APPLICATION OF THE MULTIPOINT METHOD TO THE KINETICS OF ACCELERATOR-DRIVEN SYSTEMS

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ABSTRACT

The full derivation of the multipoint method in neutron kinetics for source-driven systems is presented. The results presented show the advantages that can be attained with respect to the standard point model, even when treating relevant spatial and spectral transients. The technique is inserted into a quasi-static framework and some test results for source-driven systems are illustrated.

1. INTRODUCTION

The time-dependent neutronic analysis of accelerator-driven systems (ADS) requires the adaptation of classic methods and the developments of new numerical techniques. The reactor physics groups at ENEA Casaccia (Italy) and at Politecnico di Torino (Italy) have been collaborating in the past years in the field of the dynamics of subcritical systems. Within this collaboration, the quasi-static algorithm has been adapted to systems dominated by an external source [1] and new codes have been prepared and tested [2]. Some design proposals for ADS concern strongly decoupled configurations. In these situations the use of standard point kinetics for time-dependent evaluations can become inadequate or, on the other hand, quasi-statics can turn out to be very expensive, as frequent recalculations of the shape function may become necessary to obtain satisfactory results. The multipoint technique was proposed long ago [3] and recently revived [4]. In a recent paper [5] the multipoint equations were derived through an extension of the factorization procedure as used for the derivation of the classic point equations. The technique has shown to be very efficient in some test calculations performed on idealized configurations. This paper concerns further developments of the method and its inclusion into a quasi-static procedure. The results presented evidence the advantages that can be obtained with respect to standard quasi-statics.

2. THE GENERAL FORMULATION OF THE KINETIC MULTIPOINT METHOD

The neutron balance in a multiplying structure is rigorously described by the transport equation. To solve realistic problems for reactor applications the phase variables (space and velocity) are discretized. Hence, in general, the transport problem can be written down in the following discrete form:

$$\begin{cases} \dfrac{1}{v_m}\dfrac{d\phi_{nm}(t)}{dt} = \displaystyle\sum_{n'}\sum_{m'} k_{nm,n'm'}\phi_{n'm'}(t) + \sum_{i=1}^{6}\lambda_i\chi_{i,m}C_{i,n}(t) + S_{nm}(t), \\ \dfrac{dC_{i,n}(t)}{dt} = \beta_i \displaystyle\sum_{m'} f_{nm'}\phi_{nm'}(t) - \lambda_i C_{i,n}(t) \qquad i = 1, 2, ..., 6, \end{cases} \tag{1}$$

with the inclusion of the delayed emission phenomenon, as obviously required. The indexes n e m identify a point in the space and velocity domains, respectively, while

$$\phi_{nm}(t) = \phi(\mathbf{r}_n, \mathbf{v}_m, t) \text{ and } C_{i,n}(t) = C_i(\mathbf{r}_n, t) \tag{2}$$

are the neutron fluxes and precursor concentrations. The coefficients $k_{nm,n'm'}$ account for the coupling between the point (n', m') in the phase space and the point (n, m). These coupling coefficients can depend on the physical approximations adopted to simplify the transport problem as well as on the discretization algorithms and may be time dependent. Equations in system (1) show already a *multipoint* structure. However, in the following with the term *multipoint* it is referred to a model involving more than one but usually only *a few* points. Hence the equation for each point shall describe the balance for a relatively *large* region in the phase space, accounting for leakage, multiplication and transfer within and across the boundaries of the region itself.

The multipoint model is here obtained by supposing to subdivide the phase space domain in a (hopefully small) number of regions Γ_{NM} and to adopt the following factorization in each one of them:

$$\phi_{nm}(t) = A_{NM}(t)\varphi_{nm} \quad \forall(\mathbf{r}_n, \mathbf{v}_m) \in \Gamma_{NM}, \tag{3}$$

where A_{NM} is the region-wise amplitude function and φ_{nm} is the detailed shape of the neutron flux. To obtain a multipoint model, φ_{nm} is supposed to be constant in time and coincident with the (normalized) neutron distribution in the *reference system,* usually the initial steady-state configuration for the problem considered, solution of the steady-state problem:

$$\sum_{n'}\sum_{m'} k^{(0)}_{nm,n'm'}\varphi_{n'm'} + \sum_{i=1}^{6}\beta_i \sum_{m'}\chi_{i,m} f^{(0)}_{nm'}\varphi_{nm'} + S_{nm}(0) = 0, \qquad (4)$$

where initial system parameters are identified by superscript (0). On the other hand, quasi-statics can be generalized by letting a time dependence in φ_{nm}, as is to be seen in the following. However, in this case the time variation for φ_{nm} should be much slower than for A_{NM}. This condition can be attained only by imposing a further requirement on the shape function (normalization condition), which is also required to make the factorization (3) unique. Consequently, the quasi-static approach can be considered a two-scale approach for the time integration of the balance equations (1).

For the definition of the method, it is necessary to suppose the availability of a "weight function" w_{nm}. Although not strictly required, usually it is taken as the solution φ^+_{nm} of a suitable adjoint-importance problem, defined on the reference system:

$$\sum_{n'}\sum_{m'} k^{(0)}_{n'm',nm}\varphi^+_{n'm'} + \sum_{i=1}^{6}\beta_i \sum_{m'}\chi_{i,m'} f^{(0)}_{nm}\varphi^+_{nm'} + S^+_{nm} = 0. \qquad (5)$$

The consistent definition of the adjoint source term S^+_{nm} is still an open problem. If the neutron importance at phase-space point n, m is defined as the total number of fission neutrons produced by the injection of a neutron at $(\mathbf{r}_n, \mathbf{v}_m)$ in the subcritical system, then the adjoint source is to be defined simply as $S^+_{nm} = \nu\Sigma_f(\mathbf{r}_n, E_m)$. It is also possible to use as weighting function the solution of a sourceless equation with the introduction of a suitable eigenvalue [1].

An inner product definition is introduced for each subzone according to the following formula:

$$\langle w|\, g\rangle = \sum_n \sum_m w_{nm} g_{nm} \quad \forall(\mathbf{r}_n, \mathbf{v}_m) \in \Gamma_{NM}. \qquad (6)$$

To simplify notation, it is useful to define the following sum operations:

$$\left[\sum_m\right]_M h_{nm} = \sum_m h_{nm} \quad \forall(\mathbf{r}_n, \mathbf{v}_m) \in \Gamma_{NM},$$

$$\left[\sum_n \sum_m\right]_{NM} h_{nm} = \sum_n \sum_m h_{nm} \quad \forall(\mathbf{r}_n, \mathbf{v}_m) \in \Gamma_{NM}.$$

Henceforth, the sum in the r.h.s. of formula (6) is written in the following as:

$$\langle w|\, g\rangle = \left[\sum_n \sum_m\right]_{NM} w_{nm} g_{nm}. \qquad (7)$$

The factorization (3) is now introduced into the balance equations (1)

$$\begin{cases} \dfrac{1}{v_m}\varphi_{nm}\dfrac{dA_{NM}}{dt} + \dfrac{1}{v_m}A_{NM}\dfrac{d\varphi_{nm}}{dt} = \displaystyle\sum_{N'}\sum_{M'}\left[\sum_{n'}\sum_{m'}\right]_{N'M'} k_{nm,n'm'}\varphi_{n'm'}A_{N'M'} \\ \qquad\qquad + \displaystyle\sum_{i=1}^{6}\lambda_i\chi_{i,m}C_{i,n} + S_{nm}, \\ \dfrac{dC_{i,n}}{dt} = \beta_i \displaystyle\sum_{M'}\left[\sum_{m'}\right]_{M'} f_{nm'}\varphi_{nm'}A_{NM'} - \lambda_i C_{i,n}, \\ \qquad\qquad i = 1, 2, ..., 6, \qquad \mathbf{r}_n, V_m \in \Gamma_{NM}, \end{cases} \tag{8}$$

which are then projected on the weight function multiplying by w_{nm} and summing over NM. At this point is necessary to introduce the normalization condition:

$$\frac{d}{dt}\left[\sum_n\sum_m\right]_{NM} w_{nm}\frac{1}{v_m}\varphi_{nm}(t) = \frac{d\gamma_{NM}}{dt} = 0, \tag{9}$$

which allows to eliminate the term produced by the projection of the second term in the l.h.s. of Eq. (8) and, most important, makes the factorization unique. The transfer term needs some attention and, with a suitable reorganization of the terms, it can be cast into the following form:

$$\begin{aligned} &\left[\sum_n\sum_m\right]_{NM} w_{nm}\sum_{N'}\sum_{M'}\left[\sum_{n'}\sum_{m'}\right]_{N'M'} k_{nm,n'm'}\varphi_{n'm'}A_{N'M'} \\ &= \sum_{N'}\sum_{M'}\left[\sum_n\sum_m\right]_{NM}\left(w_{nm}\left[\sum_{n'}\sum_{m'}\right]_{N'M'} k_{nm,n'm'}\varphi_{n'm'}\right)A_{N'M'} \\ &= \sum_{N'}\sum_{M'}\gamma_{NM}K_{NM,N'M'}A_{N'M'}. \end{aligned} \tag{10}$$

Finally, the multipoint model is obtained in the form of the following system of point-like differential equations:

$$\begin{cases} \dfrac{dA_{NM}}{dt} = \displaystyle\sum_{N'}\sum_{M'}K_{NM,N'M'}A_{N'M'} + \sum_{i=1}^{6}\lambda_i C_{i,NM} + S_{NM}, \\ \dfrac{dC_{i,NM}}{dt} = \beta_i\displaystyle\sum_{M'}F_{i,NM,M'}A_{NM'} - \lambda_i C_{i,NM}, \qquad i = 1, 2, ..., 6. \end{cases} \tag{11}$$

The following straightforward definitions are introduced for the effective multipoint source:

$$S_{NM} = \frac{1}{\gamma_{NM}} \left[\sum_n \sum_m \right]_{NM} w_{nm} S_{nm}, \tag{12}$$

and the effective multipoint delayed neutron precursor concentrations:

$$C_{i,NM} = \frac{1}{\gamma_{NM}} \left[\sum_n \sum_m \right]_{NM} w_{nm} \chi_{i,m} C_{i,n}. \tag{13}$$

The equations for the effective delayed neutron concentrations require the introduction of the effective delayed neutron production coefficients:

$$F_{i,NM,M'} = \frac{1}{\gamma_{NM}} \left[\sum_n \sum_m \right]_{NM} w_{nm} \chi_{i,m} \left[\sum_{m'} \right]_{M'} f_{nm'} \varphi_{nm'}. \tag{14}$$

The multipoint model can be easily included into a quasi-static scheme (improved quasi-statics, IQS). This requires to update the neutron shape by solving on a slow scale the shape model, Eqs. (8), using an implicit Euler time discretization:

$$\begin{cases} \frac{1}{v_m} \frac{dA_{NM}(t)}{dt}\Big|_{t=T} \varphi_{nm}(T) + \frac{1}{v_m} A_{NM}(T) \frac{\varphi_{nm}(T) - \varphi_{nm}(t_0)}{\Delta t_\varphi} \\ = \sum_{N'} \sum_{M'} \left[\sum_{n'} \sum_{m'} \right]_{N'M'} k_{nm,n'm'}(T) \varphi_{n'm'}(T) A_{N'M'}(T) + \sum_{i=1}^{6} \lambda_i \chi_{i,m} C_{i,n}(T) + S_{nm}(T), \\ C_{i,n}(T) = C_{i,n}(t_0) e^{-\lambda_i \Delta t_\varphi} \\ \qquad + \int_{t_0}^{T} \beta_i \sum_{M'} \left[\sum_{m'} \right]_{M'} f_{nm'}(T) \varphi_{nm'}(t_0) A_{N'M'}(t') e^{-\lambda_i (T-t')} dt', \end{cases} \tag{15}$$

where t_0 indicates the initial time and Δt_φ the time interval for which the shape has been assumed constant. Assuming $A_{NM}(T)$ and its time derivative to be known, the system of equations (15) can be solved, using the same algorithm used for solving the reference problem, Eq. (5). However, the solution shall not obey to the normalization condition (9). Hence it is required to adjust the amplitude terms in Eq. (15) to force the shape to fulfill such condition. This can be done by assuming the amplitude to be continuous in time but letting its derivative to be discontinuous, starting an iterative procedure until convergence, as specified in the following formulae:

$$\frac{1}{v_m}\left.\frac{dA_{NM}(t)}{dt}\right|_{t=T}^{(l)}\varphi_{nm}^{(l)}(T)+\frac{1}{v_m}A_{NM}(T)\frac{\varphi_{nm}^{(l)}(T)-\varphi_{nm}(t_0)}{\Delta t_\varphi}=$$
$$\sum_{N'}\sum_{M'}\left[\sum_{n'}\sum_{m'}\right]_{N'M'}k_{nm,n'm'}(T)\varphi_{n'm'}^{(l)}(T)A_{N'M'}(T)+\sum_{i=1}^{6}\lambda_i\chi_{i,m}C_{i,n}(T)+S_{nm}(T), \tag{16}$$

where the starting ($l = 0$) value for the amplitude derivative is assumed to be the final value computed by the solution of Eqs. (11), namely:

$$\gamma_{NM}^{(l)}(T)=\left[\sum_{n}\sum_{m}\right]_{NM}w_{nm}\frac{1}{v_m}\varphi_{nm}^{(l)}(T), \tag{17}$$

$$\varphi_{nm}^{(l+1/2)}(T)=\frac{\gamma_{NM}(t_0)}{\gamma_{NM}^{(l)}(T)}\varphi_{nm}^{(l)}(T), \tag{18}$$

$$\gamma_{NM}^{(l+1/2)}(T)\left.\frac{dA_{NM}(t)}{dt}\right|_{t=T}^{(l+1)}$$
$$=\left[\sum_{n}\sum_{m}\right]_{NM}w_{nm}\left\{\sum_{N'}\sum_{M'}\left(\left[\sum_{n'}\sum_{m'}\right]_{N'M'}k_{nm,n'm'}(T)\varphi_{n'm'}^{(l+1/2)}(T)\right)A_{N'M'}\right.$$
$$\left.+\ \chi_{i,m}\lambda_iC_{i,n}(T)+S_{nm}(T)\right\}, \tag{19}$$

where by definition $\gamma_{NM}^{(l+1/2)}=\gamma_{NM}(t_0)$. Hopefully, if the shape does not undergo too large a change within a shape step, the iterative process should converge in very few iterations; otherwise, the advantage of the quasi-static scheme may be fully vanified.

The procedure for updating the shape can be repeated along the transient, as in the standard quasi-static technique.

3. RESULTS

The method is applied to two different configurations of subcritical source-driven systems, in order to evidence its merits with respect to usual quasi-statics and point kinetics. The systems are studied in one-dimensional plane geometry using multigroup diffusion. Figure 1 gives a sketch of the two configurations and the flux distributions in steady state conditions. Reactor A is characterized by material parameters as proposed for the Myrrha project [6]. Reactor B has typical parameters of the Masurca systems for the MUSE experiment

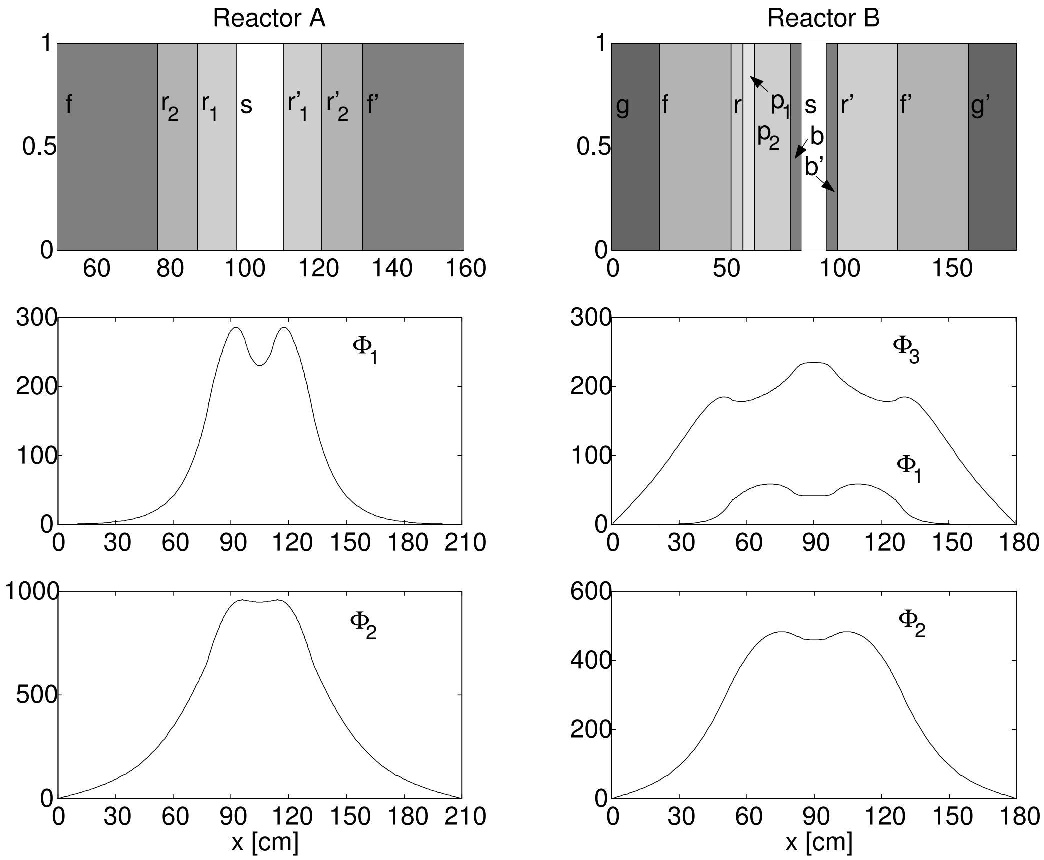

Fig. 1. Material configuration of the systems considered (top). Zone identification: s: source channel; r, r': multiplying zones, Masurca type; r_1, r_1': multiplying zones, Myrrha type (low enrichment); r_2, r_2': multiplying zone, Myrrha type (high enrichment); f, f': reflectors; b,b': lead buffers; p_1, p_2: perturbed zones; g, g': shields. Initially, r, p_1 and p_2 have the same properties as r'. Flux distributions in steady state, calculated in two-group diffusion for reactor A and in three-group diffusion for reactor B (bottom).

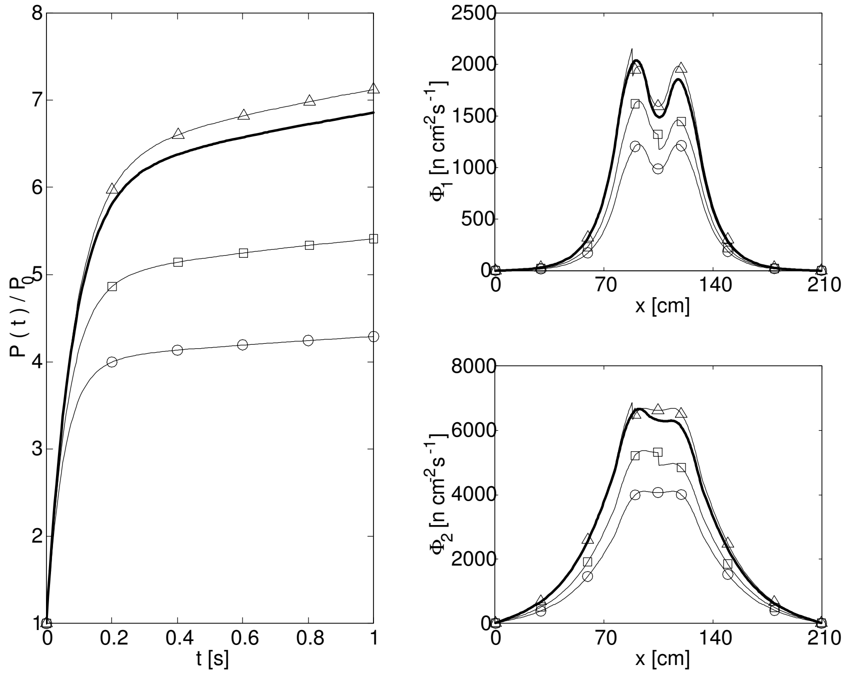

Fig. 2. Transient caused by a $\delta\Sigma_{a1} = -0.1\delta\Sigma_1$ in the r_2 zone of system A. The multiplication constant becomes $k_{eff} = 0.998447$. On the right, the fluxes at the last time considered in the transient ($t = 1$ s) are plotted.

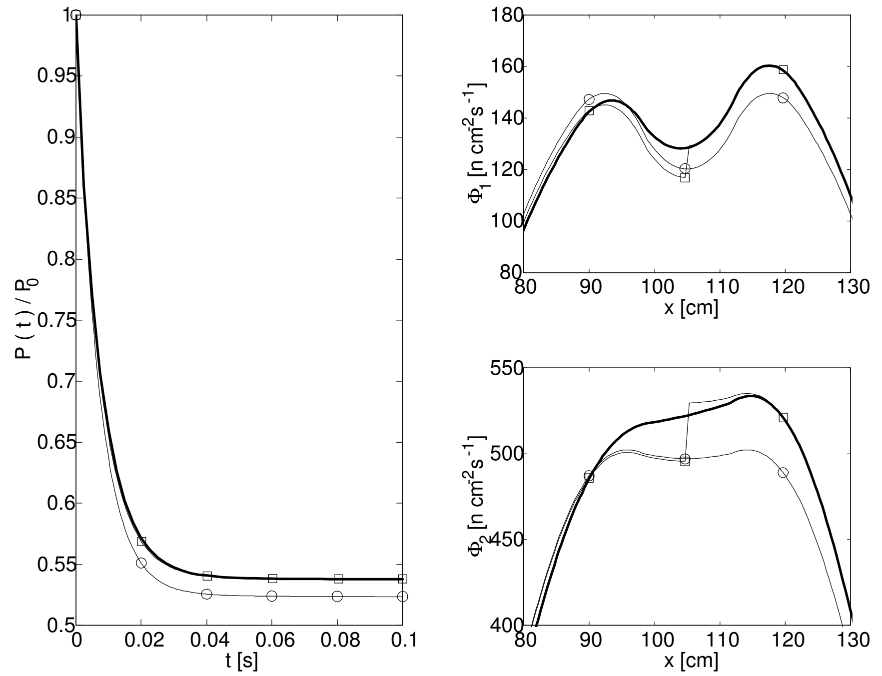

Fig. 3. Transient caused by a $\delta\Sigma_{a1} = +0.05\delta\Sigma_1$ in the r_1 and r_2 zones of system A. The multiplication constant becomes $k_{eff} = 0.961767$. Final fluxes on the right.

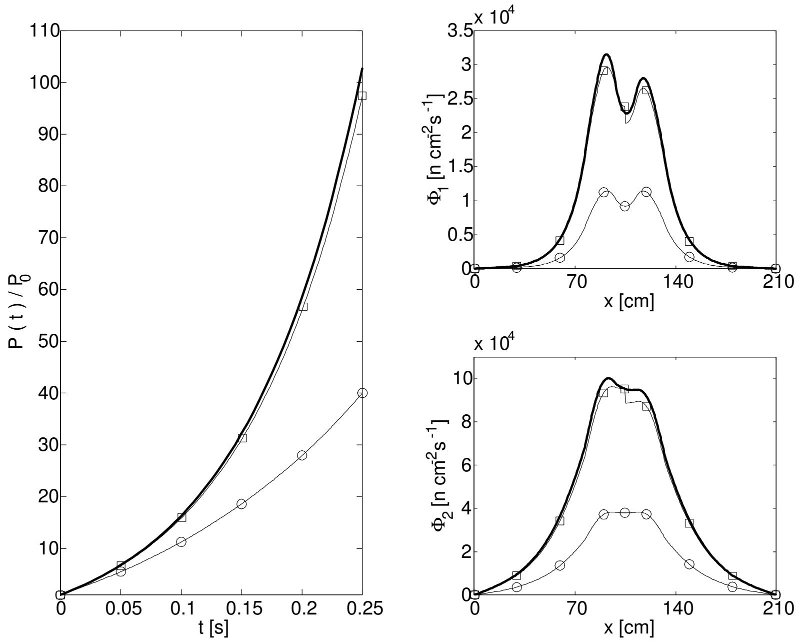

Fig. 4. Transient caused by a $\delta\Sigma_{a1} = +0.1\delta\Sigma_1$ in the r_1 zone of system A. The multiplication constant becomes $k_{eff} = 1.00489$.

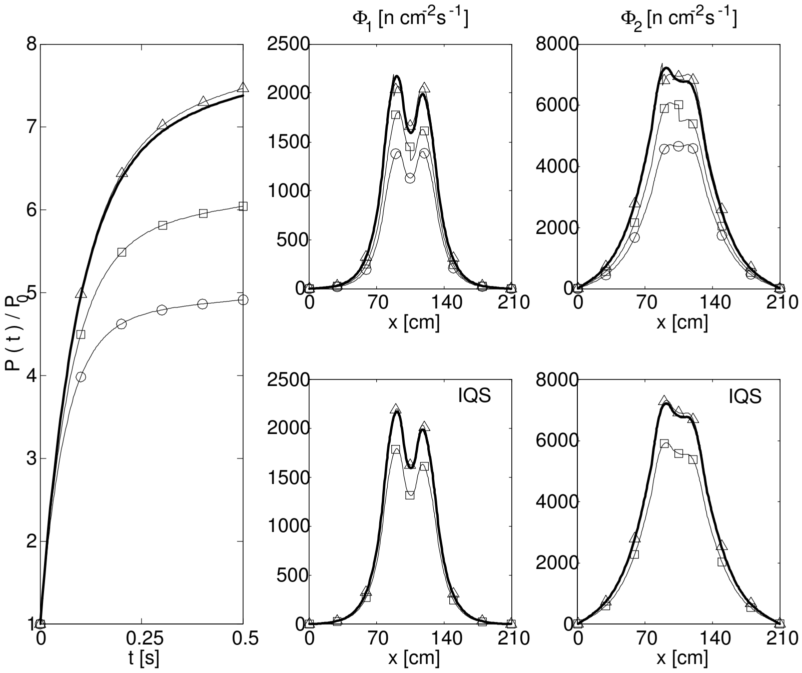

Fig. 5. Transient caused by a $\delta\Sigma_{a1} = -0.05\delta\Sigma_1$ and $\delta\Sigma_{a2} = -0.1\delta\Sigma_2$ in the r_2 zone of system A. The multiplication constant becomes $k_{eff} = 0.998850$. In the right on the top, comparison of reference, point and two-point kinetics can be seen. At the bottom, IQS results are reported.

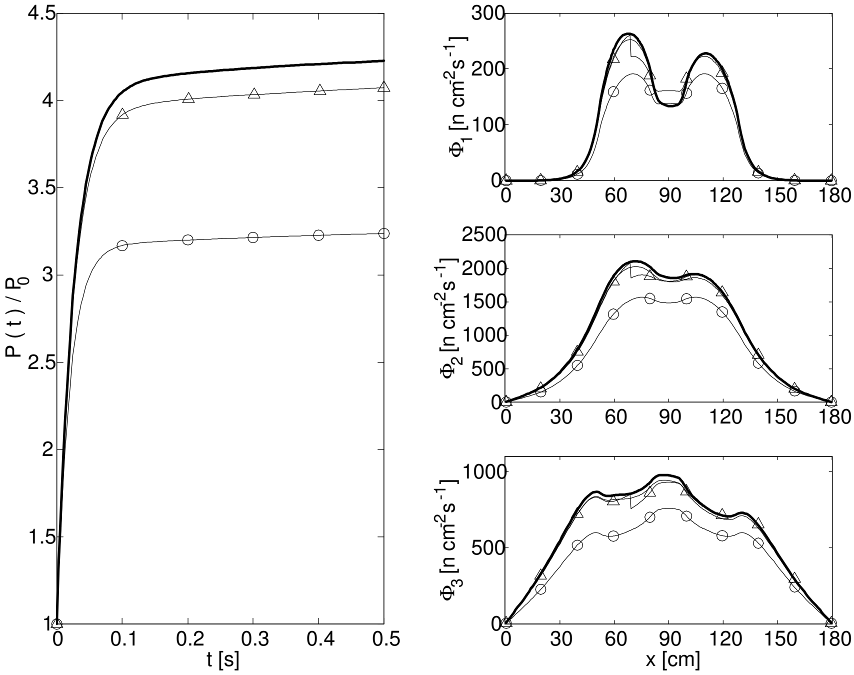

Fig. 6. Transient caused by a $\delta\Sigma_{ag} = -0.2\delta\Sigma_g$ for all groups in the p_1 zone of system B. The multiplication constant becomes $k_{eff} = 0.994264$.

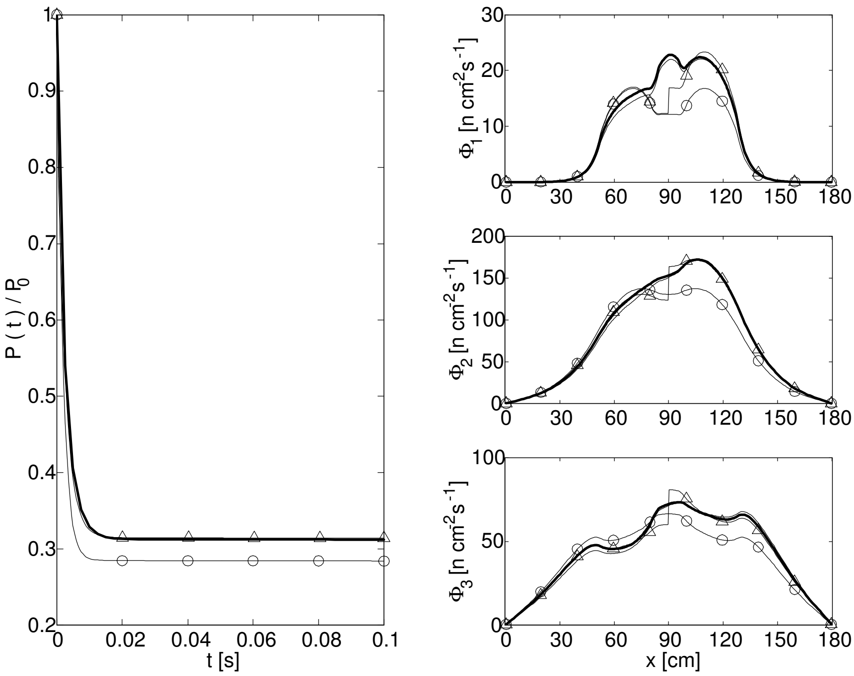

Fig. 7. Transient caused by a $\delta\Sigma_{ag} = +0.2\delta\Sigma_g$ for all groups in the p_2 zone of system B. The final multiplication constant is $k_{eff} = 0.912024$.

[7]. Reactor A is treated in a two-group diffusion model, yielding k_{eff}=0.98000, and it is driven by a source having intensity 0.7119 in the first group and 0.2881 in the second one. For reactor B a three-group model is adopted, k_{eff}=0.97060, and a unitary source is assumed in the first group only. Steady-state source-driven fluxes are also reported in the figure.

The following figures present some selected results for transient evolutions. In all the figures the curves identified by circles report results produced by point kinetics, those identified by squares and triangles concern multipoint kinetic calculations and the solid bold line the reference obtained by direct numerical solution of the diffusion system of equations [2]. Figure 2 shows the transient following a localized perturbation of the capture cross section in a Myrrha-type system. It can be seen that point kinetics is completely unable to give acceptable results and it is grossly on the side of unsafety, since the power evolution is highly underestimated. Results for two two-spatial-point kinetics are shown. In the first case, the reactor is subdivided into two equal zones. The curve identified by squares shows an improvement with respect to one-point kinetics but seems to be still unsatisfactory. To produce the curve identified by triangles the reactor is subdivided into two zones again, but in one of them only regions f and r_2 are included. The significant improvement can be clearly seen. Also, results for the power evolution are conservative.

It must be noted that, in Fig. 2 as in the following ones, since different amplitudes are supposed in each region, at the end of the transient the spatial distribution of the neutron flux as foreseen by the multipoint method can be spatially discontinuous.

A negative-reactivity transient is reported in Fig. 3. In this case, the error in the power is smaller for point kinetics. The two point model assumes a subdivision into two symmetric zones, which seems adequate in view of the spatial nature of the perturbation. Large differences in the spatial flux distribution do not affect the power evolution significantly, as they take place in the non-fissile source-channel zone. Figure 4 shows the efficiency of the multipoint model in following even a supercritical evolution.

In Fig. 5, also the improved quasi-static technique is analyzed, by recalculating the neutron shape at 0.5 s. The curves identified by squares are produced by a symmetric subdivision of the reactor, while those identified by triangles by a subdivision as for Fig. 2. Both recalculations of the shape converge in less than four iterations, to relative errors smaller than 10^{-4} on the values of the normalization constants γ_{NM}.

Figures 6 and 7 concern the Masurca-like calculation. In both transients the excellent performance of the multipoint method can be appreciated. IQS results for both the spatial flux distributions and the powers are indistinguishable from the reference calculation.

4. CONCLUDING REMARKS

The multipoint method is applied for the kinetics of accelerator-driven systems. In many configurations typical of accelerator-driven systems its performance is by far more efficient than standard point kinetics. Therefore, the multipoint technique can be proposed for introduction into quasi-static procedures, to highly enhance their performances.

Future developments include the implementation of the IQS technique to multidimensional problems and to the transport discrete ordinate model.

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