

## DERIVATION OF $\nu_d$ DATA FROM $\beta_{eff}$ MEASUREMENTS: INTEREST AND LIMITS

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### ABSTRACT

The  $\nu_d$  data for the three important isotopes,  $^{235}\text{U}$ ,  $^{238}\text{U}$ , and  $^{239}\text{Pu}$ , included in the JEF2 library have been calculated, as a function of energy, by using the so-called LENDEL's model. In the framework of a general benchmarking of the JEF2 file, these evaluated data have been used to calculate 22 effective delayed neutron fraction ( $\beta_{eff}$ ) measurements in the thermal and fast energy ranges.

In order to make the calculated  $\beta_{eff}$  value dependent on the  $\nu_d$  values alone, the ERALIB1 set of cross section data adjusted for the main nuclear parameters but the  $\nu_d$  has been used in the neutronics calculations for these 22 benchmarks.

A reanalysis of the experimental conditions has been performed in order to produce a realistic estimate of the covariance matrix on the one hand and consistent values for the calculated part  $Pc$  of the "experimental" value of  $\beta_{eff}$  on the other hand. It appears that all  $\beta_{eff}$  data but one (experiment SNEAK 9C2) are reproduced by the calculation using the JEF2  $\nu_d$  data with a discrepancy lower than 3%, which is the performance required by Reactor Physicists.

A statistically consistent adjustment procedure has been used to quantify the small corrections to apply to the evaluated  $\nu_d$  data.

Recommended  $\nu_d$  values have been derived for  $^{235}\text{U}$ ,  $^{238}\text{U}$  and  $^{239}\text{Pu}$ . They are affected by an uncertainty lower than 3% except for  $^{238}\text{U}$  (~4%). This result raises the question of the capability of current calculational systems to calculate accurately the  $^{238}\text{U}$  fission rates.

Further measurements and basic investigations are highly encouraged to improve the situation for the higher Pu isotopes and the minor Actinides.

## 1. INTRODUCTION

The experimental delayed neutron yield ( $\nu_d$ ) data are not very numerous and they are somewhat inaccurate. For example the uncertainties given by KRICKS and EVANS [18] for Pu-239, U-235 and U-238 are of the order of 10%, while the requirements are for 3%.

In addition, the rare models to calculate them are not based on a fine description of the physical phenomenon which produces the delayed neutrons but instead on a set of semi-phenomenological laws in which the present knowledge is concentrated. Consequently these models do not have the predictive capabilities which are required by Reactor Physicists.

The need for an additional source of information is therefore obvious. This justifies, among other arguments, the use of integral data, in particular effective delayed neutron fraction ( $\beta_{eff}$ ) measurements via an adjustment procedure. The first practical demonstrations of the performances of this procedure can be found in the numerous works by D'ANGELO and FILIP [1].

However, for that purpose, several strict conditions have to be observed, some of them imposed by the diversity of these integral data:

- The neutronics calculations have to be performed with consistent and well-validated codes and nuclear data sets,
- the integral data have to be measured in 'clean' cores and be as numerous and as independent of one another as possible,
- different techniques should be employed to identify and quantify possible systematic errors,
- the uncertainties should be assessed in a realistic way with clear identification of systematic and statistical components.

Apart from the most recent data obtained in cooperative experimental programs (BERENICE, FCA) , the last two conditions are generally poorly respected. As a matter of fact, the 'old' data have been obtained at different periods of time , in different contexts. The uncertainties have not always been properly estimated. In addition it has to be noted that the so-called experimental  $(\beta_{eff})_{exp}$  values necessarily include calculated parameters, so that they are partly calculational model dependent.

In order to derive recommended  $\nu_d$  data by a statistical adjustment technique, we first had to prepare a homogeneous set of integral data and a realistic integral covariance matrix. So we had to carefully reanalyse the primary data related to the experimental values and their uncertainties and to use a single calculational procedure to quantify the calculated parts ( $P_c$ ) of  $(\beta_{eff})_{exp}$ .

This task, essentially based on a recent thesis work [2], is described here. We present also recommended  $\nu_d$  data for U-235,U-238 and Pu-239 in the thermal and fast ranges. The  $\nu_d$

values for these 3 major nuclei represent all the information provided by the presently available integral data base. In particular, it was not possible to derive any information about the higher Pu isotopes even though they are important nuclei in some applications. In the same way, if the minor actinides become important for future applications, they should also be the target of an experimental effort.

Since  $\beta_{eff}$  measurements are difficult and costly experiments, the integral data base will never be of a large size so that the adjustment results will bear a strong dependence on the a priori evaluated data which have still to be improved. Such improvements can be achieved by new measurements, renewed interest on systematics, and a larger effort for better predictive, Physics-based models.

It appears that part of the required efforts are underway today (see separate contributions to this conference).

## 2. CALCULATIONAL METHODS

The well accepted KEEPIN's formalism [20] is generally adopted to calculate the  $\beta_{eff}$  as a quasi-static parameter (independent of  $\alpha_i, \lambda_i$ , see Equation 9 below):

$$\beta_{eff} = \frac{\sum_i \overline{v_{d_i}} \int_V \left\{ \int_0^\infty \sum_{f_i} (E, \vec{r}) \cdot \Phi(E, \vec{r}) dE \cdot \int_0^\infty \chi_{d_i} (E') \cdot \Phi^* (E', \vec{r}) dE' \right\} d\vec{r}}{\sum_i \int_V \left\{ \int_0^\infty v_{t_i} (E) \cdot \sum_{f_i} (E, \vec{r}) \cdot \Phi(E, \vec{r}) dE \cdot \int_0^\infty \chi_{t_i} (E') \cdot \Phi^* (E', \vec{r}) dE' \right\} d\vec{r}} \quad (1)$$

$\beta_{eff}$  depends on the various average  $\overline{v_{d_i}}$  and on neutronics parameters, such as the neutron flux  $\Phi(E, \vec{r})$ , its adjoint  $\Phi^*(E, \vec{r})$  and the fission cross sections  $\Sigma_{f_i}(E, \vec{r})$  at each point  $\vec{r}$  and for each fissioning nuclide  $i$ , which are basic characteristics of a reactor core. In other words, a correct calculation of ( $\beta_{eff}$ ) requires a correct calculation of the whole core. If these neutronics parameters are calculated with validated codes and data sets with minimized biases and uncertainties (like data sets adjusted on large and complete integral data bases), then the errors in the  $\beta_{eff}$  calculation become primarily dependent on errors in the  $\overline{v_{d_i}}$ : any discrepancy in a calculated  $\beta_{eff}$  with respect to its measured value can be directly related to discrepancies in the  $\overline{v_{d_i}}$  values. This is the basic assumption made here.

All the experiments have been calculated with the ERANOS-ERALIB1 [3],[4] system of codes and data especially designed for fast systems calculations but having also demonstrated good performances in the thermal range. The ERALIB1 data set, in 1968 groups, has been derived from the JEF2 library by a statistical adjustment on an integral data base made of 364 data which included in particular all the cores reviewed in this

$(\beta_{eff})_{exp}$  analysis with the exception of MISTRAL1, MISTRAL2 and FCA cores. For thermal systems, the ERANOS calculations were checked against APOLLO2 [5] calculations. Both APOLLO2 and ERANOS systems of codes are based on deterministic methods. Although checks with respect to Monte-Carlo methods have been made as often as possible and in spite of the large number of integral data used to adjust ERALIB1, we cannot totally exclude the presence of some (hopefully minimized) biases, especially for what concerns the rates of threshold fission cross-sections like  $\Sigma_f$  for U-238.

The performances of the ERANOS-ERALIB1 system in the fast range are excellent, as demonstrated by critical mass and reaction rate predictions :

-The critical mass is predicted, on an average, with a bias of  $80 \pm 130 pcm$  and an uncertainty of  $180 pcm$  ( $1 \sigma$ ).

- The spectral indices are differently predicted depending on whether threshold reactions are involved or not. For example :

a/ for the Pu-239 fission over U-235 fission index  $\frac{F9}{F5}$  we obtain:

bias = -0.0016 ; uncertainty = 0.019

b/ for the U-238 fission over U-235 fission index  $\frac{F8}{F5}$  :

bias = -0.005 ; uncertainty = 0.028

( The sign - means an overprediction)

On the average the F8 fission is slightly over-predicted but the uncertainty is quite high. This means that some F8 rates are over-predicted and some are under-predicted. So when speaking about U-238 fission, one has to be specific and mention explicitly the core and the position of the measurement.

### 3. INTEGRAL DATA

In order to validate the  $\nu_d$  data, we used 22 integral data which can be divided into 2 different subsets:

- A subset of recent experiments comprising :
  - the so called MISTRAL1 and MISTRAL2 experiments performed in the framework of a joint French-Japanese programme in the EOLE facility. They bring valuable information in the thermal range on the  $\nu_d$  of the major nuclei : U-235, U-238, Pu-239.
  - the experiments performed also in the framework of an international cooperation specifically set up with the objective to validate the  $\nu_d$  data for the major nuclei in the fast range. These are the BERENICE and FCA programmes. The experiments have been performed in MASURCA (CEA/Cadarache) and FCA (JAERI/Tokai-Mura) facilities.  $(\beta_{eff})_{exp}$  data have been obtained and analyzed by different teams using different experimental techniques. In some cases, for the same core, the same

technique (Californium source method or noise method) was applied by different teams. These were very informative situations.

- A subset of less recent experiments:

They were performed in the SNEAK (Karlsruhe, Germany) and ZPR (Argonne, USA) facilities for those which are related to the fast energy range; a Japanese experiment, SHE8, very representative of the Japanese effort in the thermal range, was also considered.

The main characteristics of this data base is to be essentially populated with fast reactor data and it would have been worthwhile to include more thermal data ,in particular those of the SHE program. Unfortunately this was not possible for a simple reason: lack of manpower.

Among the 22 experiments, 11 involve U isotopes solely, while one involves Pu isotopes solely (PuCSS). The 10 remaining experiments give information on U and Pu isotopes together. The data were obtained by the following experimental techniques :

- Californium Source : Experiments R2, ZONA 2 performed in MASURCA  
7A, 7B, 9C1, 9C2 performed in SNEAK  
XIX-I,XIX-3 performed in FCA.
- Covariance : Experiments CRef, PuCSS, RSR, U9, UFe-Ref,  
UFe-Leak performed in ZPR  
XIX-I,XIX-3 performed in FCA.
- Frequencies : Experiments XIX-I, XIX-3 performed in FCA  
MISTRAL1, MISTRAL2 performed in  
EOLE  
R2, ZONA2 performed in MASURCA
- $\alpha$ -Rossi : Experiments R2 performed in MASURCA  
XIX-I, XIX-3 performed in FCA.

The experiments R2 and ZONA2 performed in MASURCA and also the experiments (XIX-1,XIX-2 and XIX-3) performed in FCA in the framework of the BERENICE international benchmark program involving experienced teams (IPPE, FCA, CEA) are particularly attractive since 3 or more different measurement techniques were used. In addition, interesting situations occurred when the same technique was used for the same core (Californium source for R2 and ZONA2, Frequencies method for XIX-1 and XIX-3). For these situations, systematic differences (up to 4%) are observed in the experimental

component  $P_m$  of the  $(\beta_{eff})_{exp}$  values, the data proposed by the Cadarache team being the highest. However, these differences do not alter the general statistical consistency of the data. In a general way, the analysis of the differences in  $(\beta_{eff})_{exp}$  could help to quantify the systematic errors relevant to each experimental method. All the experiments considered in the integral data base can be considered as 'clean'. A few depart more than the others from this required quality, though. This is, in particular, the case of the FCA experiments which are characterized by more cell heterogeneity effects, a situation that requires careful calculations leading to corrective factors which significantly depart from unity, especially for what concerns the spectral indices F8/F5 and F8/F9 [6].

For each experimental technique, the so called «experimental»  $(\beta_{eff})_{exp}$  value actually depends on a calculated part, so that  $(\beta_{eff})_{exp}$  may be written as:

$$(\beta_{eff})_{exp} = P_m \cdot P_c \text{ for the Californium source method}$$

or  $(\beta_{eff})^2 = P_m \cdot P_c$  for the Frequencies,  $\alpha$ -Rossi and Covariance methods.

$P_m$  and  $P_c$  represent, respectively, the measured and the calculated parts in the expression of  $(\beta_{eff})_{exp}$ . A complete definition of the  $P_m$  and  $P_c$  terms can be found in the References [2] and [7].

For the same method, the  $P_c$  term ( and hence the  $P_m$  term) may differ according to the particular implementation of the experimental method. For example, concerning the method of Frequencies, the  $P_c$  term used in the BERENICE program is expressed as :

$$P_c = \frac{2D \sum_i F_i^{cal}(\vec{0})}{F} \quad (2)$$

(  $D$  :complete DIVEN factor,  $F_i^{cal}(\vec{0})$  : isotope i fission rate at the core center,  $F$  total fission rate over the whole core.)

while it is expressed as follows in the MISTRAL experiments :

$$P_c = \frac{\sigma_{fU235}^{FC}}{\sigma_{fU235}^{core}} \cdot a \cdot b \cdot D_v \cdot \frac{F^{cell}}{F_{U235}^{FC}} \quad (3)$$

(  $D_v$  :component of the DIVEN factor related to the prompt neutron emission ; a, b :axial and radial form factors ;  $F^{cell}$  :total fission rate divided by the number of cells ;  $F_{U235}^{FC}$  : fission rate in the fission chamber in the center of the core )

The  $\sigma$ 's are the average fission cross-sections in the fission chamber or over the whole core). This diversity illustrates the amount of work needed to revisit the published  $(\beta_{eff})_{exp}$  data, a task all the more difficult as the published information is scarce.

It is important that  $(\beta_{eff})$  and the  $Pc$  term of  $(\beta_{eff})_{exp}$  be calculated by using the same calculational tools for the sake of internal consistency necessary in an adjustment procedure.

A careful analysis of the experimental methods and published data [2] led to revisited uncertainties and modified  $(\beta_{eff})_{exp}$  values through a recalculation of the  $Pc$  terms. This work proved to be essential since it provided the ingredients to construct a realistic covariance matrix associated to a consistent integral data base. The resulting set of  $(\beta_{eff})_{exp}$  and associated standard deviation values is displayed in Table 1:

		$\beta_{eff}$ (pcm)	Standard deviation (%)
<b>R2</b>	Cf Source	723.5 (739, 708)	<b>3.4</b>
	Frequencies	726.4	<b>2.3</b>
	$\alpha$ -Rossi	745.0	<b>1.8</b>
<b>ZONA2</b>	Cf. Source	353.7 (358.6, 348.7)	<b>3.5</b>
	Frequencies	350.0	<b>2.2</b>
<b>7A</b>	Cf. Source	395.0	<b>2.8</b>
<b>7B</b>	Cf. Source	429.0	<b>2.8</b>
<b>9C1</b>	Cf. Source	748.0	<b>4.2</b>
<b>9C2</b>	Cf. Source	416.0	<b>4.6</b>
<b>Cref</b>	Covariances	383.6	<b>2.2</b>
<b>PuCSS</b>	Covariances	223.4	<b>2.3</b>
<b>RSR</b>	Covariances	337.3	<b>2.2</b>
<b>U9</b>	Covariances	712.2	<b>2.1</b>
<b>UFeRef</b>	Covariances	670.8	<b>2.1</b>
<b>UFeLeak</b>	Covariances	675.8	<b>2.1</b>
<b>XIX-1</b>	Frequencies	742 (742,742)	<b>2.4</b>
<b>XIX-3</b>	Frequencies	249.1 (252, 246.2)	<b>2.5</b>
<b>MISTRAL-1</b>	Frequencies	789.7	<b>1.6</b>
<b>MISTRAL-2</b>	Frequencies	372.5	<b>1.6</b>
<b>SHE-8</b>	Cinetique	696.0	<b>4.6</b>

Table 1 : Recommended Values for  $(\beta_{eff})_{exp}$  and Uncertainties.

Note that the data in parentheses refer to values respectively obtained by CEA and FCA in the international benchmark programme already mentioned. The recommended values are arithmetic averages.

The uncertainties have been (re)calculated by using the error propagation law, the re-estimated uncertainties in the primary parameters and the sensitivity coefficients. Calculation details (indirect effect terms ignored) can be found in Reference [2].

The complete integral covariance matrix  $[V_{\beta_{eff}^{exp}}]$  is expressed as :

$$[V_{\beta_{eff}^{exp}}] = \sum_k V_{\beta_{eff}^{exp}}^{meas,k} + \sum_l V_{\beta_{eff}^{exp}}^{calc,l} \quad (4)$$

In this expression  $\sum_k V_{\beta_{eff}^{exp}}^{meas,k}$  is the sum of the covariance matrices related to the k measured parameters ( the Californium source  $S_{Cf}$ , the reactivity change  $\Delta\rho$ , the fission rate in the center of the core  $F_{ref}^{exp}(\vec{0})$ , the spectral power density DSP,.....), while  $\sum_l V_{\beta_{eff}^{exp}}^{calc,l}$  is the sum of the covariance matrices related to the calculated parameters (the relative Integral of normalization INR, the neutron  $D_v$  and spatial  $D_s$  components of the DIVEN factor, the so called  $K_{cal} = \frac{F_{ref}^{cal}}{F}$  factor) .

Off-diagonal terms exist in both sets of covariance matrices. As a matter of fact :

- A full correlation exists between the errors in  $Pc$  terms.
- The following rules have been adopted to establish the covariances related to the measured parameters :
  - Full correlation for the fission rates measured in the same reactor by the same team using the same detector. These rates are so high that the statistical part of them can be neglected.
  - Full correlation to a part of the uncertainty in  $S_{Cf}$ , part taken as 1% when not mentioned otherwise.

No correlation has been considered for the uncertainties in the other parameters, with the consequence of a possible slight underestimation of the covariance terms of the ‘measured’ matrices.

For a better readability,  $[V_{\beta_{eff}^{exp}}]$  is presented in the following relationship:

$$[V_{\beta_{eff}^{exp}}] = [\Sigma].[CORR].[\Sigma] \quad (5)$$

$[\Sigma]$  stands for the diagonal standard deviation matrix whose terms can be found in Table1.

The correlation matrix  $[CORR]$ , of Table 2, is straightforwardly obtained from the previous equation .

Legend :

- |              |                |                        |            |               |              |              |
|--------------|----------------|------------------------|------------|---------------|--------------|--------------|
| 7            | 2: R2Fréq      | 3 : R2- $\alpha$ Rossi | 4 : Z2-Cf  | 5 :Z2Fréq     | 6 : 7A       | 7 : 7B       |
| 8 : 9C1      | 9 : 9C2        | 10 : Cref              | 11 : PuCSS | 12 : RSR      | 13 : U9      | 14 : UFeRef  |
| 15 : UfeLeak | 16 : Mistral-1 | 17 : Mistral-2         | 18 : She8  | 19 :XIX-1Freq | 20:XIX-3Freq | 21:9C2-U8/U5 |
| 22:9C2Pu9/U5 |                |                        |            |               |              |              |

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	
1	1.	0.28	0.36	0.5	0.29	0	0.	0.	0.														
2		1.	0.65	0.27	0.6														0.3	0.33			
3			1.	0.35	0.74														0.38	0.43			
4				1.	0.29																		
5					1.														0.31	0.35			
6						1.	0.7	0.46	0.43														
7							1.	0.46	0.43														
8								1.	0.28														
9									1.														
10										1.	0.83	0.83	0.79	0.82	0.82	0.4	0.4						
11											1.	0.85	0.80	0.84	0.84	0.44	0.44						
12												1.	0.8	0.83	0.83	0.42	0.42						
13													1.	0.77	0.77	0.35	0.35						
14														1.	0.83	0.29	0.29						
15															1.	0.38	0.38						
16																1.	0.6						
17																	1.						
18																		1.					
19																			1.	0.6			
20																				1.			
21																						1	0.2
22																							1.

Table 2 : Uncertainty Correlation Matrix for the  $(\beta_{eff})_{exp}$  Values

#### 4. MICROSCOPIC DATA AND ( $\beta_{eff}$ ) CALCULATIONS

Two sets of microscopic data have been considered for U-235, U-238 and Pu-239: JEF2.2 and ENDF/B-VI. This last set presents a rough (classical) energy dependence illustrated by three segments of a straight line. In JEF2.2 the energy dependence is more finely described since LENDEL's model [8] has been used. This semi-empirical model corresponds to an average description of the macroscopic physical effects, some of them (the odd-even effect as an example) being energy dependent. In addition, it takes into account an explicit competition between the prompt and the delayed neutron emission. (Actually, for U-238 JEF2.2 has been taken from JENDL3. Calculations performed with LENDEL's model produced very close results).

In anticipation of future adjustments, the pointwise  $\nu_d(E)$  data have been treated in 5 energy groups in the interval :  $10^{-5} eV - 19.68 MeV$  .

These groups are :

Group number	Boundaries	Range of expected information
Group 1	$10^{-5} eV - 10 KeV$	: Thermal and epithermal range (LWR).
Group 2	$10 KeV - 500 KeV$	:Fast reactor range + fission spectrum (FBR).
Group 3	$500 KeV - 4 MeV$	:Fission spectrum, fast reactor range (Spheres, FBR).
Group 4	$4 MeV - 7 MeV$	:1st and 2nd chance fission competition (Spheres)
Group 5	$7 MeV - 20 MeV$	:Multichance fission competition-Poor information

The standard deviations derived from LENDEL's parameters uncertainties and taking into account the  $\nu_d(E)$  uncertainties given in ERALIB1 are so high (13%-22%) that they have not been considered.

The error bars of the 3 main isotopes have been estimated on the basis of BLACHOT's estimations in the thermal range [9] and have been increased to take into account both :

- the maximum spread between the different evaluations, namely 2% for U-235, 2.5 % for U-238, 3% for Pu-239

- the energy dependence of the uncertainty due to the various fission chances.

For the higher Pu isotopes we referred to the experimental information only.

The adopted a priori uncertainties are quoted in Table 4 below :

Isotopes	group 1 0 - 10 keV	group 2 10 keV-500 keV	group 3 500 keV - 4 MeV	group 4 4 MeV - 7 MeV	group 5 7 MeV - 20 MeV
<b>U235</b>	3. %	3 %	4. %	6. %	7. %
<b>U238</b>	6. %	6. %	7. %	9. %	10. %
<b>Pu239</b>	4 %	4 %	5. %	7. %	8. %
<b>Pu240</b>	15 %	15 %	16 %	18 %	19 %
<b>Pu241</b>	12 %	12 %	13 %	15 %	16 %
<b>Pu242</b>	20 %	20 %	21 %	23 %	24 %

Table 4: Standard deviations by energy group assigned to the reviewed nuclei

The same correlation matrix has been assigned to each nucleus.

Group	1	2	3	4	5
1	1.	0.20	0.04	0.0	0.0
2	0.20	1.	0.20	0.02	0.0
3	0.04	0.20	1.	0.10	0.02
4	0.0	0.02	0.10	1.	0.05
5	0.0	0.0	0.02	0.05	1.

Tableau 5 : Standard Deviation Correlation Matrix for any Nucleus.

The medium range correlations, which have been assumed, are justified by the following arguments :

- the use of a model,
- the experimental technique to measure  $\nu_d(E)$  is the same in the thermal and the fast ranges with the same systematic errors,
- $\nu_d(E)$  is anti-correlated to  $\nu_p(E)$  which is calculated on a theoretical basis and exhibits very long range correlations.

In addition, a full correlation was assumed inside a group. This is not strictly true for Group 1 which covers both the thermal and epithermal ranges since the fluctuations, which are suspected in the resonance range, are related to variations in the fission yields for each resonance and cannot be predicted from the  $\nu_d$  thermal value.

To take into account the energy dependence of  $\nu_d(E)$ , KEEPIN's original formalism has been modified by A.FILIP [10] in a way that keeps the basic meaning of ( $\beta_{eff}$ ) as the ratio of the delayed neutron and the total neutron productions.

$$\beta_{eff} \equiv \sum_i \beta_{eff_i} = \frac{\sum_i \int_V \left\{ \int_0^\infty v_{d_i}(E) \cdot \sum_{f_i}(E, \vec{r}) \cdot \Phi(E, \vec{r}) dE \cdot \int_0^\infty \chi_{d_i}(E') \cdot \Phi^*(E', \vec{r}) dE' \right\} d\vec{r}}{\sum_i \int_V \left\{ \int_0^\infty v_{t_i}(E) \cdot \sum_{f_i}(E, \vec{r}) \cdot \Phi(E, \vec{r}) dE \cdot \int_0^\infty \chi_{t_i}(E') \cdot \Phi^*(E', \vec{r}) dE' \right\} d\vec{r}} \quad (6)$$

The 22 integral data have been calculated with both the JEF2.2 and ENDF.B-VI data, using this formalism.

The differences between experimental (E) (Table 1) and calculated (C) data are given in Table 6 (JEF2.2) and in Table 7 (ENDF/B-VI).

Mock-up	Technique	C (pcm)	E (pcm)	(E-C)/C (%)
<b>R2</b>	Cf. Source	741.2 ± 3.4%	723.5 ± 3.4 %	<b>-2.38 ± 4.8 %</b>
<b>R2</b>	Frequencies	741.2 ± 3.4%	726.4 ± 2.3 %	<b>-1.84 ± 4.1 %</b>
<b>R2</b>	α-Rossi	741.2 ± 3.4%	745.0 ± 1.8 %	<b>0.51 ± 3.8 %</b>
<b>Zona2</b>	Source Cf.	348.7 ± 3.8 %	353.7 ± 3.5 %	<b>1.43 ± 5.2 %</b>
<b>Zona2</b>	Frequencies	348.7 ± 3.8 %	350.0 ± 2.2 %	<b>0.37 ± 4.4 %</b>
<b>7A</b>	Cf. Source	387.5 ± 4.2 %	395.0 ± 2.8 %	<b>1.94 ± 5.0 %</b>
<b>7B</b>	Cf. Source	437.6 ± 4.5%	429.0 ± 2.8 %	<b>-1.98 ± 5.3 %</b>
<b>9C1</b>	Cf. Source	748.4 ± 3.5%	748.0 ± 4.2 %	<b>-0.06 ± 5.5 %</b>
<b>9C2</b>	Cf. Source	399.1 ± 4.1%	416.0 ± 4.6 %	<b>4.23 ± 6.2 %</b>
	$\frac{v_d^8}{v_d^5}$ pile oscillator	2.84 ± 7.4%	2.66 ± 3. %	<b>-6.44 ± 8.0 %</b>
	$\frac{v_d^9}{v_d^5}$ pile oscillator	0.397 ± 6.2%	0.404 ± 3. %	<b>1.63 ± 6.9 %</b>
<b>Cref</b>	Covariances	380.8 ± 4.7%	383.6 ± 2.2 %	<b>0.72 ± 5.2 %</b>
<b>PuCSS</b>	Covariances	221.8 ± 4.8 %	223.4 ± 2.3 %	<b>0.72 ± 5.3 %</b>
<b>RSR</b>	Covariances	328.6 ± 4.6 %	337.3 ± 2.2 %	<b>2.65 ± 5.1 %</b>
<b>U9</b>	Covariances	725.5 ± 4.3 %	712.2 ± 2.1 %	<b>0.81 ± 4.8 %</b>
<b>UfeRef</b>	Covariances	674.4 ± 3.7 %	670.8 ± 2.1 %	<b>-0.53 ± 4.3 %</b>
<b>UfeLeak</b>	Covariances	674.3 ± 3.7 %	675.8 ± 2.1 %	<b>0.22 ± 4.3 %</b>
<b>Mistral-1</b>	Frequencies	808.2 ± 3. %	789.7 ± 1.6 %	<b>-2.33 ± 3.4 %</b>
<b>Mistral-2</b>	Frequencies	370.7 ± 3.4%	372.5 ± 1.6 %	<b>0.49 ± 3.8%</b>
<b>SHE8</b>	Kinetics	694.25 ± 3.2%	696.0 ± 4.6%	<b>0.25 ± 5.6%</b>
<b>XIX-1</b>	Frequencies	763.3 ± 3.5 %	742. ± 2.4 %*	<b>-2.79 ± 4.6 %μ</b>
<b>XIX-3</b>	Frequencies	253.6 ± 3.5 %	249.1 ± 2.5 %*	<b>-1.77 ± 4.3 %</b>

Table 6 : (E-C)/C Values Obtained Using JEF2.2  $v_d(E)$  data

The differences are small, always smaller than 3% (except for C2 experiment), that is the limit given in the HPRL [19] for what concerns the required uncertainty on ( $\beta_{eff}$ ). This means that the JEF2 data are of sufficient quality.

Mock-up	technique	C (pcm)	E (pcm)	(E-C)/C (%)
<b>R2</b>	Cf. Source	729.1	723.5 ± 3.4 %	<b>-0.76 ± 4.8 %</b>
<b>R2</b>	Frequencies	729.1	726.4 ± 2.3 %	<b>-0.37 ± 4.1 %</b>
<b>R2</b>	α-Rossi	729.1	745.0 ± 1.8 %	<b>2.18 ± 3.8 %</b>
<b>Zona2</b>	Cf. Source	337.0	353.7 ± 3.5 %	<b>4.95 ± 5.2 %</b>
<b>Zona2</b>	Frequencies	337.0	350.0 ± 2.2 %	<b>3.85 ± 4.4 %</b>
<b>7A</b>	Cf. Source	387.5	395.0 ± 2.8 %	<b>1.94 ± 4.7 %</b>
<b>7B</b>	Cf. Source	419.1	429.0 ± 2.8 %	<b>2.36 ± 5.3 %</b>
<b>9C1</b>	Cf. Source	733.5	748.0 ± 4.2 %	<b>1.98 ± 5.5 %</b>
<b>9C2</b>	Cf. Source	384.0	416.0 ± 4.6 %	<b>8.34 ± 6.2 %</b>
<b>CRef</b>	Covariances	365.5	383.6 ± 2.2 %	<b>4.95 ± 5.2 %</b>
<b>PuCSS</b>	Covariances	220.3	223.4 ± 2.3 %	<b>1.41 ± 5.3 %</b>
<b>RSR</b>	Covariances	318.3	337.3 ± 2.2 %	<b>5.98 ± 5.1 %</b>
<b>U9</b>	Covariances	694.7	712.2 ± 2.1 %	<b>5.29 ± 4.8 %</b>
<b>UFeRef</b>	Covariances	678.3	670.8 ± 2.1 %	<b>-1.10 ± 4.3 %</b>
<b>UFeLeak</b>	Covariances	678.2	675.8 ± 2.1 %	<b>-0.35 ± 4.3 %</b>
<b>XIX-1</b>	Frequencies	767.9	742. ± 2.4 %	<b>-3.37 ± 4.6 %</b>
<b>XIX-3</b>	Frequencies	241.2	251. ± 2.5 %	<b>4.06 ± 4.6 %</b>
<b>Mistral-1</b>	Frequencies	806.9	789.7 ± 1.6 %	<b>-2.13 ± 3.8 %</b>
<b>SHE-8</b>	Kinetics	700.7	696.0 ± 4.6 %	<b>-0.67 ± 5.9%</b>

Table 7 : (E-C)/C Values Obtained Using ENDF/B-VI  $\nu_d(E)$  data

The results obtained with ENDF.B-VI indicate a general underestimation by a significant amount, and they exhibit a much larger dispersion than those obtained with JEF2.2.

A first analysis leads to the conclusion that both the underestimation and the dispersion can be explained by too low U-238  $\nu_d$  data, since the data for the other nuclei (U-235 and Pu-239) are very similar to those of JEF2.

## 5. ADJUSTMENT PROCEDURE ; METHODOLOGY AND RESULTS

The statistical adjustment procedure is controlled by a generalized  $\chi^2$  minimization. Adjusted data with minimized biases are obtained when the  $\chi^2$  value after adjustment lies within a confidence interval which is defined so as to guarantee that the obtained  $\chi^2$  value is a correct estimator, with given probability, of the mean value of a (reduced) KHI2 distribution. This one is approximated by a Gaussian *normalized to 1*, with 1 as a mean value and  $\sqrt{\frac{2}{N}}$  as a standard deviation. N is the degree of freedom of the system and equals the number of integral data.

Choosing, for example,  $\alpha = 1,35.10^{-3}$ , the quantity  $1 - 2\alpha = 0.9973$  which represents the integral of the Gaussian in the interval  $[-3\sigma, +3\sigma]$ , represents also the probability for  $\chi^2$  to lie in the interval  $(1 - 3\sqrt{\frac{2}{N}} < \chi^2 < 1 + 3\sqrt{\frac{2}{N}})$ .

$$\Pr(1 - 3\sqrt{\frac{2}{N}} < \chi^2 < 1 + 3\sqrt{\frac{2}{N}}) = (1 - 2\alpha) = 0.9973 \quad (7)$$

The adjustment calculations have been performed with the AMERE code [11], a substantially modified French version of the AMARA code [12].

When using the (E-C)/C values obtained with the JEF2  $\nu_d$  data, the results are as follows :

	$1 - 3\sqrt{\frac{2}{N}}$	$\chi_r^2$	$1 + 3\sqrt{\frac{2}{N}}$
Before adjustment	0.0954	1.269	1.9045
After adjustment	0.0954	0.965	1.9045

These figures show that the adjustment has been performed in correct conditions and that there are no obvious problems (no systematically erroneous data, acceptable covariance data) concerning the input data.

Since the  $(\beta_{eff})_{exp}$  data present in the base are not very sensitive to the data of the higher Pu isotopes, the expected integral information is limited to the 3 major nuclei. Considering the 5 groups there are 15 ( actually less than that ) unknown for 22 data. From a pure statistical point of view the system is not under dimensioned.

The distribution of the residuals after adjustment is perfectly Gaussian around a quasi zero average value  $(-0.44 \pm 2.1)\%$ . This confirms the correctness of the adjustment and suggests the presence of a very small systematic (over-predictive) error on  $(\beta_{eff})$  and on  $\nu_d$ .

The numerical adjustment results are displayed in table 8 :

	Corrections (%)					Uncertainties (%) due to the adjustment only				
	Group 1	Group 2	Group 3	Group 4	Group 5	Group 1	Group 2	Group 3	Group 4	Group 5
U235	-2.0	0.43	0.33	-0.05	-0.02	1.3	1.6	3.5	5.9	6.9
U238	0.00	-0.02	0.18	-0.18	-0.64	5.9	5.9	2.4	8.4	9.8
Pu239	0.38	1.88	1.21	0.49	0.09	1.7	2.6	4.1	6.8	7.9

Table 8 : Corrections on JEF2.2  $\nu_d$  Data due to the Adjustment

The resulting corrections on the thermal and fast range averaged (fission rate) values are :

	U235	U238	Pu239
Thermal range	-1.2	-0.4	+0.7
Fast range	0.04	0.1	1.5

Table 9 : Changes (in %) in the Averaged Values due to the Adjustment.

The most noticeable impacts of the introduction of the integral information concern the thermal range for U-235 and the fast range for Pu-239. They confirm the positive 'slopes' for these 2 nuclei. It is important to note that the decrease imposed to U-235 results from the inclusion of the Japanese values obtained for the BERENICE and FCA programs (see Table 1).

If these values are ignored the adjustment suppresses the decrease but keeps the 'slope'. This situation corresponds to our conviction but has to be further investigated.

Concerning U-238 the adjustment results tend to establish a positive 'slope' contrary to what has been obtained using LENDEL's model and the JEF2  $\nu_p(E)$  evaluation. This trend is consistent with the recent measurement by Piksaikin and coworkers [13]. Before concluding to a possible weakness of the model, one has to demonstrate that the discrepancies with the recent experimental data are not the result of the use of imperfect  $\nu_p(E)$  data in the model. Calculations have been started at CEA Cadarache to reassess the  $\nu_p(E)$  curve of U-238.

From the previous figures one can recommend values in the thermal range :

$$\overline{\nu_d} \quad \begin{array}{l} \mathbf{U235 : 1.616 \pm 0.029 (1.8 \%)} \\ \mathbf{U238: 4.508 \pm 0.252 (5.6 \%)} \\ \mathbf{Pu239 : 0.640 \pm 0.014 (2.2 \%)} \end{array} \quad \text{in } 10^{-2} \text{ unit}$$

in the fast range :

$$\overline{\nu_d} \quad \begin{array}{l} \mathbf{U235 : 1.659 \pm 0.035 (1.9 \%)} \\ \mathbf{U238: 4.707 \pm 0.188 (4.0 \%)} \\ \mathbf{Pu239 : 0.656 \pm 0.014 (2.1 \%)} \end{array} \quad \text{in } 10^{-2} \text{ unit}$$

The results obtained using ENDF/B-VI confirm the conclusions of the preliminary analysis, namely the too low B-VI values for  $\nu_d$  of U-238.

## 6. CONCLUDING REMARKS AND PERSPECTIVES

Integral data are a valuable source of information for delayed neutron yield evaluation provided that a number of conditions are strictly observed. It seems this is not exactly the case since some important integral parameters, like the U-238 fission rate, may not be predicted with the needed accuracy.

This leads to input  $\frac{F8}{F5}$  experimental data in the adjustment procedure which are distributed with a large spread not fully explained by the differences in the neutron spectra of the measurement. This situation probably results from measurement difficulties; we keep in mind the results of the IRMA experiment for which a spread of 2% was observed in the measurements by different teams of the U-238 fission rate in the same place of the same reactor (MASURCA). This is why the performances of the European ERANOS-ERALIB1 system and of the FCA calculational system have to be intercompared, in particular for what concerns the fission rate predictions.

In the same way, the systematic difference which appears in the Cadarache and FCA Pm values for a same technique must be understood. This is a crucial point that should be fully investigated. The recent proposal of careful  $(\beta_{eff})_{exp}$  cross measurements in some future MASURCA cores (perhaps as a follow-up to the benchmark BERENICE) is a first step in the right direction.

Despite these restrictions, which don't dramatically impact the U-238 recommended  $\nu_d$  values, the present analyzed  $\beta_{eff}$  data set is a valuable information basis to confirm the JEF2.2 evaluations and to improve the evaluated accuracies for the 3 important isotopes.

Concerning the higher Pu isotopes and the important minor Actinides, who are of interest for possible future applications, the status is not as favourable.

However recent developments, in both the areas of experimental technique and systematics bear the promise to fill the gap.

The measurements by PIKSAIKIN and coworkers on Np-237 [14], U-238 [12] show the experimental technique is well developed and can be used for any  $\nu_d(E)$  measurement.

Concerning the systematics, the one presented also by PIKSAIKIN and coworkers [15] is developed on the basis of the average half-life  $\bar{T}$  of the delayed neutron precursors.

$$\nu_d = a.[\bar{T}(E)]^b \quad (8)$$

Although its Physics basis is unquestionable, its prediction capability seems to be limited since it is dependent on an exact knowledge of the precursor yields.

LENDEL's model [8] is also interesting because of its description of the contribution of the prompt emission (on which the precursor yields depend) to the 'direct effect' emission. Some points should be clarified which are related to the description of the 'secondary' effects, such as the odd-even effect or the contribution of the cumulative yields. What is needed is a model describing for a single fissioning nucleus all the effects and their evolution with the excitation energy. The competition between the various fission chances and related effects would be treated in a weighted sum as described below:

$$\nu_d(E) = \sum_i \alpha_i(E) \cdot \nu_d^i(E) \quad (9)$$

In this expression the  $\alpha_i$  and  $\nu_d^i$  respectively stand for the fission probability and related delayed yield of the  $i^{\text{th}}$  fissioning nucleus.

Obviously, the scientific community is waiting for a basic approach based on a modeling of the scission mechanism to derive a realistic and accurate description of the fragment charge and mass distributions at any incident energy. But this is a long term and challenging task!

To complete the scope of the Physics behind the delayed neutron emission, it is worth mentioning the existence of fluctuations in the resolved resonance range, independently of their real practical importance. The only way to treat them requires the knowledge of the fluctuations of the emitter yields. This is the way used by OHSAWA and coworkers [16] for U-235, who used the fission fragment yield fluctuations measured by HAMBSCHE [17]. Similar results are obtained at CEA Cadarache to correct the JEF2 fluctuations which resulted from a misuse of LENDEL's model.

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