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AN INVENSTAIGATION OF SUBCRITICALITY LEVEL IN ACCELERATOR-DRIVEN SYSTEM

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ABSTRACT

In order to derive a generic range of subcriticality of ADS, various fundamental sensitivities of the system performance to the subcriticality level have been assessed. First, to clarify the definition of the subcriticality in ADS, the mathematical and physical implications of the two often-used parameters (the conventional effective multiplication factor k_{eff} and the source multiplication factor k_{src}) have been addressed. Based on the fundamental premise of ADS, a concept of the minimum required and maximum allowable subcriticality levels are proposed and they have been evaluated for an LBE-cooled ADS design. The minimum subcriticality is determined by the operational and safety requirements and the maximum allowable subcriticality level is bounded by the system economy and the technical feasibility of the system. Also, with an integral safety analysis method, called balance of power, several important transients have been analyzed to assess the sensitivity of the system response to the subcriticality level. In addition, the impacts of the subcriticality level on the transmutation performance of an ADS are evaluated in terms of discharge burnups of both fuel and long-lived fission products.

1. INTRODUCTION

A major motivation of accelerator-driven system (ADS) is its potentially enhanced safety characteristics, due to the fact that the core is kept subcritical. As a result, it is generally recognized that an ADS could be used to transmute the radioactive materials such as transuranic elements (TRUs) and long-lived fission products (LLFPs).[1,2]

Concerning the ADS design, it is found that there is no common consensus on the definition of the subcriticality of the ADS, leading to a technical ambiguity. In an ADS, to define the subcriticality level consistently is very important since the system design as well as its safety is significantly affected by the value. Currently, two parameters are being frequently used as the index of the subcriticality level of ADS, one is the conventional effective multiplication factor k_{eff} and the other so-called source-multiplication factor k_{src} . The k_{src} factor is defined as the ratio of neutron production to loss for a subcritical system just in the same way for k_{eff} .[3] Physically, k_{src} denotes the degree of multiplication of the external source, while k_{eff} indicates the multiplication of the fission neutrons. Thus, the two parameters are generally different from each other, depending on the design feature of an ADS. In addition to k_{eff} and k_{src}, a recent work[4] proposed another concept of subcriticality for the ADS, adding to the ambiguity of the definition of the subcriticality level. Recalling that the subcriticality is the very fundamental basis of ADS, it is necessary to clarify the definition of the subcriticality level in ADS.

When it comes to ADS designs, a crucial issue is to find the optimal subcriticality. Previous researches show that the subcriticality is spread over a relatively wide range and the value is usually determined by a rule of thumb. During the early development stage of the ADS, it was generally thought that a larger subcriticality would provide a better safety performance. However, nowadays, it is a common sense that the subcriticality might negatively impact the safety of an ADS in some special situations.[5,6] Thus, it is necessary to account for the positive and negative effects of the subcriticality level on the performance of ADS in determining the subcriticality. Basically, it can be said that the safety requirement of ADS should determine the lower bound of the subcriticality level. On the other hand, when designing an ADS, the economy and technical feasibility of the system should also be taken into account. Concerning the economy of a typical ADS, a rather detailed analysis[7] showed that the k_{eff} value should be greater than at least 0.95 for an economic operation. In the meantime, if the subcriticality is too large, the required accelerator power would be too high, subsequently it might be very difficult/costly to develop the accelerator itself and to couple the accelerator and the subcritical core. These economic and technical constraints might determine an upper bound of the subcriticality level.

In addition to safety and economy, the transmutation performance of an ADS transmuter could be affected by the subcriticality level. Basically, the subcriticality of ADS determined the degree of surplus neutrons which can be used for the LLFP transmutation.[8] Obviously, the fuel inventory of the subcritical core generally depends on the level of subcriticality. Therefore, the discharge burnup of the fuel and LLFPs is dependent on the subcriticality. Clearly, the transmutation efficiency of nuclear waste needs to be accounted for in determining the subcriticality of an ADS.

The objective of this paper is to provide a generic guideline for the subcriticality level of an ADS. In Section 2, the definition of the subcriticality of an ADS is reviewed. A lower and upper bounds of the subcriticality level are derived based on a general properties of an ADS in Section 3. Section 4 contains a generic safety analysis results for a specific ADS core and Section 5 shows the sensitivity of the transmutation performance with respect to the subcriticality level. Finally, conclusions are drawn in Section 6.

2. DEFINITION OF SUBCRITICALITY IN ADS

Recalling the implication of the word ‘subcriticality’, it is clear that the subcriticality should mean the ‘distance from a criticality’, i.e., a measure of off-criticality. Obviously, criticality indicates that the neutron chain reaction is maintained in a system without any external source. In other words, the criticality condition is independent of an external source. This implies that the off-criticality could be measured in terms of k_{eff}, due to its very definition. *Conventionally, when a system A has a larger subcriticality than the other one B, it is presumed that A has a greater degree of off-criticality than B.* Therefore, a quantity $(1-k_{eff})$ could be used as a definition of the subcriticality of an ADS, as is already often utilized. Or, following the conventional definition of the so-called 'static reactivity (ρ_s)', a quantity $-\rho_s = (1-k_{eff})/k_{eff}$ could also be used as a useful definition of the subcriticality.

Below is the mathematical and physical comparison of the effective multiplication factor k_{eff} and the source multiplication factor k_{src}. Let us consider a subcritical system with an extraneous source *S*

$$A\phi_s = F\phi_s + S\,, \tag{1}$$

where *A* and *F* are neutron destruction and production operators, respectively, and ϕ_s is the neutron flux. Conventionally, the source multiplication factor in an ADS is defined by

$$k_{src} = \frac{< F\phi_s >}{< A\phi_s >} = \frac{< F\phi_s >}{< F\phi_s > + < S >}\,, \tag{2}$$

where < · > denotes the integration over the phase space.[3]

For the system in Eq. (1), the forward and adjoint fluxes (ϕ and ϕ^*) of the associated homogeneous system can be found by solving

$$A\phi = \frac{1}{k_{eff}} F\phi\,, \tag{3}$$

$$A^*\phi^* = \frac{1}{k_{eff}} F^*\phi^*, \tag{4}$$

where A^* and F^* are the adjoint operators of *A* and *F*, respectively.

Multiplying Eq. (1) by ϕ^* and integrating over the energy and space, and using the properties of the adjoint operators, one can get

$$k_{eff} = \frac{< \phi^*, F\phi_s >}{< \phi^*, A\phi_s >} = \frac{< \phi^*, F\phi_s >}{< \phi^*, F\phi_s > + < \phi^*, S >}\,. \tag{5}$$

Comparing Eqs. (2) and (5), it is clear that k_{eff} and k_{src} should be generally different from each other, since ϕ^* is highly space- and energy-dependent in ADS. Note that ϕ^* is interpreted as a neutron importance function.[9,10] Actually, the difference between k_{eff} and k_{src} can be quite large depending on characteristics of core and source. For example, if the source multiplication is very inefficient in a subcritical system, i.e., very small $< F\phi_s >$, the k_{src} value could be very small, although k_{eff} is close to unity. Note that the k_{eff} value is independent of the external source. On the contrary, k_{src} could be significantly larger than k_{eff} if an external source is located at a high 'neutron importance position' and its energy is much higher than the fission neutron energy. Thus, it is possible that two systems with the same k_{eff} value could have rather arbitrary k_{src}. Of course, it is also possible that k_{src} of a system A could be greater or smaller than that of another system B, regardless of the difference of k_{eff} and k_{src} between the two systems. Consequently, on the premise that the subcriticality of an ADS should be measured in terms of off-criticality, k_{src} in Eq. (2) might not be consistently used as an index of the subcriticality.

Concerning the definition of the subcriticality level of an ADS, Kobayashi and Nishihara[4] proposed to use the following multiplication factor:

$$k_{sq} = \frac{< G, F\phi_s > + < G, S >}{< F\phi_s > + < S >}, \tag{6}$$

$$A^* G \ = F^* G + \nu\Sigma_f, \tag{7}$$

where ν and Σ_f are the number of neutrons per fission and the fission cross section, respectively. According to the definition, *G(r, E)* denotes the number of fission neutrons produced by a source neutron at position *r* and energy *E*. Due to this physical meaning, the function *G(r, E)* is often called the source neutron importance function.[9,10] Clearly, the k_{sq} factor is also highly dependent on the multiplication of the source. Thus, it is also subject to the same inconsistency problem as in the k_{src} factor. Based on the above discernation, the subcriticality level of an ADS is represented by using the k_{eff} factor throughout this paper.

Although k_{src} cannot be consistently used to define the subcriticality in ADS, it might be otherwise useful in characterizing the ADS. Previously, Soule et al[3] introduced the so-called source efficiency (φ^*) using k_{eff} and k_{src}:

$$\varphi^* = {\frac{k_{src}}{1-k_{src}}} \Big/ {\frac{k_{eff}}{1-k_{eff}}} . \tag{8}$$

Note that φ^* means the efficiency of the source multiplication relative the multiplication of the fission neutrons. Recalling that $k_{src}/(1-k_{src})$ is the number of fission neutrons produced by a single source neutron, the fission power (P_{fiss}) of an ADS might be estimated by

$$P_{fiss} = \frac{E_f \varphi^*}{\nu} \frac{k_{eff}}{1-k_{eff}} < S >, \tag{9}$$

where E_f =energy per fission reaction, ν =average number of neutrons produced by a fission reaction. Also, letting n_{sp} =number of spallation neutrons produced by a proton, the required proton current (I_p) in ampere for an ADS with a fission power P_{fiss} (in MW) can be estimated by

$$I_p = \frac{\nu}{\varphi^* n_{sp}} \left(\frac{1-k_{eff}}{k_{eff}} \right) \frac{P_{fiss}}{E_f} \ \text{[A]}. \tag{10}$$

Eqs. (9) and (10) indicate that, given φ^* for an ADS, the system power or accelerator current can be evaluated with only the k_{eff} value. Note that E_f should be in MeV in Eq. (10). Generally, the source efficiency highly depends on the source characteristics such as energy spectrum, spatial distribution. Also, it is affected by the power distribution in the fuel blanket. In spite of the fundamental dependency of φ^* on the core design, the source efficiency change is quite small if the perturbation is not very large for a specified system. Thus, a scoping evaluation for an ADS could be done effectively with the concept of the source efficiency.

3. MINIMUM AND MAXIMUM SUBCRITICALITY IN ADS

3.1 GENERAL CONSIDERATION

In the ADS design, a fundamental premise is that the core should be subcritical in both reloading and operational regimes, without any active intervention of the operator, e.g., control rod insertion. If the core reaches a critical state, the surmised advantages in terms of the safety could be seriously hampered and the motivation for such a core design can hardly be justified. Therefore, the subcriticality level of an ADS core should be sufficient enough to accommodate a possible positive reactivity insertion during the operation including the fuel loading stage. This implies that a minimum required subcriticality could be determined for a specific ADS design using the core characteristics and the design philosophy. In this work, it is assumed that the ADS core is designed and operated based on the conventional concepts adopting a fixed cycle length and a solid fuel.

Generally, an ADS core has a negative power defect, i.e., power reduction from a 'hot' full power condition to a 'hot' zero power state results in a positive reactivity insertion to the core. Additionally, a further temperature decrease from a zero power to a reloading stage generally increases the core reactivity. Thus, the subcriticality at a full power state should be sufficiently large to compensate for the positive temperature defect (δk_{eff}^{TD}) resulting from the temperature swing from the 'hot' full power state to a 'cold' reloading state. In addition to this operational requirement for the subcriticality (the power defect), it might be required that the core should remain still subcritical even in the case of an accidental insertion of a positive reactivity (δk_{eff}^{AC}). These two reactivity change δk_{eff}^{TD} and δk_{eff}^{AC} may determine the minimum required subcriticality in ADS.

On the other hand, there could be a maximum permissible level of subcriticality in an ADS core. As is already mentioned, the system economy demand that k_{eff} should be greater than 0.95 for an economic operation ($k_{eff}^{E\min}$). Meanwhile, technologies (accelerator-related and/or coupling accelerator and reactor) may determine the lower bound of the k_{eff} value ($k_{eff}^{T\min}$). This technical constraint generally depends on the design specifications of ADS of interest such as the power level, external source configuration, etc. For example, a very high power system may require an accelerator current that is unavailable even with an advanced accelerator technology. Especially, if a beam window concept is utilized to couple the accelerator and the subcritical core, the window integrity may set a more stringent barrier to the permissible accelerator current.

Consequently, in the ADS core, the effective multiplication factor k_{eff} at any full power condition should satisfy the following inequality relation:

$$\max(k_{eff}^{E\min} = 0.95, k_{eff}^{T\min}) < k_{eff} < 1 - \delta k_{eff}^{TD} - \delta k_{eff}^{AC} - \delta k_{eff}^{UM} - \delta k_{eff}^{UC}, \tag{11}$$

where δk_{eff}^{UM} and δk_{eff}^{UC} indicate the uncertainties associated with the reactivity measurement and calculations, respectively.

Given a system, the power defect can be readily evaluated. However, concerning the positive accidental reactivity insertion, it is very difficult to determine the required quantity δk_{eff}^{AC} since several hypothetical accidents might be involved. If an ADS is designed such that all the possible accidental reactivity insertions could be allowed, the system would likely violate the constraint imposed by the economics and technical requirements. In a fast spectrum ADS loaded with a dedicated fuels such as TRU or MA (Minor Actinide), it is well known that a coolant voiding in the core generally induces a large amount of positive reactivity insertion. Besides, a TRU (or MA) transmuter core is known to contain a fuel mass many times larger than the critical mass.[11] Thus, a severe core disruptive accident involving a core compaction may lead to a huge positive reactivity insertion, which cannot be compensated for by the initial subcriticality from the practical viewpoint. Thus, for practical and realizable ADS design, a compromise needs to be made for the determination of δk_{eff}^{AC}, depending on the design features.

Based on our belief that the probability for a hypothetical accident incurring a large reactivity insertion to occur is negligibly low in the ADS design, we propose the following strategy to evaluate the quantity δk_{eff}^{AC} in Eq. (11). In the typical liquid target ADS design, there is a relatively vulnerable equipment whose failure may produce some positive reactivity, i.e., the rupture of the beam delivery tube or the beam window failure. In this case, the beam tube may be filled with the target material, and this could increase the core reactivity. Also, possibly, a control rod could be used in the ADS core to regulate the power level and power distribution. In this situation, an inadvertent ejection of the rod should not make the core critical. These two accidents may be considered as basic cases for determining the minimum required subcriticality of an ADS.

In case of a TRU transmuter, the k_{eff} value decreases almost linearly over a burnup period, without any reactivity-regulating device. If the burnup reactivity swing is δk_{eff}^{BU} $(= k_{eff}^{BOC} - k_{eff}^{EOC}$: difference between beginning of cycle (BOC) and end of cycle (EOC)), the maximum k_{eff} at BOC should satisfy the following relation:

$$\max(k_{eff}^{E\min}, k_{eff}^{T\min}) + \delta k_{eff}^{BU} < k_{eff} < 1 - \delta k_{eff}^{TD} - \delta k_{eff}^{AC} - \delta k_{eff}^{UM} - \delta k_{eff}^{UC}, \qquad (12)$$

On the other hand, if there is a reactivity increase during the burnup, the BOC k_{eff} value should be determined such that the relation Eq. (11) could be satisfied at the burnup point with the maximum k_{eff} value.

3.2 REPRESENTATIVE EVALUATION OF SUBCRITICALITY

All the parameters involved in the subcriticality range Eqs. (11) and (12) are dependent on the design characteristics of an ADS core. For a representative estimation of the minimum and maximum subcriticalities, a model ADS called HYPER (Hybrid Power Extraction Reactor)[12,13] was considered in this work. Basic design features and core characteristics of the HYPER system are provided in the Appendix.

In HYPER, the coolant inlet temperature (T_{in}) is 340 °C and the coolant temperature rise (ΔT_c) at the full power is 170 °C. Assuming that the fuel loading is done at a coolant temperature of 240 °C (T_{load}) and the average fuel temperature (T_f) is 600 °C, the temperature defect $\delta\rho^{TD}$ from a full power condition to a cold reloading state can be estimated with the reactivity coefficients in Table A.II in the following way [14]:

$$\delta\rho^{TD} = -(\alpha_D + \alpha_E)\times(T_f - T_{in} - \frac{\Delta T_c}{2}) - (\alpha_D + \alpha_E + \alpha_{LBE} + 2\alpha_R)\times\frac{\Delta T_c}{2}$$
$$+ (\alpha_D + \alpha_E + \alpha_{LBE} + \alpha_R)\times(T_{load} - T_{in}) = 454\, pcm$$

In the current HYPER design, no control rods are utilized for the reactivity control. Thus, the safety requirement quantity $\delta\rho^{AC}$ was evaluated for the beam window failure case only. In this case, the beam tube is assumed to be filled with the LBE (Lead-Bismuth Eutectic) coolant. As shown in Table A.II, the window failure results in a significant increase of the reactivity, +753 pcm: $\delta\rho^{AC} = 753\, pcm$ for the current HYPER core. As a result, neglecting the uncertainty terms, one can find a minimum required subcriticality, (the maximum allowable k_{eff}) $k_{eff}^{\max}$ < 0.988 for the HYPER core. For more practical estimation of the upper bound of the k_{eff} value, both the measurement and calculational uncertainties should be taken into account. It is well known that the calculation uncertainty for the reactivity change is quite large in the conventional fast reactor design. Also, it is expected the measurement uncertainty would be rather noticeable in ADS, depending on the measurement method. A recent research on an experimental subcritical core reported that the reactivity could be measured quite accurately, within a few percent error for subcriticality level down to k_{eff} =0.9.[3] For the purpose of illustration, let us assume that the calculational uncertainty is 30% of the calculated values and the measurement one is 100 pcm. It should be mentioned that the actual uncertainties could be quite different from those values. These assumptions provide the new maximum allowable k_{eff} value of 0.984 for the HYPER core.

As shown in Fig. A.1, a single external source is used for the simplification of the core design and the beam window is used. A preliminary evaluation for the beam window of HYPER shows that the maximum allowable proton current should be smaller than 20mA.[15,16] Thus, assuming that the accelerator power is achievable up to 20 MW, the k_{eff} value should not be smaller than the value requiring a proton current 20 mA. The technical lower bound of k_{eff} can be approximately estimated by using Eq. (10). For the current HYPER core design, the source efficiency is about 1.05 and n_{sp} was evaluated to be about 28 for the target configuration. Thus, letting ν =2.9, E_f = 200 MeV, one can find that the minimum required k_{eff} is about 0.961 for the current HYPER design. In Fig. 1, the technical lower bound of k_{eff} is plotted as a function of the maximum allowable proton current for several power levels.

From Fig. 1, one can note that k_{eff} should be greater than 0.9705 (or 0.9801) if the maximum proton current is 15mA (or 10mA) for the 1000 MWth power HYPER. Actually, taking into account the current accelerator performance, the maximum allowable proton current 20mA might be considered as a high one. This implies that, for some practical ADS design, the minimum k_{eff} should be fairly high and a high power system would be very challenging in the ADS with a beam window, from the viewpoints of accelerator technology and coupling of accelerator and reactor. A windowless target or a multiple source scheme could be adopted to alleviate the window problem. However, these design concepts might results in a very complicated system design and may require a higher accelerator current.

As shown in Table A.I, the EOC subcriticality of the current HYPER design is k_{eff} =0.9511, so it can be said that the current HYPER core design cannot satisfy the technical boundary condition imposed by the beam window. Thus, further design modifications are required. Obviously, reducing the burnup reactivity swing could be a possible solution to satisfy the technical constraint. Also, the beam window design could be modified to allow for a larger proton current. It is clear that a larger beam delivery tube would results in a larger maximum allowable proton current with some compromise of the core neutronic performance. Otherwise, the power level should be appropriately reduced.

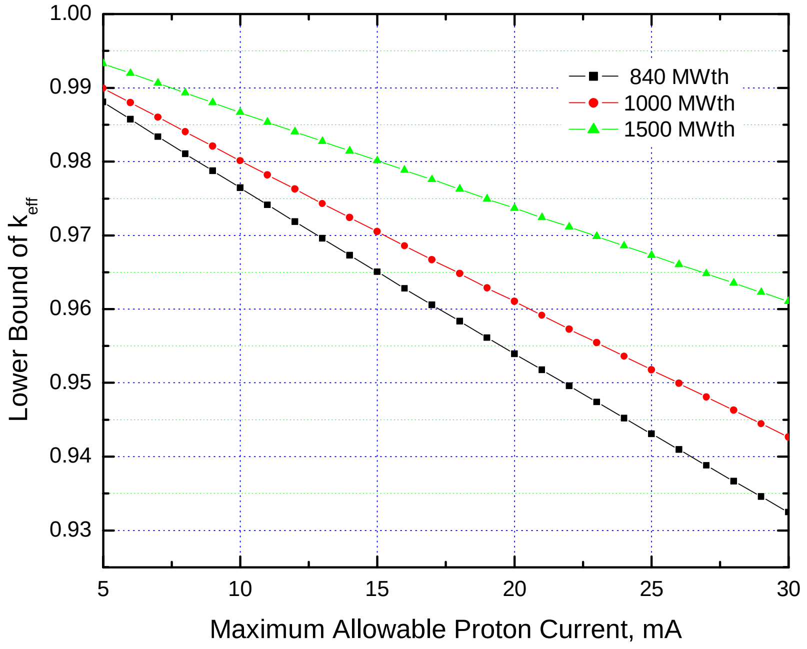

Fig. 1. Minimum allowable k_{eff} as a function of maximum accelerator current in HYPER.

4. IMPACTS OF SUBCRITICALITY ON ADS SAFETY

4.1 BALANCE OF POWER (BOP) METHOD

Safety of an ADS system is directly affected by the subcriticality level, since the response of the system during a transient is quite sensitive to the subcriticality. For an accurate safety analysis, a wide range of detailed calculations should be done. However, in this paper, a generic safety feature of an ADS is evaluated using an integral deterministic method for several important sever accident cases. In the late 80's, an effective integral safety analysis method, called "Balance of Reactivity (BOR)", was developed for critical fast reactors and extensively used for the Integral Fast Reactor (IFR).[14] Recently, based on the concept of the BOR method, Gandini et al[5] proposed a similar integral, asymptotic method, so-called "Balance of Power" for the ADS. In the BOR method, it is assumed that the reactor core asymptotically approaches to a new critical state after a limited transient. However, any subcritical state can be virtually achieved in ADS in a similar circumstance, if the incidental transient does not lead to a critical state. The BOP method is briefly reproduced in for the sake of fidelity of this paper.

In the BOP method, the reactivity change resulting from a transient (ρ) is represented by (at an equilibrium state),

$$\rho = (P-1)A + (P/F-1)B + \delta T_{in} C + \delta\rho_{ext}, \quad (13)$$

where *P* and *F* are normalized power and flow rate, respectively, and

$(A+B)$ = reactivity coefficient experienced in going to full power and flow from zero power isothermal at coolant inlet temperature,
B = power/flow reactivity coefficient,
C = inlet temperature reactivity coefficient,
δT_{in} = inlet temperature change from normal value T_{in},
$\delta\rho_{ext}$ = external reactivity insertion.

Meanwhile, if ρ_0 =initial reactivity of the system ($\rho_0 = 1 - 1/k_{eff}^0$, k_{eff}^0 =initial effective multiplication factor), the fission power (P_{fiss}) of ADS can be evaluated by (see Eq. (9))

$$P_{fiss} = E_f \frac{\varphi^*}{\nu} \frac{< S > + < \delta S >}{(-\rho_0 - \rho)}, \tag{14}$$

whrere $< S >$ and $< \delta S >$ are the initial source strength and its deviation from the initial value, respectively. In Eq. (14), assuming the quantity $E_f \varphi^* / \nu$ is constant, the normalized power *P* (normalized to unity at the initial state) can be written as

$$P = \frac{\rho_0}{< S >} \frac{< S > + < \delta S >}{(\rho_0 + \rho)}. \tag{15}$$

Rearranging Eq. (15) for ρ and inserting into Eq. (13), one can obtain the following BOP balance equation:

$$P[\rho_0 + (P-1)A + (P/F-1)B + \delta T_{in} C + \delta\rho_{ext}] - \rho_0 (1 + \frac{\delta i}{i}) = 0, \tag{16}$$

where $< \delta S > / < S >$ was replaced by $\delta i / i$, the fractional change of the accelerator current *i*.

It is noteworthy that Eq. (16) converted to the original BOR equation by letting $\rho_0 = 0$, i.e., critical condition. Also, the validity of Eq. (16) is limited by the approximation $E_f \varphi^* / \nu$ =constant. Obviously, E_f / ν could nicely be assumed to be a constant. However, the source efficiency φ^* may change during the transient, depending on the type of transient. It generally depends on the geometrical configuration of the core, power distribution, source neutron characteristics, etc. For example, introducing a strong absorber around the source region can significantly reduce the source efficiency. On the contrary, inserting the fuel material between the source zone and the fuel blanket generally increases the efficiency. Of course, the core melting would result in a drastic change of φ^*, thus the BOP method cannot be used in such situations. Nevertheless, our experiences show that the variation of φ^* is rather small, within several percent, for a wide range of the coolant temperature change and the power distribution change. Therefore, the assumption could be utilized for many important transients without a big compromise in the accuracy. Furthermore, it is important to note that for a specific transient the change of φ^* tends to show a very similar trend for different initial subcriticality levels. Thus, the above BOP method could be properly used for a relative comparison of the transient responses of ADSs with different subcriticalities, in spite of the possible noticeable error of the absolute values.

From Eq. (16), one can easily obtain the following asymptotic power level

$$P = \frac{A + B - \rho_0 - \delta T_{in} C - \delta\rho_{ext} - \sqrt{[A + B - \rho_0 - \delta T_{in} C - \delta\rho_{ext}]^2 + 4\rho_0 (A + \frac{B}{F})(1 + \frac{\delta i}{i})}}{2(A + \frac{B}{F})} . \quad (17)$$

In the BOP method, the change of coolant outlet temperature from the nominal value T_{out} can be defined by

$$\delta T_{out} = \delta T_{in} + (P / F - 1)\Delta T_c , \quad (18)$$

where ΔT_c is the coolant temperature rise at the full power/flow condition. It is important to note that the overall heat balance of the whole system including the secondary one is not modeled in the above BOP method. Basically, the inlet temperature is affected by the secondary side behavior in the real system. Thus, it can be stated that the BOP method is based on the assumption that there is no intervention in the secondary system. In general, a change of the core outlet temperature results in a change of the inlet one with a significant time lag, which is usually several tens seconds depending on the system design. In the meantime, the core approaches quite fast to an asymptotic state in most transients, compared to the time lag. Thus, it can be a reasonable assumption that the inlet temperature does not change during a time period shorter than the time lag. For a realistic long-term behavior, a dynamic heat balance equation should be solved.

4.2 APPLICATION TO MODEL PROBLEM

With the BOP method, several important transient analyses were performed for the HYPER system to assess the impacts of the subcriticality level on the safety. From the basic reactivity coefficients in Table A.II, one can estimate the three reactivity-related parameters *A*, *B*, and *C* in the BOP method with the following relations[14]:

$$A = (\alpha_D + \alpha_E)\Delta T_{FC} = -97.3\, pcm$$
$$B = (\alpha_D + \alpha_E + \alpha_{LBE} + 2\alpha_R)\Delta T_c / 2 = -208.5\, pcm$$
$$C = (\alpha_D + \alpha_E + \alpha_{LBE} + \alpha_R) = -1.482\, pcm / {}^\circ C$$

In the relation for *A*, ΔT_{FC} means the difference between average fuel and coolant temperatures, which is assume to be 175 °C in this paper. Also, *A* is calculated on the assumption that the fuel expands independently of the cladding. The above A, B, and C values are reasonably assumed to be independent of the subcriticality level of the system.

The BOP method was applied to four types of transient, which are IOR (Insertion of Reactivity), TOC (Transient of Current), CIT-WS (Chilled Inlet Temperature without Scram), LOHS-WS (Loss of Heat Sink without Scram), and LOF-WS (Loss of Flow without Scram). In any accident scenario, shut-off of the accelerator current should result in a zero power state, if the core is always maintained subcritical during the event. Thus, in this paper, the analysis was performed on the assumption that the current is not shut-off. In each transient, the asymptotic power level and the outlet coolant temperature were evaluated for several representative subcriticality levels (k_{eff} =0.96, 0.97, 0.98, 0.99, 0.995) and the results are summarized in Table I.

Table I. Impacts of subcriticality on accident response ($T_{in} = 340$, T_{out} =510 °C)

k_{eff}	IOR (270 pcm insertion)*	TOC (100% increase of current)	CIT-WS	LOHS-WS	LOF-WS (10% natural circulation)
0.995	P =1.41 T_{out} =579 °C	P =1.52 T_{out} =598 °C	P =1.20 T_{out} = 445 °C	T_{out} >>1000 °C	P =0.44 T_{out} =1083 °C
0.99	P =1.24 T_{out} =551 °C	P =1.66 T_{out} =623 °C	P =1.12 T_{out} = 431 °C	T_{out} >>1000 °C	P =0.54 T_{out} =1254 °C
0.98	P =1.13 T_{out} =532 °C	P =1.79 T_{out} =644 °C	P =1.07 T_{out} = 421 °C	T_{out} >>1000 °C	P =0.65 T_{out} =1441 °C
0.97	P =1.09 T_{out} =525 °C	P =1.85 T_{out} =653 °C	P =1.05 T_{out} =418 °C	T_{out} >>1000 °C	P =0.71 T_{out} =1551 °C
0.96	P =1.06 T_{out} =521 °C	P =1.88 T_{out} =659 °C	P =1.05 T_{out} =418 °C	T_{out} >>1000 °C	P =0.76 T_{out} =1625 °C

* corresponding to 1$

In case of IOR, it is assumed that the accelerator current is maintained constant during the incident, i.e., $\delta i = 0$. Also, the inlet temperature is assumed not to change during the event: the increased power is absorbed in the secondary system. As is expected, in this IOR case, a large subcriticality is favorable, although the difference in the power change is rather small for the subcriticality range $0.96 \le k_{eff} \le 0.98$. It can be said that the larger the subcriticality is, the better the system response would be.

For the TOC event, the accelerator current was doubled relative to the initial one. This large variation of the current is to account for the large burnup reactivity swing of a TRU-loaded ADS core. Note that the 100% current jump roughly corresponds to 2%Δk for an initial k_{eff} =0.98. Within a relatively short or intermediate time period, the system response can be evaluated with $\delta T_{in} = 0$. For the short/mid term period, Table I shows that a small subcriticality (high k_{eff}) is more favorable than a large subcriticality. This is because the power level is dominantly governed by the source strength in a highly subcritical core, while the negative reactivity feedback effects is relatively large in a slightly subcritical core. In this case, the outlet temperature increase is rather significant, thus the inlet temperature would start to increase as time goes on. This inlet temperature increase in turn would decrease the power level. Thus, from a long-term behavior, it is expected that the core would go slowly to a new equilibrium state with a lower power level and a higher inlet temperature. Previously, Gandini et al[5,6] concluded that a higher subcriticality is desirable in case of TOC, contrary to our observation. This contradiction is because they used an approximate formula, instead of the correct form such as Eq. (17), to evaluate the power level.

The system response for the CIT-WS case was calculated with an inlet temperature decrease of 100 °C. Table I shows that the responses for the four subcriticality levels are all acceptable in this case. It is observed that a large subcriticality provides a little better performance. This is because an inlet temperature decrease is basically equivalent to an insertion of a positive reactivity. However, the sensitivity of the response to the subcriticality level is rather weaker than in the IOR case, since the negative feedback effect is more effective in the high k_{eff} regime. It might be said that the CIT-WS accident does not incur a serious safety concern in ADS.

In the LOHS-WS case, the secondary heat exchanger assumed to fail with a constant accelerator current. Obviously, the outlet temperature would increase constantly since there is no heat loss from the primary system, in spite of the significant negative reactivity feedback due to the increased coolant temperature. During this event, the fission power would slowly decrease due to the negative reactivity feedback. Consequently, it can be said that a high k_{eff} core would have a slower rate of the coolant temperature increase, since the reactivity feedback effect is more significant in a slightly subcritical core. By the way, it is clear that the ADS core cannot achieve a deterministic safety level without an inherently passive device to stop the accelerator current.

Finally, in the LOF-WS event, it was assumed that the inlet temperature does not change while the coolant flow coasts down to a natural circulation. In this work, a 10% natural circulation was assumed for the LOF-WS event. One can see that a high k_{eff} core provides a little more favorable response than a low k_{eff} core, since the negative reactivity feedback is more effective in a slightly subcritical core. Clearly, in this event, the coolant temperature rise is practically unacceptable in all the subcriticality levels. It should be noted that if time goes on the coolant temperature would further increase and the power level would gradually decrease.

Clearly, if the accelerator beam is not shut off, the outlet temperature would be unacceptably high in both LOHS-WS and LOF-WS cases. Therefore, the ADS should be equipped with a very reliable beam shutdown system. On the other hand, in case of a liquid target ADS with a beam window such as HYPER, there is some possibility that, during LOHS-WS and LOF-WS, the window might fail, leading to an beam shut-off before the coolant outlet temperature reaches an unacceptably high point. However, this kind of fail-safe effect might not be expected in a windowless target system, since the target system is almost independent of the reactor coolant system.

5. IMPACTS OF SUBCRITICALITY ON TRANSMUTATION PERFORMANCE

The essential objective of the transmutation of radioactive materials is to minimize the release of those radiotoxic nuclides into environment. In general, the transmutation of TRUs and LLFPs is based on a repetitive recycling of the discharged material into the transmuter, since a complete transmutation is virtually impossible in a single fuel cycle. Therefore, the discharge burnup of TRUs and LLFPs should be as high as possible in order to minimize a loss of raditoxicity to the environment during the recycling stage.

In an ADS transmuter with a fixed power level, the TRU transmutation rate, i.e, TRU consumption per cycle, is almost independent of its subcriticality. However, the discharge burnup of fuel generally depends on the subcriticality since the fuel inventory changes with the level of subcriticality. If it is assumed that the subcriticality is adjusted by the fuel loading, not by an absorber movement, it is clear that a large subcriticality level would be favorable from the fuel burnup point of view due to the reduced fuel inventory. On the other hand, the transmutation capability of LLFP of an ADS is also dependent on the subcriticality level since LLFPs are transmuted through the neutron absorption reaction. It is noteworthy that if the subcriticality is controlled by the fuel inventory, a higher subcriticality provides more surplus neutrons available for the LLFP transmutation due to the increased external source. Obviously, a large subcriticality would enhance the LLFP transmutation performance of an ADS.

In order to quantify the impact of the subcriticality level on the transmutation performance, numerical analyses were performed for the HYPER core introduced in Section 2. The core analyses were done using the equilibrium cycle method of the REBUS-3[17] code system. Unlike the current design in Appendix, Tc-99 was also transmuted in the system. Note that Tc-99 is considered as one of the problematic LLFPs due to its high mobility in a geological repository. In this work, a metallic Tc-99 was co-mingled with the fuel and it is assumed that Tc-99 is completely recovered during the reprocessing stage of the fuel and recycled into the core. For a systematic comparison of the Tc-99 transmutation performance, the inventory of Tc-99 was fixed at 124kg. The results are summarized in Table II.

Table II compares the Tc-99 transmutation characteristics for three representative values of subcriticality, i.e., 0.97, 0.98, 0.99, in terms of transmutation rate and discharge burnup. One can see that a higher subcriticality provides a better transmutation performance in terms of absolute (kg/cycle) and relative (%/cycle) transmutation rates, although the differences between the three cases are rather small. Note that discharge burnups of Tc-99 and the TRU fuel for k_{eff} =0.97 are enhanced by ~8% and ~3%, respectively, compared with those of the case k_{eff} =0.99. It is worthwhile to note that the absolute transmutation rate for k_{eff} =0.99 could be made significantly larger by loading more Tc-99. However, the increased Tc-99 inventory would necessarily result in a lower discharge burnup of both Tc-99 and the fuel.

Table II. Impacts of subcriticality on the Tc-99 homogeneous transmutation

Initial k_{eff}	Initial inventory, kg Tc-99 / Fuel	Tc-99 transmutation			Fuel discharge burnup, a/o
		%/cycle	kg/cycle	Discharge Burnup, a/o	
0.97	124 / 4644	1.99	2.47	14.9	21.84
0.98	124 / 4722	1.93	2.39	14.5	21.53
0.99	124 / 4801	1.87	2.32	14.0	21.23

6. SUMMARY AND CONCLUSIONS

In an ADS, the subcriticality level could be consistently defined with the conventional effective multiplication factor since the subcriticality should be measured as a distance from the critical condition, i.e., off-criticality. Thus, it is proposed that (1-k_{eff}) or (1-k_{eff})/k_{eff} could be used as the definition of the subcriticality in ADS. The so-called source multiplication factor might be utilized as a measure of the source multiplication efficiency of the external source, relative to the fission neutrons.

The minimum required subcriticality should be determined by the operational and safety requirements of the ADS. From the operational viewpoint, the ADS core is required to remain subcritical over the whole operational regime including the fuel loading stage, without any active insertion of negative reactivity. Concerning the safety requirements, it is very difficult to derive any decisive guideline. In an ADS with dedicated fuels such as TRU and MA, it is clear that an absolute safety can hardly be achieved by the subcriticality itself, from the practical point of view. Therefore, we propose that the minimum subcriticality should be sufficiently large to accommodate the temperature defect from a hot full power to a cold fuel-loading state and the beam tube failure accident (in case of the liquid target ADS). Meanwhile, the maximum allowable subcriticality level might be determined such that the ADS design could be technically feasible and the system could be operated economically as well. For a maximum accelerator current of 20mA, it seems that k_{eff} should be larger than at least 0.961 to maintain the beam window integrity in a typical 1000 MWth, LBE-cooled ADS with a single central source. Using the proposed scheme, the maximum allowable k_{eff} turns out to be about 0.984 for a TRU-loaded model ADS.

The subcriticality of ADS does not always improve the system safety features. Obviously, the larger the subcriticality, the better system response for the reactivity insertion accident. Also, the subcriticality results in a little more favorable behavior during the chilled inlet temperature incident. However, in the cases of TOC (Transient of Current), LOHS-WS (Loss of Heat Sink without Scram), LOF-WS (Loss of Flow without Scram), a higher subcriticality might provide a worse system response. Especially, the LOHS-WS and LOF-WS accidents may lead to severe core damage if the accelerator current cannot be shut off in a passive way. Thus, the ADS should be equipped with a highly reliable beam shutdown mechanism. With respect to the TOC case, the burnup reactivity swing needs to be minimized for a better transient response. Clearly, the subcriticality of an ADS should be determined by accounting for both the positive and negative impacts of the subcriticality.

With respect to the transmutation performance of an ADS, a higher subcriticality is favorable in terms of the fuel discharge burnup. Also, the transmutation of an LLFP could be done more efficiently in a higher subcriticality core. However, the transmutation performance is relatively insensitive to the subcriticality level. Therefore, this transmutation performance might not be a crucial parameter in determining the subcriticality level of an ADS.

The ADS cannot be designed such that an absolute deterministic safety could be guaranteed. Instead, the surmised safety advantages should be addressed in a relative way. As a concluding remark, it is stated that, in a typical TRU (or MA)-loaded ADS, a practically applicable subcriticality might be found in the range $0.96 \le k_{eff} \le 0.98$, depending on the design specifications.

REFERENCES

1. A Roadmap for Developing Accelerator Transmutation of Waste (ATW) Technology, DOE/RW-0519 (1999).
2. "Accelerator-Driven Systems: Energy generation and transmutation of nuclear waste," Status report, IAEA-TECDOC-985 (1997).
3. R. Soule et al., “Validation of Neutronic Methods Applied to the Analysis of Fast Subcritical Systems: The MUSE-2 Experiments,” International Conference on Future Nuclear Systems (GLOBAL ’97), Yokohama, Japan, October 5-10, Vol. 1, p. 639 (1997).
4. K. Kobayashi and K. Nishihara, “Definition of Subcriticality Using the Importance Function for the Production of Fission Neutrons,” Nuclear Science and Engineering, **136**, 272 (2000).

5. A. Gandini, M. Salvatores, and I. Slessarev, “Balance of Power in ADS Operation and Safety,” Annals of Nuclear Energy, **27**, 71 (2000).
6. A. Gandini, M. Salvatores, and I. Slessarev, “Coupling of Reactor Power with Accelerator Current in ADS System,” Annals of Nuclear Energy, **27**, 1147 (2000).
7. R. A. Krakowski, "Accelerator Transmutation of Waste Economics," Nuclear Technology, **110**, 295 (1995).
8. M. Salvatores, “A Global Physics Approach to Transmutation of Radioactive Nuclei,” Nuclear Science and Engineering, **116**, 1 (1994).
9. J. Lewins, *Importance, The Adjoint Function*, Pergamon Press, New York (1964).
10. Y. H. Kim and W. S. Park, “Characterization of Source Importance Function in Accelerator-Driven System,” Submitted to Nuclear Science and Engineering (2001).
11. W. Maschek et al., “Safety Analyses for ADS Cores with Dedicated Fuel and Proposals for Safety Improvements,” Proceedings of the IAEA Technical Committee Meeting on Core Physics and Engineering Aspects of Emerging Nuclear Energy Systems for Energy Generation and Transmutation, Argonne National Laboratory, Nov. 28-Dec. 1, 2000 (to be published).
12. W. S. Park et al., "Fission Product Target Design for HYPER System," 6th Information Exchange Meeting on Actinide and Fission Product Partitioning and Transmutation, Madrid, Spain, December 9-14 (2000).
13. Y. Kim et al., " Core Design Characteristics of the HYPER System," to be presented at 7th Information Exchange Meeting on Actinide and Fission Product Partitioning and Transmutation, Jeju, Korea, October (2002).
14. D. Wade, “LMR Core Design for Inherent Safety,” NEACRP Meeting Proceedings, Paris, September (1986).
15. T. Y. Song et al., "Thermal and Stress Analysis of HYPER Target System", 6th Information Exchange Meeting on Actinide and Fission Product Partitioning and Transmutation, Madrid, Spain, December 9-14 (2000).
16. N. I. Tak et al., “Numerical Studies on Thermal-Hydraulics of HYPER,” Proceedings of Accelerator Applications/Accelerator-Driven Transmutation Technology and Application ‘01(AccApp/ADTTA ’01), Reno, Nevada, USA, November 12-15 (2001).
17. B. J. Toppel, “A User’s Guide to the REBUS-3 Fuel Cycle Analysis Capability,” ANL-83-2, Argonne National Laboratory, 1983.
18. K. L. Derstine, “DIF3D: A Code to Solve One-, Two, and Three-Dimensional Finite Difference Diffusion Theory Problems,” ANL-82-64, Argonne National Laboratory, April 1984.

APPENDIX

Core Characteristics of the HYPER system

The HYPER (Hybrid Power Extraction Reactor) system is being studied in Korea for the transmutation of both TRUs and long-lived fission products. HYPER is a 1,000 MWth LBE (Lead-Bismuth Eutectic)-cooled ADS with a single central spallation source and a beam window is used to separate the proton beam delivery tube (30cm diameter) from an LBE target, as shown in Fig. A.1. In HYPER, 1GeV proton beam impinges on the LBE target and generates the spallation neutrons. Currently, the maximum allowable subcriticality is set to k_{eff} =0.98.

The core is loaded with a ductless fuel assembly containing a TRU dispersion fuel, in which TRU-10Zr fuel particles are dispersed in a Zr matrix. It is assumed that TRU elements are obtained by removing all fission products and 99.95% uranium from the PWR spent fuel of 33 GWD/MTU burnup. Consequently, uranium is about 4.6 w/o in the fuel composition. All structural materials are HT-9 steel in the HYPER core. The P/D (Pitch-to-Diameter) ratio of the fuel lattice is 1.5 and the active core height is 160cm. In HYPER, a relatively high core height is adopted to maximize the multiplication efficiency of the external source.

In the ductless fuel assembly, 13 non-fuel rods are used as the tie rod to maintain the assembly mechanical integrity. However, the conventional duct is used for the reflector and shield assemblies to adjust the coolant flow rate. The reflector assembly is composed of a Pb-filled HT-9 rod to improve the neutron economy and to minimize generation of the radioactive materials, too. Although the core is subcritical, 6 safety assemblies are placed for the emergency case. Besides, the HYPER core is equipped with a special safety system (auxiliary shutdown system in Fig. A.1) which can make the source efficiency very small. In case of an emergency, a thick B_4C absorber is inserted into the buffer zone and surrounds the spallation source region, reducing the fission power to several percent level without shutting off the accelerator current. This kind of safety system can serve as an independent core shut-down system in ADS.

An equilibrium cycle analysis was performed with the REBUS-3[17] code for the HYPER core with a half-year cycle length (140 full power day). In this analysis, the scattered fuel reloading scheme was applied to the three core zones (inner, middle, outer), separately. In the inner zone, a 7-batch fuel management was used and an 8-batch reloading scheme was utilized for both middle and outer zones. In the HYPER core, each fresh fuel assembly is assumed to have different TRU enrichment to control the power peaking factor. In the REBUS-3 analysis, it is assumed that all the fuel elements are completely recovered and recycled into the core and 5% of the rare earth elements are carried over during the fuel reprocessing/fabrication processing.

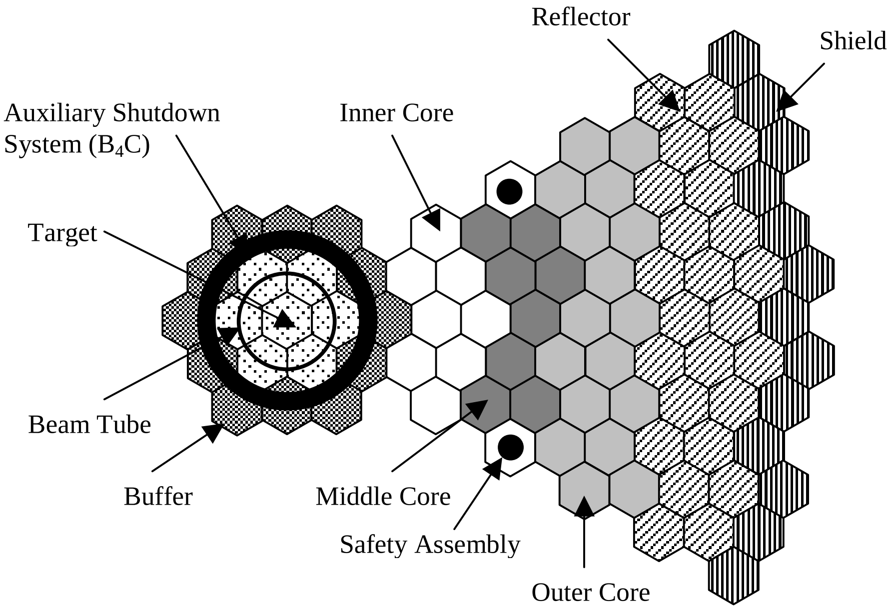

Fig. A.1. Schematic configuration of the HYPER core.

In general, the burnup reactivity swing is very large in a TRU-loaded core since the fuel contains a small amount of fertile (uranium) elements. To reduce the reactivity change during the core burnup, a B_4C burnable absorber is used in the HYPER core. For efficient depletion of the B-10 absorber, the burnable absorber is loaded only in the central part (92cm long) of the 13 tie rods of each assembly. Also, the burnable absorber is not applied in the inner-most fuel ring since it can hamper the source multiplication efficiency. In the current HYPER design, the contents of the B_4C absorber was determined such that the burnup reactivity swing is roughly equal to 3.0% Δk. It is worthwhile to note that the reactivity change is over 5%Δk in the HYPER core without the burnable absorber. Table A.I summarizes the main results of the equilibrium analysis of the HYPER core. In Table A.II, important reactivity coefficients and some reactivity effects are provided. Those reactivity-related quantities were evaluated at the beginning of cycle (BOC) of the HYPER core with the DIF-3D[18] code. More detailed analysis results for HYPER can be found in Ref. 13.

Table A.I. Equilibrium cycle performance of the HYPER core

Average Fuel Weight Fraction	Inner Zone	39.1
	Middle Zone	45.3
	Outer Zone	48.7
Effective Multiplication Factor (k_{eff})	BOC	0.98042
	EOC	0.95111
Burnup Reactivity Loss (%Δk)		2.93
Core-Average Power Density (kW/*l*)		137
3-D Power Peaking Factor	BOC	1.67
	EOC	1.95
Average Fuel Discharge Burnup (a/o)		21.9
Average B-10 Discharge Burnup (a/o)		46.0
Net TRU Consumption Rate (kg/year)		290
Equilibrium Loading (kg/year)	LWR TRU	290
	Recycled TRU	1036
	Total TRU	1326
Heavy Metal Inventory (kg)	BOC	4642
	EOC	4497

Table A.II. Reactivity coefficients and effects of the HYPER system

LBE coolant density variation, α_{LBE}	+0.045 pcm/°C
Fuel Doppler effect at nominal temperature, α_D	–0.031 pcm/°C
Radial core expansion, α_R	–0.971 pcm/°C
Axial fuel element expansion, α_E	–0.525 pcm/°C
Reactivity change due to the window failure	+753 pcm
LBE void reactivity (in active core only)	+2,745pcm
Reactivity change due to complete coolant loss	–24,834 pcm