:

$$f = R_c^e \big/ R_c^c \quad . \tag{1}$$

The superscripts e and c denote the experimental and calculated data, and the subscript c denotes the critical assembly. The neutronics characteristics R_r^c calculated for the real target core (the subscript r denotes the real target core) is modified by using the bias factor as follows:

$$\tilde{R}_r^c = R_r^c \times f \quad . \tag{2}$$

The above method is the conventional bias factor method.

Now, let us consider the case where there are different experimental data for several critical assemblies with different void ratios or with different moderator to fuel volume ratios, etc. For such a case, we can determine the bias factor as a linear combination of the individual bias factors. We assume that there are N experimental data. For each data, the bias factor is calculated by

$$f_i = R_{ci}^e \big/ R_{ci}^c \quad . \tag{3}$$

The generalized bias factor is determined by

$$\tilde{f} = \sum_{i=1}^{N} C_i \, f_i \quad , \tag{4}$$

where C_i is the weighting factor for the i'th experiment. When N=1, the above equation reduces to the conventional bias factor method. However, in this case, the weighting factor C_i must equal to unity. So, when there are N bias factors, the weighting factors are not independent. The weighting factors have to satisfy the relation:

$$\sum_{i=1}^{N} C_i = 1 \quad . \tag{5}$$

The neutronics characteristics of the target core is calculated by

$$\tilde{R}_r^c = R_r^c \times \tilde{f} \quad . \tag{6}$$

The R_r^c has uncertainty due to the cross section error $\Delta\sigma$ and the method error ΔM_{cr}, therefore it is expressed by

$$R_r^c = R_r (1 + S_r \, \Delta\sigma + \Delta M_{cr}) \quad , \tag{7}$$

where R_r is the true value of the neutronics characteristics, and S_r is the sensitivity coefficient of the target core. The cross section error $\Delta\sigma$ and the sensitivity coefficient S are defined as follows:

$$\Delta\sigma = \frac{d\sigma}{\sigma} \quad , \tag{8}$$

$$S = \frac{dR}{R} \bigg/ \frac{d\sigma}{\sigma} . \tag{9}$$

In a similar manner, R_{ci}^c is expressed as follows.

$$R_{ci}^c = R_{ci} (1 + S_i \, \Delta\sigma + \Delta M_{ci}) \quad , \tag{10}$$

where R_{ci} is the true neutronics characteristics of the i'th critical assembly.

The experimental data R_{ci}^e has the experimental error ΔE_i, and is expressed by

$$R_{ci}^e = R_{ci} \left(1 + \Delta E_i\right) \quad . \tag{11}$$

Introducing Eqs.(4), (7), (10), and (11) into Eq.(6), we obtain

$$\tilde{R}_r^c = R_r (1 + S_r \, \Delta\sigma + \Delta M_{cr}) \times \sum_{i=1}^{N} \left(C_i \, \frac{1 + \Delta E_i}{1 + S_i \, \Delta\sigma + \Delta M_{ci}} \right) \quad . \tag{12}$$

Here we assume that the relative errors $S_i \Delta\sigma$ and ΔM_{ci} are small compared with unity. In such a case, Eq.(12) reduces to

$$\tilde{R}_r^c = R_r \left\{ 1 + (S_r - \sum_{i=1}^{N} C_i S_i)\, \Delta\sigma + (\Delta M_{cr} - \sum_{i=1}^{N} C_i \, \Delta M_{ci}) + \sum_{i=1}^{N} C_i \, \Delta E_i \right\} \quad . \tag{13}$$

One can assume that there is no correlation among the cross section errors, the method errors and the experimental errors. Of course, there may be correlations between individual method errors and between individual experimental errors. Under the assumption, the variance of $\tilde{R}_r^c$ is given by

$$\begin{aligned} V\left(\tilde{R}_r^c\right) = R_r^{\,2} \times \Bigg[& \left\{ S_r - \sum_{i=1}^{N} C_i \, S_i \right\} V_x \left\{ S_r - \sum_{i=1}^{N} C_i \, S_i \right\}^t \\ & + V\left(\Delta M_{cr} - \sum_{i=1}^{N} C_i \, \Delta M_{ci} \right) \quad , \\ & + \sum_{i=1}^{N} \sum_{j=1}^{N} C_i \, C_j \, V\left(\Delta E_i \cdot \Delta E_j \right) \Bigg] \end{aligned} \tag{14}$$

where V_x is the cross section covariance matrix and superscript t denotes the transposed matrix.

Our purpose is to improve the prediction accuracy of the calculated neutronics characteristics of the target core. For this purpose, we minimize Eq.(14). Therefore, the coefficient C_i is determined to minimize Eq.(14). The differentiation of Eq.(14) by C_i is set to zero:

$$\frac{d\, V\left(\tilde{R}_r^c\right)}{d\, C_i} = 0 \quad . \qquad (i = 2,3,\cdots,N) \tag{15}$$

The coefficient C_i are solved by using Eqs.(5) and (15). Furthermore, if C_i's are negative we put $C_i = 0$, and if C_i's are larger than 1, we put $C_i = 1$. The C_i means the relative importance of i'th experiment. By eliminating C_1 using Eq.(5), Eq.(15) reduces to

$$
\begin{aligned}
&(S_r - S_1)V_x(S_i - S_1)^t + V(\Delta E_1) - V(\Delta E_1 \cdot \Delta E_i) \\
&+ V(\Delta M_{c1}) + V(\Delta M_{cr} \cdot \Delta M_{ci}) - V(\Delta M_{cr} \cdot \Delta M_{c1}) - V(\Delta M_{c1} \cdot \Delta M_{ci}) \\
&= \sum_{j=2}^{N} C_j \{ (S_j - S_1)V_x(S_i - S_1)^t \\
&+ V(\Delta E_1) - V(\Delta E_1 \cdot \Delta E_j) - V(\Delta E_1 \cdot \Delta E_i) + V(\Delta E_i \cdot \Delta E_j) \\
&+ V(\Delta M_{c1}) - V(\Delta M_{c1} \cdot \Delta M_{cj}) - V(\Delta M_{c1} \cdot \Delta M_{ci}) + V(\Delta M_{ci} \cdot \Delta M_{cj}) \} \\
&\qquad (i = 2,3,\cdots,N)
\end{aligned}
\quad . \qquad (16)
$$

The above equations determine C_i's.

The uncertainty reduction (UR) [9] of neutronics characteristics is defined by :

$$
UR = 1 - \frac{\Delta S V_x \Delta S^t + V\left(\Delta M_{cr} - \sum_{i=1}^{N} C_i \, \Delta M_{ci}\right) + \sum_{i=1}^{N}\sum_{j=1}^{N} C_j C_j V(\Delta E_i \cdot \Delta E_j)}{S_r V_x S_r^{\,t} + V(\Delta M_{cr})} \quad , \qquad (17)
$$

$$
\Delta S V_x \Delta S^t = \left(S_r - \sum_{i=1}^{N} C_i \, S_i\right) V_x \left(S_r - \sum_{i=1}^{N} C_i \, S_i\right)^t \quad . \qquad (18)
$$

Of course, the critical experiments become useful when $UR \to 1.0$.

3. Numerical Example

3.1 Calculational Model

The tight-lattice core was selected as a real target core. This reactor has a high conversion ratio and a high burn-up up to 100GWd/t. In order to archive a high conversion ratio, the core contains tight-pitch MOX fuel rods. The void fraction is increased compared with conventional BWR core. [10] This core has the void distribution from 0% to 80% and the average void ratio is about 60%. Figure 1 shows the axial core configuration.

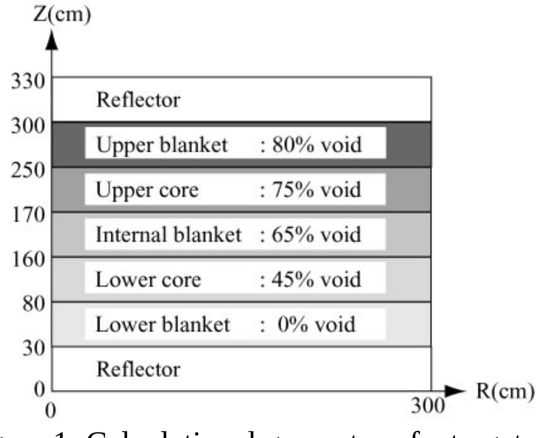

Figure 1: Calculational geometry of a target core

To obtain the bias factors, three critical experiments were selected. These experiments were performed using the FCA core. [10] The three assemblies have the different void fractions of 45% void (C45V), 65% void (C65V) and 95% void (C95V). [10]

The evaluated neutronics characteristics are the neutron multiplication factors (k_{eff}) and the reaction rate ratios of ^{238}U capture to ^{239}Pu fission (C28/F49). To analyze the target core and critical assemblies, the 70-group cross section set processed from JENDL-3.3 [2] was utilized. The sensitivity coefficients were calculated using the generalized perturbation theory code SAGEP. [11] The cross section covariance matrix was taken from JENDL-3.3.

In calculations, we assumed that the method errors and the experiment errors are zero.

3.2 The Neutron Multiplication factor (k_{eff})

The results of the uncertainty reduction are shown in Table 1.

Table 1: Comparison of uncertainty reduction UR for k_{eff} by using the conventional and present method

	Critical experiment	UR	C_i
Conventional method	C45V C65V C95V	0.340 0.533 0.473	——— ——— ———
Present method	C95V C65V	0.562	$C_{C95V} = 0.30$ $C_{C65V} = 0.70$
	C65V C45V	0.533	$C_{C65V} = 1.00$ $C_{C45V} = 0.00$

Using the conventional method, the uncertainty reduction is 0.533 at maximum. When one uses two critical experiments C95V and C65V in the present method, the uncertainty reduction is 0.562. The sensitivity coefficients of ^{238}U capture cross section and ^{239}Pu fission cross section for k_{eff} in the target core and the critical assemblies are shown in Fig. 2 and Fig. 3.

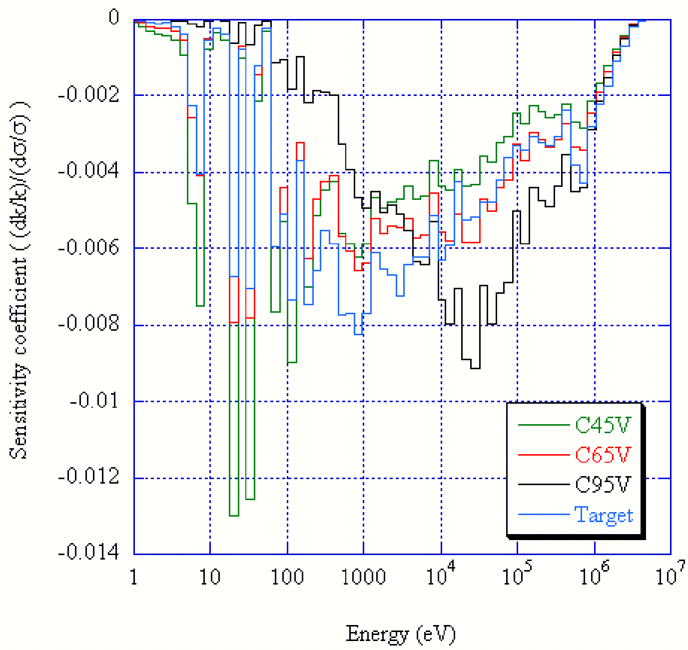

Figure 2: Sensitivity coefficients for k_{eff} in each core with respect to ^{238}U capture cross section

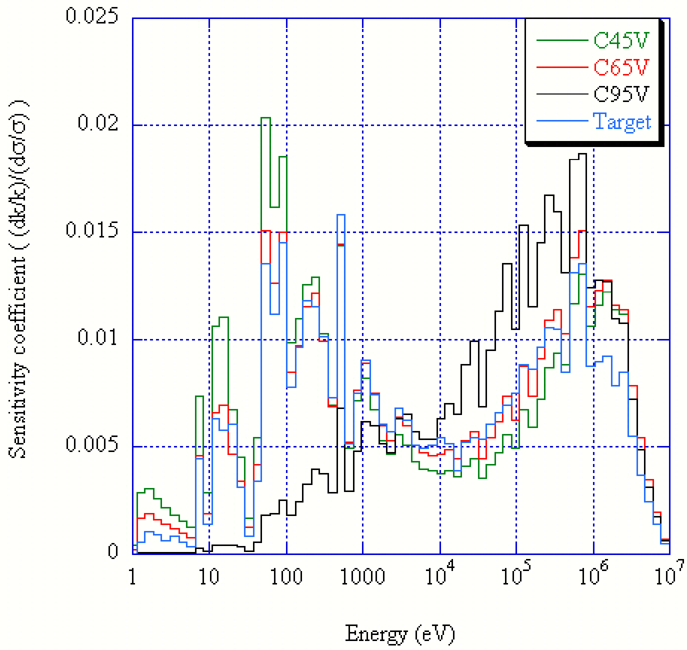

Figure 3: Sensitivity coefficients for k_{eff} in each core with respect to ^{239}Pu fission cross section

The sensitivity coefficients for C65V are similar to those for the target core. Therfore, the weighting factor C for C65V (C_{C65V}) is large (0.7) compared to that for C95V in the present method. An improvement is achieved in the present method, though the difference is small.

3.3 The Reaction Rate Ratio (C28/F49)

The target core has an axially heterogeneous configuration. In this section, we evaluate the uncertainty reductions of the reaction rate ratio (C28/F49) in each region and the whole core of the target core. The uncertainty reductions of the blanket regions were calculated using the experimental data at the core region. Table 2 shows the uncertainty reductions by the present method, and Table 3 shows the uncertainty reductions by the conventional method.

Table 2: Uncertainty reduction for C28/F49 by using the present method

Region	Critical experiment	UR	C_i
Upper blanket	C95V C65V	0.654	$C_{C95V} = 0.00$ $C_{C65V} = 1.00$
Upper core	C95V C65V	0.871	$C_{C95V} = 0.57$ $C_{C65V} = 0.43$
Inner blanket	C95V C65V	0.810	$C_{C95V} = 0.22$ $C_{C65V} = 0.78$
Lower core	C65V C45V	0.830	$C_{C65V} = 0.72$ $C_{C45V} = 0.28$
Lower blanket	C65V C45V	0.191	$C_{C65V} = 0.00$ $C_{C45V} = 1.00$
Whole core	C95V C65V	0.826	$C_{C95V} = 0.31$ $C_{C65V} = 0.69$
Whole core	C65V C45V	0.770	$C_{C65V} = 1.00$ $C_{C45V} = 0.00$

Table 3: Uncertainty reductions for C28/F49 by using the conventional method

Region	Critical experiment	UR
Upper blanket	C65V	0.654
Upper core	C65V	0.657
Inner blanket	C65V	0.805
Lower core	C45V	0.631

Lower blanket	C45V	0.191
Whole core	C65V	0.770

In the upper core region, the uncertainty reduction is 0.871 when we use the two critical experiment data (C95V and C65V). In the lower core region, the uncertainty reduction is 0.830 by using C65V and C45V data. In the conventional method, the uncertainty reduction is 0.657 and 0.631, respectively. Thus a remarkable improvement is obtained. For other regions, the uncertainty reductions are almost the same as that of the conventional method.

The uncertainty reduction for the whole core is 0.826 by using the C95V and C65V data in the present method. On the other hand, when one uses the C65V and C45V data, the uncertainty reduction is 0.770, and this result is the same as the conventional bias factor method because $C_{C65V} = 1.0$. Thus the improvement of the uncertainty reduction was seen for the present method in evaluating the reaction rate ratio for the core regions, and for the whole core. The present method has an advantage that one does not need to select most useful critical experiment, the weighting factors are automatically calculated.

4. Conclusion

The generalized bias factor method was proposed to use several experiment data for the accurate design of a target core. The generalized bias factor contains the conventional bias factors for individual critical assemblies. The method of calculating the weighting factors of individual bias factors was derived, and the uncertainty of the neutronics characteristics of the target core was estimated. The uncertainty reductions of k_{eff} and C28/F49 in the target tight-pitch core were evaluated.

Neglecting the method errors and the experimental errors, the uncertainty reduction of neutronics characteristics for k_{eff} was 0.562 using the two experimental data, C65V and C95V. On the other hand, using the conventional method, the uncertainty reduction was 0.533 at most.

The uncertainty reductions of the reaction rate ratio of ^{238}U capture and ^{239}Pu fission at each region and the whole core of the target core were evaluated. In the upper core region, the uncertainty reduction is 0.871 by the present method and 0.657 for the conventional bias factor method. In the whole core, the uncertainty reduction is 0.826 for the present method, and 0.770 for the conventional method.

The results indicated that the uncertainty is improved by the present method. Furthermore, the present method has an advantage that one does not need to select a relevant experimental data for a target core, the weighting is automatically calculated.

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