COMET Solutions to Whole Core CANDU-6 Benchmark Problems

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Abstract

In this paper, the coarse mesh transport code COMET is used to solve CANDU-6 benchmark problems in two and three dimensional geometry. These problems are representative of a simplified quarter core reactor model. The COMET solutions, the core eigenvalue and the fuel pin fission density distribution, are compared to those from the Monte Carlo code MCNP using two-group cross sections.

COMET decomposes the core volume into a set of non-overlapping sub-volumes (coarse meshes) and uses precomputed heterogeneous response functions that are constructed using Legendre polynomials as boundary conditions to generate a user selected whole core solution (e.g., the core eigenvalue and fuel pin fission density distribution). These response functions are precomputed by performing fixed source calculations with a modified version of MCNP in only the unique coarse meshes in the core.

Reference solutions are calculated by MCNP5 with a two-group energy library generated with the HELIOS lattice code. In the 2-D problem, the angular current on the coarse mesh interfaces in COMET is expanded to 2nd order in both spatial and angular variables. The COMET eigenvalue error is 0.09%. The corresponding average error in the fission density over all 3515 fuel pins is 0.5%. The maximum error observed is 2.0%. For the 3-D case, with 4th order expansion in space and azimuthal angle and 2nd order expansion in the cosine of the polar angle, the eigenvalue differs from the reference solution by 0.05%. The average fission density error over the 42180 fuel pins is 0.7% with a maximum error of 3.3%.

KEYWORDS: Heterogeneous Coarse Mesh Transport Method, CANDU Benchmark Problems, Transport Theory, Reactor Core Simulation

1. Introduction

Recently, the coarse mesh radiation transport code (COMET) was extended to 3-D geometry [1] and was later tested on small PWR benchmark problems [2]. COMET uses precomputed heterogeneous response functions for unique coarse meshes in the core to generate the detailed whole core solution such as the fuel pin fission density distribution. This is achieved by converging on the coarse mesh interface partial currents that are approximated by truncated Legendre polynomials in the angular and spatial variables. It is believed that COMET would be

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ideal for obtaining efficient pin fission density distributions in large CANDU-6 reactor cores. The over-moderated design makes the angular partial current very smooth in phase space at the coarse mesh (lattice cell) interfaces. The smooth partial current can be well approximated by low-order polynomial expansions. In this paper, the feasibility of using COMET to solve a CANDU-6 reactor core is studied by testing the code in two and three dimensional CANDU-6 quarter core configurations. The COMET method is briefly described in section 2. A description of both benchmark problems including the MCNP and COMET results are given in section 3. Concluding remarks and the direction of future work are presented in section 4.

2. Method

This section contains a brief overview of the coarse mesh transport method. A more detailed description can be found in the work of Mosher and Rahnema [3] and Forget and Rahnema [4].

2.1 Domain Decomposition

COMET solves the integro-differential neutron transport equation defined in a reactor core. The core volume is decomposed in a set of non-overlapping sub-volume elements (*e.g.* coarse meshes), for which, the boundary conditions are defined as the angular partial fluxes (or currents) exiting the neighboring meshes. Using standard notation, the transport equation in sub-volume V_i can be written as

$$\begin{aligned}\hat{\Omega}.\nabla\psi_i\left(\vec{r},\hat{\Omega},E\right)+\sigma_t\left(\vec{r},E\right)\psi_i\left(\vec{r},\hat{\Omega},E\right)-\int_0^\infty dE'\int_{4\pi}d\Omega'\,\sigma_s\left(\vec{r},\hat{\Omega}',E'\rightarrow\hat{\Omega},E\right)\psi_i\left(\vec{r},\hat{\Omega}',E'\right)\\ =\frac{1}{4\pi}\chi\left(\vec{r},E\right)\int_0^\infty dE\int_{4\pi}d\Omega\frac{\nu\sigma_f\left(\vec{r},E'\right)}{k}\psi_i\left(\vec{r},\hat{\Omega}',E'\right)\end{aligned},\tag{1}$$

with the following boundary condition

$$\psi_i^-\left(\vec{r}_{ij},\hat{\Omega},E\right)=\psi_j^+\left(\vec{r}_{ij},\hat{\Omega},E\right)\text{ where }\vec{r}_{ij}\in\left\{V_i\cap V_j\right\}\text{ for all }V_j\text{ bounding }V_i\tag{2}$$

In equation (2), k is the system eigenvalue and ψ_i is the angular flux within the sub-volume element V_i and V_j represents all the sub-volume elements sharing a common boundary with V_i. The superscripts "+" and "-"indicate the outgoing and incoming neutron directions.

This decomposition leads to a series of fixed source problems which together are entirely equivalent to the neutron transport equation expressed over the whole core. Since the problem eigenvalue (k) and the incident angular fluxes at each coarse mesh interface are not known, a two-level iteration process is used in COMET.

Solving the transport equation directly in large three dimensional heterogeneous systems (*e.g.* a nuclear reactor core) is computationally prohibitive. However, such methods have proven to be very accurate and efficient for solving lattice cell problems (*e.g.* fuel assembly). Thus, combining the efficiency and accuracy of current methods for solving small fixed source problems with the global problem decomposition can lead to efficient and accurate solutions in large heterogeneous systems with a repeated lattice cell (fuel assembly/bundle) structure (e.g., heavy and light water reactor cores).

At first glance, the decomposition is fairly similar to the more traditional response matrix or interface current techniques. However, the main difference is in the treatment of the fission source and the interface angular fluxes. In the previous methods, the fission source is treated explicitly [5], thus requiring surface to surface, surface to volume (where fission may occur), volume to surface and volume to volume first-flight collision probabilities. They also require an approximation in terms of the number of neutron generations that are tracked. COMET treats the fission source implicitly meaning that the neutron transport equation is solved over a coarse mesh in which the fission source is scaled by the core eigenvalue, as shown in equation 1. In this case, only surface to surface calculations need to be performed. Outer iterations are thus performed directly on the eigenvalue instead of the fission source. This leads to a significant improvement in terms of overall efficiency. In addition, this formulation makes no approximation regarding the shape of the fission source distribution and with the number of neutron generations, which is necessary with methods that employ in-volume responses. However, with the implicit source treatment, the magnitude of the source has to be controlled by the incident flux in the local fixed source calculation, thus requiring an external normalization in order to converge on the global system solution.

2.2 Approximation

Since the partial angular flux (or current) distribution in angle, space and energy at the boundary of each coarse mesh is not known, an approximation must be introduced. Mosher and Rahnema [3] demonstrated that a broad class of functions could be used to approximate the angular fluxes (or currents) at the coarse mesh interfaces. Ilas and Rahnema [6] used surface Green's function to expand the angular flux in a discrete-ordinates context which proved to be too cumbersome for extension to 2-D geometry. Mosher and Rahnema [3] used discrete Legendre polynomials in the discrete-ordinates context to extend the method to multi-dimension. Forget and Rahnema [7] expanded the angular current instead of the angular flux to enable the use of continuous Legendre polynomials in the Monte Carlo context. This expansion was initially tested in 2-D and later in 3-D geometry. The expansion is a tensor product of Legendre polynomials in all phase space variables.

Using orthogonal functions to represent the angular current at the coarse mesh interface allows precomputation of the solutions of equation 1 as a response to a unit boundary condition of various expansion orders in all variables. These solutions are known as response functions. Given the incoming currents and the response functions, the solution (e.g., the outgoing angular currents at the interface) in each coarse mesh is found by linear superposition. The response functions used by the heterogeneous coarse mesh transport method can be generated with any available fine-mesh method. As in the previous work by the current authors [7], this paper uses response functions generated by a Monte Carlo method. In particular, the MCNP4C [8] code is modified (1) to accommodate the scaling of the fission source by $1/k$ (an input variable) and (2) to sample from the source boundary condition expanded in Legendre polynomials. This 3-D modification in space and angles (polar and azimuthal) is an extension of the 1-D polar expansion technique proposed by Griesheimer and Martin [9]. Each individual tallied response function is the response of the quantity of interest (e.g., outgoing angular current, fuel pin fission density) to each expansion order of the incoming angular current at the coarse mesh surface. Using a stochastic method to generate the response functions carries the burden of precision and uncertainty. The coarse mesh method inherits the statistical uncertainties of its response functions and must propagate them appropriately in the calculation. However, unlike the full core stochastic calculations, the cost of greater precision in the coarse mesh method is entirely shifted to the one-time precomputational phase (the response function library generation).

3. Results

Two benchmark problems representative of a simplified upper right hand corner of an operating CANDU-6 reactor in two and three dimensional ¼ core configurations are used to benchmark COMET. For benchmarking purposes, these configurations are simplified to limit the number of burnup points and macroscopic cross section regions and energy groups. It is noted that COMET is not limited in these regards. The ¼ core configuration for the 2-D problem is shown in Fig. 1. The corresponding configuration for the 3D case is shown in Fig. 2. The 2-D core is the fourth axial slice of the 3-D core. There are 95 fuel bundles in the 2-D core and 1140 fuel bundles in the 3-D core. These are exact representations of the 37 fuel element CANDU-6 bundle as shown in Fig. 3. Specular reflective boundary conditions are used on the west and south surfaces and vacuum is used on the other external surfaces in both configurations. The authors are well aware that periodic rotational symmetry instead of specular reflective boundary condition is more appropriate for an accurate representation of the reactor. However the MCNP code used to obtain the reference solutions does not allow rotational symmetry.

As mentioned earlier, for simplicity, four different bundle averaged burnups (800 kWd/t, 2700kWd/t, 5000 kWd/t, 7000 kWd/t) are chosen to represent a typical CANDU-6 core burnup distribution [10] as closely as possible. Using the lattice depletion code HELIOS [11] with its 47-group production cross section library is used to perform single bundle depletion calculations with specular reflection, These calculations generate detailed (region dependent) two-group macroscopic cross section data for the response function and full core MCNP calculations. To simplify the benchmark problem, spatial homogenization is used to obtain one set of averaged macroscopic cross sections for all fuel pins in the bundle. Homogenization is also used to obtain macroscopic cross sections for the moderator, coolant and cladding. The gap between the pressure and calandria tubes is treated as vacuum in both HELIOS and MCNP.

The 2-D MCNP reference simulation used 350,000 particles per cycle with a total of 3900 active cycles and 100 skipped cycles. This case started from a converged fission source distribution generated with an initial MCNP run using 1.4 billion particles. The reference eigenvalue for this case is 1.01943 ± 0.00001 with an average and a maximum pin fission density uncertainty of 0.06% and 0.10%, respectively. The calculation took approximately 4 days to complete. It took two additional days to converge the fission source.

The 3-D MCNP reference calculation used 600,000 particles per cycle with a total of 2700 active cycles and 800 skipped cycles. This calculation used a converged fission source distribution generated by a separate MCNP simulation with 2.1 billion particles. The reference eigenvalue is 1.02173 ± 0.00001 with an average and a maximum pin fission density uncertainty of 0.24% and 0.59% respectively. This calculation took 3 days to converge the fission source and 7 additional days to generate the reference solution. All reference calculations were performed on a 15-node cluster with dual core 2.8 MHz processors.

For the 2-D coarse mesh (COMET) calculation, low-order response functions are computed using a modified version of MCNP. Each response function calculation uses 3 million active particles. This leads to 6 minutes of MCNP calculation in each unique fuel assembly and 1 minute of run time in the moderator assembly. The COMET results are presented in Tab. 1. It should be noted that all computational times for COMET listed in the table are obtained using a personal computer with a 2.8 GHz processor with 1GB of random access memory.

Table 1: Coarse Mesh Transport Results for the 2-D Benchmark Problem

	{0,0,0}	**{1,0,0}**	**{2,2,2}**
Eigenvalue RE (%)	-0.60	0.41	0.09
Pin fission density AVG (%)	3.0	0.5	0.5
Pin fission density RMS (%)	3.6	0.7	0.6
Pin fission density MRE (%)	2.8	0.5	0.4
Pin fission density MAX (%)	9.2	2.4	2.0
CPU time (s)	2.0	2.9	31.7

RE: Relative Error

AVG: Average relative error

$$avg\ RE = \frac{\sum_N |e_n|}{N}$$

where N is the number of fuel pins and e_n is the calculated per cent error for the nth pin fission density, p_n.

RMS: Root Mean Square Error

$$RMS = \frac{\sqrt{\sum_N e_n^2}}{N}$$

MRE: Mean Relative Error

$$MRE = \frac{\sum_N |e_n| \cdot p_n}{N . p_{avg}}$$

MAX: Maximum Error

{a,b,c}: a^{th} order expansion in space
b^{th} order expansion in cosine(polar angle)
c^{th} order expansion in azimuthal angle

As can be seen from Tab. 1, COMET produces very accurate results in the 2-D problem even with a low-order expansion of the interface angular current. The eigenvalue error is 0.09% with a standard deviation of about 0.03%. The pin fission density average error is 0.5% with a maximum error of 2.0%. The uncertainties associated with the pin results are all around 0.3%. It is also interesting to note that the 0^{th} order expansion in angles with 1^{st} order expansion in space yields a very accurate fission density distribution. The average pin fission density error is also 0.5% with a maximum of 2.4%. As seen from Tab. 1, very accurate pin fission density distribution can be achieved with a very low order expansion set {1,0,0}. In comparison, the computational time of the 2^{nd} order calculation (the last column) is around 32 seconds. Note that the COMET solution can be accelerated by using low-order solutions. Results of this acceleration are presented in Tab. 2.

Table 2: Coarse Mesh Computational Times after Low-Order Acceleration*

	$\{2,2,2\}^{\{0,0,0\}}$	**$\{2,2,2\}^{\{1,0,0\}}$**
CPU time (s)	16.4	18.3

* the {a’,b’,c’} exponent indicates the expansion order of the low-order acceleration

By using the 0^{th} order solution to accelerate the 2^{nd} order calculation, the same accuracy as in Tab. 1 is achieved with a 50% reduction in the computational time.

For the 3-D COMET calculations, each response function is generated using 8 million active particles. Each calculation takes approximately 5 minutes for the lattice cell and 1 minute for the moderator coarse mesh on the same computer mentioned earlier. COMET results for the 3-D benchmark problem are presented in Tab. 3. The precomputation of the 3-D response functions is performed in approximately 3 days on 15 dual-core 2.8 MHz processors.

Table 3: Coarse Mesh Transport Results for the 3-D Benchmark Problem

	{2,2,2,2}	**{4,4,2,4}**
Eigenvalue RE (%)	0.14	0.05
Pin Fission density AVG (%)	0.8	0.7
Pin Fission density RMS (%)	1.0	0.8
Pin Fission density MRE (%)	0.7	0.6
Pin Fission density MAX (%)	3.5	3.3
CPU time (hour)	4.0	21.8

{a,b,c,d}: a^{th} order expansion in space
b^{th} order expansion in space
c^{th} order expansion in cosine of the polar angle
d^{th} order expansion in azimuthal angle

Once again, with a low-order approximation of the interface partial angular currents, very good accuracy is obtained in the 3-D problem. With a 4^{th} order expansion in both spatial variables and the azimuthal angle and a 2^{nd} order expansion in the cosine of the polar angle, the eigenvalue differs from the MCNP reference solution by 0.05%. The pin fission density error distribution is also very accurate. The average relative error is 0.7% with a maximum of 3.3%. The mean relative error, which corresponds to a pin fission density weighted average error, indicates that the higher errors occur in low fission density regions. Out of the 42180 pins in the benchmark problem only 21% of the pins have errors greater than 1% and only 11 pins have errors greater than 3%. The pin fission density uncertainties (one standard deviation) of the COMET solution is on average 0.2% with a maximum of 0.3%

The computational time required for the coarse mesh solution of the 3-D problem is considerably larger than that for the 2-D problem. This difference is attributed mainly to poor memory management in the code and also to a slow convergence rate. However, COMET still offers a substantial computational gain over full core Monte Carlo calculations. In full core Monte Carlo CANDU core calculations, a great deal of computational effort is required to converge the fission source distribution. This is not an issue in COMET since the fission source is implicitly treated in the response functions calculations. Another advantage of COMET is that it can provide solutions with lower statistical uncertainties than the full core Monte Carlo while using response functions generated with substantially smaller number of histories. In the 3-D example, the Monte Carlo reference solution has a maximum pin fission density uncertainty of 0.59%, while the COMET solution has a maximum uncertainty of 0.3%.

4. Conclusion

The coarse mesh transport method presented in this paper decouples the full core problem into a collection of sub-volume elements (*e.g.* coarse meshes). This shifts most of the computation time to a priori calculations of response functions for the unique coarse meshes (fuel bundle types) in the system. That is, the method takes advantage of the repeated structure found frequently in large problems such as nuclear reactor cores (*e.g.* fuel bundles). Response functions can be generated with any available existing stochastic or fine-mesh deterministic codes. In this study, MCNP was chosen to generate the low-order response functions in which the angular currents were expanded in terms of a tensor product of Legendre polynomials.

The method was tested on 2-D and 3-D CANDU-6 benchmark problems using two-group fuel-pin-wise macroscopic cross sections. These problems are typical of a simplified quarter core operating reactor composed of 95 fuel bundles for the 2-D model and 1140 for the 3-D model. COMET is benchmarked against full core MCNP results. For the 2-D core, very accurate results are obtained with a second order Legendre polynomial expansion of the interface currents in the spatial and angular variables. The eigenvalue differs from the reference solution by 0.09 %. The pin fission density average error over all pins is 0.5% with a maximum of 2.0%. Using 0^{th} order acceleration, the 2-D core is solved in less than 17 seconds. For the 3-D core, interface currents are expanded to second order expansion in the cosine of the polar angle and fourth order in the other phase space variables. This approximation yields an eigenvalue error of 0.05%. The pin fission density average error is 0.7% with a maximum error of 3.3%.

Future work will be focused on accelerating the 3-D solution through better memory management, improved sweeping method, optimized programming and response function reduction techniques. The COMET code will also be tested in the case where the adjuster rods are inserted into the core. Also, continuous refueling and variable burnup (interpolation) configurations will be considered in the future.

5. Acknowledgements

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6. References

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7. Appendix

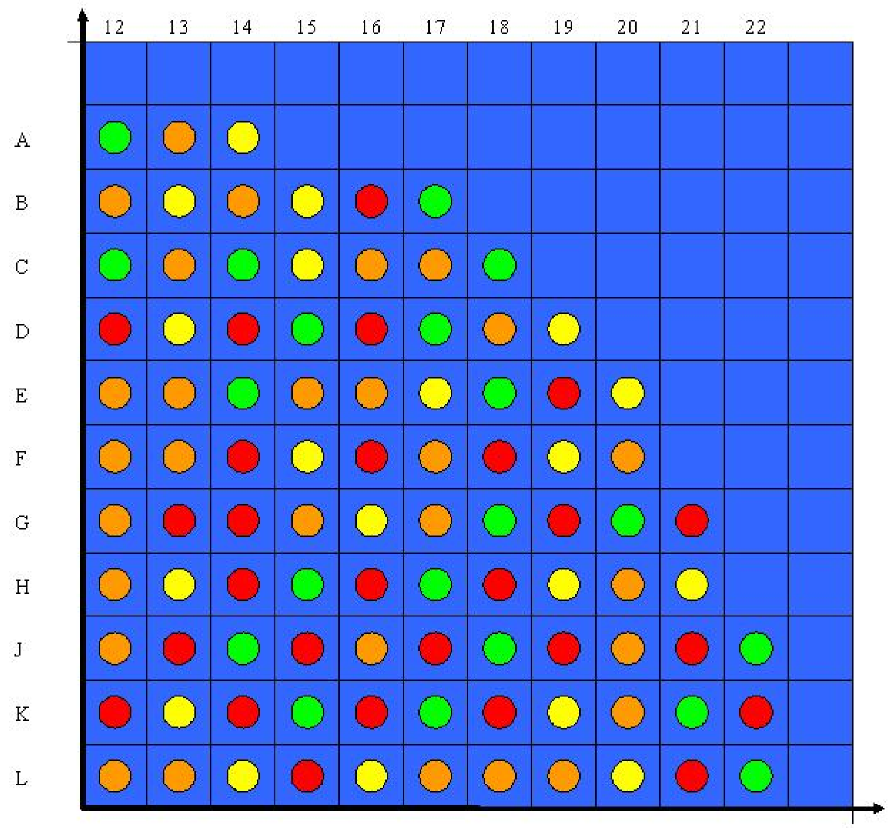

Figure 1: Quarter core representation of the 2-D Benchmark Problem (see Tab. 4 for the burnup distribution scheme)

Axial Plane 1

5	5	5	5	5	5	5	5	5	5	5	5
5	5	5	5	5	5	5	5	5	5	5	5
5	5	5	5	5	5	5	5	5	5	5	5
5	5	5	5	5	5	5	5	5	5	5	5
5	5	5	5	5	5	5	5	5	5	5	5
5	5	5	5	5	5	5	5	5	5	5	5
5	5	5	5	5	5	5	5	5	5	5	5
5	5	5	5	5	5	5	5	5	5	5	5
5	5	5	5	5	5	5	5	5	5	5	5
5	5	5	5	5	5	5	5	5	5	5	5
5	5	5	5	5	5	5	5	5	5	5	5
5	5	5	5	5	5	5	5	5	5	5	5

Axial Plane 2

5	5	5	5	5	5	5	5	5	5	5	5
1	4	1	5	5	5	5	5	5	5	5	5
4	1	4	1	4	1	5	5	5	5	5	5
1	3	1	3	1	3	1	5	5	5	5	5
4	1	4	1	4	1	4	1	5	5	5	5
1	3	1	4	1	3	1	4	1	5	5	5
4	1	4	1	4	1	3	1	4	5	5	5
2	4	2	4	1	4	1	4	1	4	5	5
4	1	4	1	4	1	4	1	4	1	5	5
2	4	1	4	1	4	1	4	1	4	1	5
4	1	4	1	4	1	4	1	4	1	4	5
1	4	1	4	1	4	1	4	1	4	1	5

Axial Plane 3

5	5	5	5	5	5	5	5	5	5	5	5
1	4	2	5	5	5	5	5	5	5	5	5
4	2	4	1	4	1	5	5	5	5	5	5
1	3	1	3	2	3	1	5	5	5	5	5
4	2	4	1	4	1	4	1	5	5	5	5
2	3	1	4	2	3	1	4	2	5	5	5
4	2	4	2	4	2	4	1	3	5	5	5
3	4	3	3	2	3	1	4	1	4	5	5
4	1	4	1	4	1	4	2	4	2	5	5
3	4	1	4	2	4	1	4	2	4	1	5
4	2	4	1	4	1	4	1	4	1	4	5
2	4	1	4	2	3	2	4	2	4	1	5

Axial Plane 4

5	5	5	5	5	5	5	5	5	5	5	5
1	3	2	5	5	5	5	5	5	5	5	5
3	2	3	2	4	1	5	5	5	5	5	5
1	3	1	2	3	3	1	5	5	5	5	5
4	2	4	1	4	1	3	2	5	5	5	5
3	3	1	3	3	2	1	4	2	5	5	5
3	3	4	2	4	3	4	2	3	5	5	5
3	4	4	3	2	3	1	4	1	4	5	5
3	2	4	1	4	1	4	2	3	2	5	5
3	4	1	4	3	4	1	4	3	4	1	5
4	2	4	1	4	1	4	2	3	1	4	5
3	3	2	4	2	3	3	3	2	4	1	5

Axial Plane 5

5	5	5	5	5	5	5	5	5	5	5	5
1	3	3	5	5	5	5	5	5	5	5	5
3	3	2	2	4	1	5	5	5	5	5	5
1	2	1	1	3	3	1	5	5	5	5	5
3	2	4	2	3	1	2	2	5	5	5	5
3	2	1	3	3	2	1	3	3	5	5	5
2	3	3	3	4	3	3	2	2	5	5	5
4	4	4	2	2	2	2	4	1	4	5	5
2	2	3	1	3	1	4	2	2	3	5	5
3	4	1	4	3	4	1	3	3	4	2	5
2	3	3	1	3	1	3	2	3	1	3	5
3	2	2	4	2	2	4	2	2	4	1	5

Axial Plane 6

5	5	5	5	5	5	5	5	5	5	5	5
2	3	3	5	5	5	5	5	5	5	5	5
2	3	1	2	3	1	5	5	5	5	5	5
1	2	1	1	3	3	1	5	5	5	5	5
3	3	3	2	3	2	2	2	5	5	5	5
4	2	1	2	3	1	1	3	3	5	5	5
2	3	2	3	3	4	3	2	2	5	5	5
4	3	4	1	2	2	2	4	1	3	5	5
1	2	2	1	3	1	4	2	1	3	5	5
4	4	1	3	3	4	1	3	3	4	2	5
2	3	2	1	3	2	3	2	2	1	3	5
3	1	2	3	2	1	4	2	3	3	1	5

Axial Plane 7

5	5	5	5	5	5	5	5	5	5	5	5
2	3	3	5	5	5	5	5	5	5	5	5
2	3	1	2	3	2	5	5	5	5	5	5
1	2	1	1	4	3	1	5	5	5	5	5
3	3	3	2	3	2	2	2	5	5	5	5
4	2	1	2	4	1	1	3	3	5	5	5
2	3	2	3	3	4	3	2	2	5	5	5
4	3	4	1	2	2	2	4	1	4	5	5
1	2	2	1	3	1	4	3	1	4	5	5
4	4	2	3	3	4	1	3	4	4	2	5
2	3	2	1	3	2	3	2	2	1	4	5
3	1	2	4	2	1	4	2	3	4	1	5

Axial Plane 8

5	5	5	5	5	5	5	5	5	5	5	5
2	3	3	5	5	5	5	5	5	5	5	5
2	3	1	2	3	1	5	5	5	5	5	5
1	2	1	1	4	3	1	5	5	5	5	5
3	3	3	2	3	2	2	2	5	5	5	5
4	2	1	2	4	1	1	3	3	5	5	5
2	3	2	3	3	4	3	2	2	5	5	5
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1	2	2	1	3	1	4	3	1	4	5	5
4	4	2	3	3	4	1	3	4	4	2	5
2	3	2	1	3	2	3	2	2	1	4	5
3	1	2	4	2	1	4	2	3	4	1	5

Axial Plane 9

5	5	5	5	5	5	5	5	5	5	5	5
2	3	3	5	5	5	5	5	5	5	5	5
2	3	1	2	3	1	5	5	5	5	5	5
1	2	1	1	3	3	1	5	5	5	5	5
3	3	3	2	3	2	2	2	5	5	5	5
4	2	1	2	3	1	1	3	3	5	5	5
2	3	2	3	3	4	3	2	2	5	5	5
4	3	4	1	2	2	2	4	1	3	5	5
1	2	2	1	3	1	4	2	1	3	5	5
4	4	1	3	3	4	1	3	3	4	2	5
2	3	2	1	3	2	3	2	2	1	3	5
3	1	2	3	2	1	4	2	3	3	1	5

Axial Plane 10

5	5	5	5	5	5	5	5	5	5	5	5
2	2	3	5	5	5	5	5	5	5	5	5
2	3	1	3	3	2	5	5	5	5	5	5
2	2	1	1	4	2	1	5	5	5	5	5
3	3	3	2	2	2	2	2	5	5	5	5
4	2	2	2	4	1	2	3	3	5	5	5
1	4	2	3	3	4	3	2	2	5	5	5
4	3	4	1	3	1	2	3	2	3	5	5
1	2	2	2	2	2	3	3	1	4	5	5
4	4	2	3	4	4	2	3	4	3	2	5
2	4	2	2	3	2	2	3	2	1	3	5
4	1	3	4	3	1	4	2	3	3	1	5

Axial Plane 11

5	5	5	5	5	5	5	5	5	5	5	5
2	2	4	5	5	5	5	5	5	5	5	5
2	3	1	3	2	2	5	5	5	5	5	5
3	2	2	1	4	2	2	5	5	5	5	5
2	4	3	3	2	3	1	3	5	5	5	5
4	2	3	2	4	1	2	2	3	5	5	5
1	4	2	4	3	4	2	3	1	5	5	5
4	3	4	1	4	1	3	3	2	2	5	5
1	2	2	3	2	2	3	3	1	4	5	5
4	4	3	3	4	4	3	2	4	3	3	5
1	4	2	3	2	3	2	3	2	2	2	5
4	1	4	3	4	1	4	1	4	2	2	5

Axial Plane 12

5	5	5	5	5	5	5	5	5	5	5	5
3	1	4	5	5	5	5	5	5	5	5	5
1	4	1	4	2	4	5	5	5	5	5	5
4	1	3	1	4	2	3	5	5	5	5	5
2	4	2	3	2	3	1	3	5	5	5	5
4	1	3	1	4	1	3	2	4	5	5	5
1	4	2	4	2	4	2	4	1	5	5	5
4	2	4	1	4	1	4	2	3	2	5	5
1	2	1	3	2	3	2	4	1	4	5	5
4	3	4	2	4	3	3	2	4	2	4	5
1	4	2	4	2	4	2	4	1	3	2	5
4	1	4	3	4	1	4	1	4	2	3	5

Axial Plane 13

5	5	5	5	5	5	5	5	5	5	5	5
3	1	4	5	5	5	5	5	5	5	5	5
1	4	1	4	1	4	5	5	5	5	5	5
4	1	3	1	4	1	3	5	5	5	5	5
1	4	1	4	1	4	1	4	5	5	5	5
4	1	4	1	4	1	3	1	4	5	5	5
1	4	1	4	1	4	1	4	1	5	5	5
4	1	4	1	4	1	4	1	4	1	5	5
1	1	1	4	1	3	1	4	1	4	5	5
4	2	4	1	4	2	3	1	4	1	4	5
1	4	1	4	1	4	1	4	1	3	1	5
4	1	4	1	4	1	4	1	4	1	4	5

Axial Plane 14

5	5	5	5	5	5	5	5	5	5	5	5
5	5	5	5	5	5	5	5	5	5	5	5
5	5	5	5	5	5	5	5	5	5	5	5
5	5	5	5	5	5	5	5	5	5	5	5
5	5	5	5	5	5	5	5	5	5	5	5
5	5	5	5	5	5	5	5	5	5	5	5
5	5	5	5	5	5	5	5	5	5	5	5
5	5	5	5	5	5	5	5	5	5	5	5
5	5	5	5	5	5	5	5	5	5	5	5
5	5	5	5	5	5	5	5	5	5	5	5
5	5	5	5	5	5	5	5	5	5	5	5
5	5	5	5	5	5	5	5	5	5	5	5

Figure 2: 3-D Core layout (see Tab. 4 for the burnup distribution scheme)

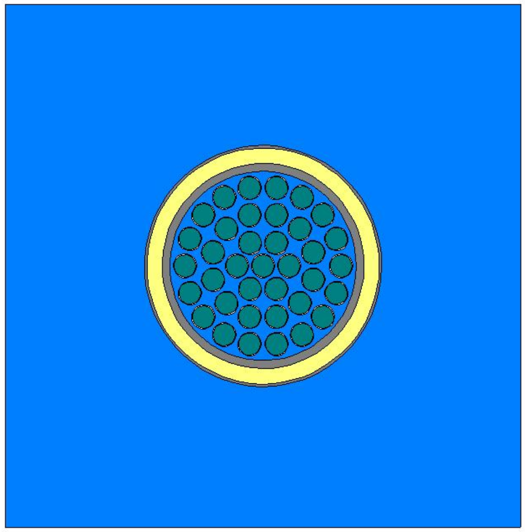

Figure 3: CANDU-6 Lattice Representation

Table 4: Burnup Distribution Scheme for Benchmark Problems

Color Scheme (2-D Benchmark)	Numbering Scheme (3-D Benchmark)	Burnup (kWd/t)	Material Properties
Green	1	800	Fuel
Yellow	2	2700	Fuel
Orange	3	5000	Fuel
Red	4	7000	Fuel
Blue	5	---	Moderator