THEORY AND ANALYSIS OF THE FEYNMAN-ALPHA METHOD FOR DETERMINISTICALLY AND RANDOMLY PULSED NEUTRON SOURCES

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ABSTRACT

In future planned accelerator driven subcritical systems (ADS), as well as in some recent related experiments, the neutron source to be used will be a pulsed accelerator. For such cases the application of the Feynman-alpha method for measuring the reactivity is not straightforward. The dependence of the Feynman $Y(T)$ curve (variance-to-mean minus one) on the measurement time T will show quasi-periodic ripples, corresponding to the periodicity of the source intensity. Correspondingly, the analytical solution will become much more complicated. One can perform such a pulsed Feynman-alpha measurement in two different ways: either with synchronising the start of each measurement block with the pulses ("deterministic pulsing") or with not synchronising ("random pulsing"). The variance-to-mean has been determined analytically for such cases recently and reported very briefly in previous publications. Their properties as well as their advantages and disadvantages will be analysed here in detail. It is found that the stochastic pulsing leads to an analytic solution which is much simpler than that for the deterministic case, and the relationship between the pulsed and continuous source is much more straightforward than in the deterministic case. However, the amplitude of the ripples, constituting a deviation of the pulsed Feynman Y curve from the smooth curve corresponding to the traditional constant source case, is much larger for the stochastic pulsing than for the deterministic one. The reasons for this are also analysed in the paper. Some comparisons with measurements, made by other groups in the EC-supported project MUSE, are referred to, and they confirm the trends found theoretically.

1. INTRODUCTION

Accelerator driven subcritical reactors (ADS) represent many new challenges in general, and for fluctuation based reactivity measurement methods in particular. Continuous monitoring of the subcritical reactivity is desired in such systems, and it is natural to investigate the applicability of the known methods, previously used in traditional systems. One such method is the Feynman-alpha or variance-to-mean method ([1] and [2]).

The theory of the Feynman-alpha method was originally elaborated for continuous, single emission sources, i.e. for traditional radioactive sources that emit one neutron at a time, and whose intensity is constant. Future ADS will, however, operate with sources that deviate from the simple radioactive sources. In a commercial ADS, a spallation source will be used, which emits several, correlated neutrons per source emission. This aspect of the Feynman-alpha method has already been treated theoretically in recent work ([3] and [4]).

Another anticipated difference between an ADS and a traditional subcritical system with a source will be that the accelerator driven source, either a spallation or a simple D-D or D-T neutron generator, will probably be driven in a pulsed mode. A periodically driven source is time-dependent, thus the corresponding Feynman-alpha formula will again differ from the continuous one, which has to be derived. Actually, one can perform or evaluate such pulsed measurements in two different ways. One is when each measurement gate time for the neutron counters is synchronised with the start of a pulse. Such an alternative will be called deterministic pulsing. The other alternative is when the gate start time is not synchronised with the pulsing, rather it is at random times as compared to the pulse start times. This variant will be called stochastic pulsing (or just stochastic source), for obvious reasons.

The theory of the Feynman-alpha measurements, corresponding to these two cases, was worked out recently ([5]-[7]). Both methods have advantages as well as disadvantages. Originally, the deterministic case was considered since performing the measurement in a synchronised way is straightforward, and the formulation of the corresponding theory and solution methods is also straightforward. Not surprisingly, however, it was found that the resulting Feynman Y-curve is not smooth, rather it contains ripples, corresponding to the source periodicity. Such a source (and thus the induced neutron noise) is not stationary (i.e. not invariant to an arbitrary time shift, only to a shift equal to a multiple of the pulsing period). However, as it is easy to show, such a source remains uncorrelated in time. Due to the non-stationarity, the solution is given as a piecewise analytic function, which does not appear easily suitable for estimating the prompt neutron time constant from a measurement with curve fitting.

Hence the idea arose to open the counter time gates in an uncorrelated, random way in comparison with the pulses. In such a case the source, and thus also the induced neutron noise, become stationary. Correspondingly, by applying complex function techniques, a solution was obtained which is, in one way, “smoother” than the solution corresponding to the deterministic pulsing. However, this solution will too contain ripples, although they can be described by simple periodic functions. Compared to the deterministic pulsing, the amplitude of the ripples, i.e. the deviation from the smooth curve of the traditional Feynman-alpha method, will be larger for the case of stochastic pulsing. As it will be shown in this paper, the reason for this can be traced back to the fact that although the emission of a neutron is still completely independent from the emission of another neutron, due to the source intensity modulation, temporal correlations will exist between the source neutrons. The integral of these correlations is positive, thus the variance of such a source will be over-Poisson, hence the larger ripples in the Feynman-alpha or $Y(T)$ curve.

In the original publications containing the derivation of the pulsed Feynman-alpha method ([5]-[7]), only the solution methods and the resulting formulas are given. The purpose of the present paper is to supply a comparative analysis of the two methods, and to give a discussion of the underlying physics and statistical properties of the basic processes. This way a better insight is given into the characteristics of the resulting formulas, and hence also into their advantages and disadvantages. Finally, comparisons with experiments are also possible at this stage, even if at a very preliminary form. These measurements were performed some other research groups, participating in the EC-supported project MUSE.

The paper consists of two main parts. In Section 2, the concept of the deterministic and stochastic pulsing is introduced, and the basic statistical properties of such pulses are derived. In Section 3 the variance-to-mean for the deterministic and stochastic pulsed subcritical systems is recalled from the recent publication. A comparative analysis of the two methods is made, based on the results about the source properties. Comparison with recent measurements made in the frame of the MUSE project confirms the theoretical findings.

2. BASIC SOURCE PROPERTIES AND STATISTICS

We have to make certain assumption and definitions regarding the nature and the statistical properties of the source. We shall assume that the source emission of a single neutron is a Poisson process, i.e. the probability of the emission of a neutron during an infinitesimal time interval dt is equal to

$$P_1(dt) = S(t)dt \tag{1}$$

where $S(t)$ is the (possibly time-dependent) source intensity. For a radioactive decay, this assumption works reasonably well with a constant $S(t)$, at least for times much shorter than the half-life of the decaying nuclide. This also incurs that the neutron emissions at different infinitesimal time intervals around t_1 and t_2 are uncorrelated (although, in general, not independent for time-dependent sources):

$$C(t_1, t_2) \equiv \langle n(t_1)n(t_2)\rangle - \langle n(t_1)\rangle\langle n(t_2)\rangle = S(t_1)S(t_2) - S(t_1)S(t_2) = 0 \tag{2}$$

where $n(t)dt$ is the probability of emitting one particle within dt around t For deterministic source pulsing (or for any deterministic function $S(t)$), $n(t)$ is equal to $S(t)$, as the above relationship also shows.

The vanishing of the correlations between emissions at different times is also equivalent with the fact that the process has a Poisson variance, i.e. the variance of the number of neutrons emitted during a time interval T is equal to the expected value of emissions. This follows from the known fact that [8]

$$\sigma_N^2(T) = N(T) + \int_0^T C(t_1, t_2)dt_1 dt_2 \tag{3}$$

Eqn (3) shows that with a vanishing correlation the variance will become Poisson. Eqn (3) also shows that the variance can be a Poisson one even if the correlation function is non-vanishing, provided its integral over $(t_1, t_2) \in R_2$ is equal to zero. Positive correlations (which are generated e.g. in the case of branching) lead to an over-Poisson variance $(\sigma_N^2 > N)$ whereas negative correlations, usually resulting from exclusive processes such as those due to conservation relations, have under-Poisson variance. Of course, in the present case not only the variance is Poisson, but the whole number distribution of the emitted particles is a Poisson distribution with a time dependent parameter $S(t)$, as long as $S(t)$ is a deterministic function. The relationship between the correlations and the variance was only recalled here because it will be made use of in the interpretation of the results of the stochastically pulsed source.

In the experiments that we shall analyse with respect to the Feynman-alpha method, the source is a periodically pulsed neutron generator. It is customary to assume that in the continuous mode of operation with constant ion current, the production of neutrons is a Poisson process described by (1). This is equivalent with the assumption that the generation of deuterium ions, as well as their reaction rate in the target, follow Poisson statistics. The intensity is though not constant, rather a periodic function as is the case with a pulsed neutron generator. The pulsing is achieved by chopping the deuterium beam electronically. This is how the GENEPI neutron generator will be used in the EC-supported project MUSE [9]. It is a fairly good assumption that the chopping does not affect the statistics of the deuterium beam (and thus the statistics of the generated neutrons) by any other means than imposing a time-dependent intensity function $S(t)$. The pulsing can then be approximated by a train of square pulses:

$$S(t) = S\sum_{n=-\infty}^{\infty}\{H(t - nT_0) - H(t - W - nT_0)\} \tag{4}$$

where W is the pulse width and T_0 the repetition period. In reality, the pulses will not be square, rather have a smoother form such as a bell-shaped pulse [9], but (4) is a good first approximation.

Eqn (4) describes the case of deterministic pulsing. Physically, this is always the case with the pulse generation, since the generator cannot be pulsed randomly. However, in order that (4) can be used in calculation of the Feynman-alpha function, the measurements always need to start at the same time in relation to the pulses, say at the beginning of a pulse.

For the deterministic pulsing, as was shown earlier, the variance of the number of emitted neutrons during a time interval T is equal to the mean, i.e. the relative variance is unity. However, neither mean nor the variance is a smooth function, rather they are piecewise smooth. This can be seen by starting with the full probability distribution of the time-dependent Poisson process:

$$P_N(t) = \frac{e^{-\int_0^T S(t)dt}\left[\int_0^T S(t)dt\right]^N}{N!}, \tag{5}$$

or its probability generating function

$$G(Z, t) = \sum_{N=0}^{\infty} Z^N P_N(t) \tag{6}$$

which is given from (5) as

$$G(Z, T) = e^{(Z-1)\int_0^T S(t)dt} \tag{7}$$

From either expression it follows that

$$\langle N(t)\rangle \equiv N(t) = \int_0^T S(t)dt =$$

$$= \begin{cases} SnW + S\dfrac{W}{T_0}T & \text{for} \quad nT_0 \le T \le nT_0 + W \\ S(n+1)W & \text{for} \quad nT_0 + W \le T \le (n+1)T_0 \end{cases} \tag{8}$$

The mean value, and hence also the mean, is thus a piecewise smooth function with linear and constant sections alternating. The variance-to-mean is, on the other hand, constant and equal to unity.

When such a source is applied to drive a subcritical system, the statistics of the detector counts will not obey pure Poisson statistics, as it was shown in [5] and will also be recited below. In that case also both the mean and the variance become complicated piecewise analytic functions. Such a case, being the consequence of the non-stationarity of the deterministically pulsed source, clearly has some disadvantages both from the viewpoint of measurement techniques as well as from the viewpoint of evaluating the measurement.

It was for these reasons that the idea of “randomizing” the source was suggested. This randomisation is achieved in practice by choosing the start of each measurement block at random (i.e. not synchronised) relative to the time of the pulses. If for each measurement one sets the (random) gate start time equal to zero, then the beginning of the pulse train is at random times. This amounts to having a “stochastically pulsed” source (or for short: a stochastic source). Such a source can be described by the source function

$$S(t,\xi) = S\sum_{n=-\infty}^{\infty}\{H(t-nT_0-\xi)-H(t-W-nT_0-\xi)\} \tag{9}$$

where ξ is a random variable, distributed uniformly in $(0, T_0)$, i.e.

$$p(\xi) = \frac{1}{T_0}. \tag{10}$$

A source emission process, described by the above source function, is called a doubly random Poisson process [10]. In general, any Poisson process with a parameter which itself if a random process or a random variable is called a doubly random Poisson process. The advantage of the stochastically pulsed source of the type (9) is that it is stationary, i.e. its distribution is invariant to an arbitrary time shift. In particular, the expected number of source emissions is a smooth linear function of time.

On the other hand, doubly random Poisson processes, in general, have over-Poisson variance, which we will also find in our case. This is related to the fact that the stochastic source becomes correlated, with the positive correlations dominating. The non-zero correlations, that will be calculated concretely soon, can be illustrated by noticing that for a time difference of

$$W < |t_2 - t_1| < T_0, \tag{11}$$

$$C(t_1, t_2) \equiv \langle n(t_1)n(t_2)\rangle - \langle n(t_1)\rangle\langle n(t_2)\rangle = -\langle n(t_1)\rangle\langle n(t_2)\rangle = -\left(\frac{SW}{T_0}\right)^2 \neq 0 \tag{12}$$

since for such time lags,

$$\langle n(t_1)n(t_2)\rangle = 0 \tag{13}$$

because either t_1 or t_2 must lie outside a pulse whatever value the random variable ξ takes. The last equality in (12) follows from (16) below. For the case (11), the correlations are negative, but for most of the complementing time lag intervals the correlations are positive, as we will see soon. From these qualitative points it is also clear that the correlations are a periodic function of $|t_2 - t_1| \equiv u$, and thus the variance-to-mean of the stochastic source will also be oscillating ([6] and [7]).

Quantitatively, the generating function of the distribution of the doubly stochastic Poisson process, defined by (9), is obtained from a simple master equation as [6]

$$G(Z, T, \xi) = e^{(Z-1)\int_0^T S(t,\xi)dt} \tag{14}$$

Evaluating the moments of such a doubly stochastic process includes averaging over the various realisations of the random variable ξ. This means that the corresponding moments of the number of emitted particles have to be calculated from the generating function

$$G(Z, T) = \int G(Z, T, \xi)p(\xi)d\xi = \frac{1}{T_0}\int G(Z, T, \xi)d\xi \tag{15}$$

with $G(Z, T, \xi)$ given by(14). It is assumed that the order of the integrals w.r.t. t and ξ is interchangeable.

For the first moment of the number of neutrons emitted by the random source, from (15) one obtains

$$\langle N(t)\rangle = \frac{d}{dZ}G(Z,T)\Big|_{Z=1} = \frac{1}{T_0}\int_0^T\int_0^{T_0} S(t,\xi)d\xi dt = \frac{SW}{T_0}T \quad (16)$$

For $T = nT_0$, the expected value of the stochastic source, eqn (16), equals to that of the deterministic source, eqn (8). That is, the mean value of the stochastic source represents a lower envelope of the piecewise smooth function of the mean value of the deterministic source.

The second moment of the stochastic source can be calculated from

$$\langle N(N-1)\rangle = \frac{d^2}{dZ^2}G(Z,T)\Big|_{Z=1} = \frac{1}{T_0}\int_0^T\int_0^T\int_0^{T_0} S(t',\xi)S(t'',\xi)dt'dt''d\xi$$
$$\equiv \int_0^T\int_0^T K(t',t'')dt'dt'' \quad (17)$$

where

$$K(t',t'') \equiv \frac{1}{T_0}\int_0^{T_0} S(t',\xi)S(t'',\xi)d\xi \quad (18)$$

The kernel $K(t', t'')$ can easily be calculated as a function of $|t' - t''| \equiv u$ by substituting (9) into (18). The dependence of $K(u)$ on u is shown in Fig. 1 below, and the full dependence of $K(t', t'')$ on both

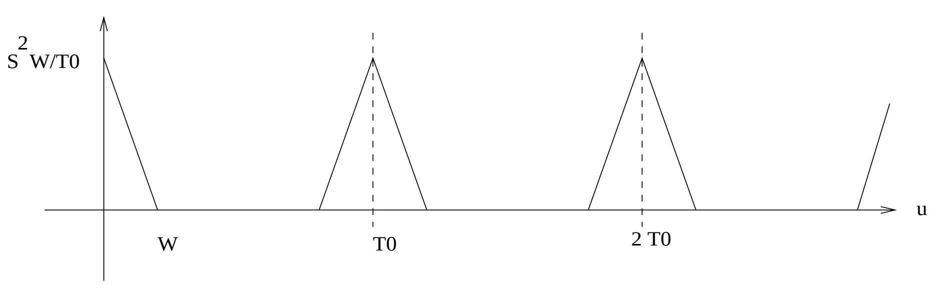

Figure 1. The dependence of the kernel $K(u)$ of Eqn (18) on u

of its arguments is shown in Fig. 2. After the kernel is calculated, the double integral in (17) can also be evaluated, and the second moment is obtained. This latter is though a much more complicated task, and can be solved by Laplace transform and complex function techniques. The details of the calculations will not be given here; these can be found in the internal report [6] and will also be published in a more detailed journal publication. Here only the final result will be given as follows

$$\langle N(N-1)\rangle = \left(\frac{SW}{T_0}\right)^2 T^2 + 2\sum_{n=1}^{\infty}\frac{S^2T_0^2}{n^4\pi^4}\left(\sin\frac{n\pi T}{T_0}\right)^2\left(\sin\frac{n\pi W}{T_0}\right)^2 \quad (19)$$

From here it follows that the variance of the stochastic source is over-Poisson, since

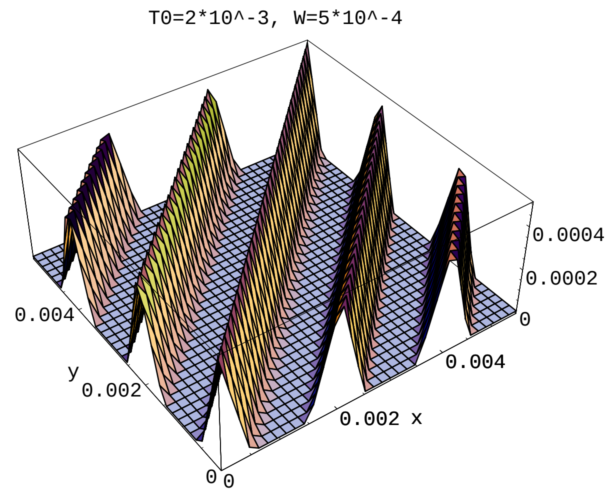

Figure 2. The dependence of $K(t', t'')$ on its arguments

$$\sigma_N^2(T) - N = \langle N(N-1)\rangle - N^2 = 2\sum_{n=1}^{\infty}\frac{S^2T_0^2}{n^4\pi^4}\left(\sin\frac{n\pi T}{T_0}\right)^2\left(\sin\frac{n\pi W}{T_0}\right)^2 > 0 \qquad (20)$$

Another way of putting the result is to express the variance-to-mean of the stochastic source, which is trivially obtained from the above as

$$\frac{\sigma_N^2}{N} = 1 + \frac{2ST_0^3}{W\pi^4}\sum_{n=1}^{\infty}\frac{1}{n^4T}\left(\sin\frac{n\pi T}{T_0}\right)^2\left(\sin\frac{n\pi W}{T_0}\right)^2 \qquad (21)$$

Eqns (20) and (21) show the over-Poisson variance of the stochastically pulsed source. In general, doubly stochastic processes have over-Poisson variance; there are examples of this fact in Ref. [10]. The over-Poisson character can also be explained by the positive integral of the correlation function, and hence, ultimately, by the fact that stochastic pulsing introduces positive correlations between the source particles. From a comparison between (3), (12), (16) and (17) it is clear that

$$C(t', t'') = K(t', t'') - \left(\frac{SW}{T_0}\right)^2. \qquad (22)$$

From the above and from Figures 1 and 2 one can see that the correlations are, on the whole, positive. Eqns (20) and (21) also show that for the cases of $W = T_0$ and $W = 0$ the variance reverts to Poisson; the first case represents the continuous source, whereas the second case is the pathological limit of no particles in the system.

3. THE FEYNMAN-ALPHA FORMULAE FOR DETERMINISTIC AND STOCHASTIC PULSING

In this section we quote the results for the cases of deterministic and stochastic pulsing from [5] and [6], [7], and perform a comparative analysis. In order that the application of the Feynman technique with pulsed sources be feasible, the pulse repetition time should be shorter than $(1/\alpha)$ where α is the prompt neutron time constant and $(1/\alpha)$ the time it takes for the prompt part of the Feynman-alpha curve to go into saturation. If this condition is fulfilled, the pulsed Feynman-alpha (or the Feynman Y) curve will have several oscillations within $(1/\alpha)$, so that the Y curve, corresponding to the continuous one, can be abstracted as some mean value or envelope of the pulsed curve. For the MUSE experiments, this condition is fulfilled, even if not with large margins. The maximum pulse repetition rate of GENEPI is 5 kHz, which gives a pulse repetition time of 200 μsec. This is several times, or at least a few times, shorter than the $(1/\alpha)$ of the MASURCA reactor used in the experiments, even for deep subcritical reactivities.

All calculations in References [5] - [7] were made in a simplified model. The most important restriction of the model is that the delayed neutrons are neglected, only prompt neutrons are assumed. Hence the results are only valid for the prompt part of the Y-curve; it is assumed that the results can be interpolated for the case when delayed neutrons are accounted for by a proper substitution of the prompt neutron time constant.

The Feynman Y-functions given in the following subsections refer to the modified variance-to-mean of the detected neutron counts in a time interval T as

$$Y(T) \equiv \frac{\sigma_Z^2(T)}{\langle Z \rangle} - 1 \tag{23}$$

where Z is the random number of detections during a time gate period T

3.1 DETERMINISTIC PULSING

The solution for this case was derived by classical methods of treating piecewise constant or smooth functions. The results are quoted here from [5] as follows. Let T denote the time as $T = KT_0 + r$, $0 \le r < T_0$, then the Feynman Y curve is given as

$$(T) = \frac{\lambda_d \lambda_f}{\alpha^2} \langle \nu\,(\nu - 1) \rangle \left(1 - \frac{V(T)}{\langle Z(T) \rangle (\lambda_d N_0)} \right) \tag{24}$$

Here the following notations and functions were introduced:

$$\begin{aligned} V(T) &= 2e^{-\alpha T} \left[\frac{(e^{\alpha K T_0} - 1)}{(e^{\alpha T_0} - 1)} R_1 + e^{\alpha K T_0} F_1(r) \right] - \\ & e^{-2\alpha T} \left[\frac{(e^{2\alpha K T_0} - 1) + (1 - e^{\alpha T})^2}{(e^{2\alpha T_0} - 1)} R_2 + e^{2\alpha K T_0} F_2(r) \right], \end{aligned} \tag{25}$$

R_1 and R_2 are constants, whereas F_1, F_2 and $Z(T)$ are piecewise analytical functions, e.g.

$$F_1(r) = \begin{cases} -f_1(\alpha, r) + r g_1; & 0 \le r(t) < W \\ -f_1(\alpha, W) + W g_1 + (r - W) g_2; & W \le r(t) < T_0 \end{cases} \tag{26}$$

with

$$f_1(x, y) = \frac{1 - e^{xy}}{x}. \tag{27}$$

Similar expressions can be derived for F_2 and $\langle Z(T) \rangle$, and thus the whole solution for the variance-to-mean of the deterministic pulsing case is available.

As is seen from the above, the solution is a piecewise smooth function, with a relatively complicated analytic expression. The Feynman Y-curve, corresponding to two different pulse repetition times T_0 and pulse widths W are shown in Figs 3a and 3b. The Y-curve corresponding to a continuous source

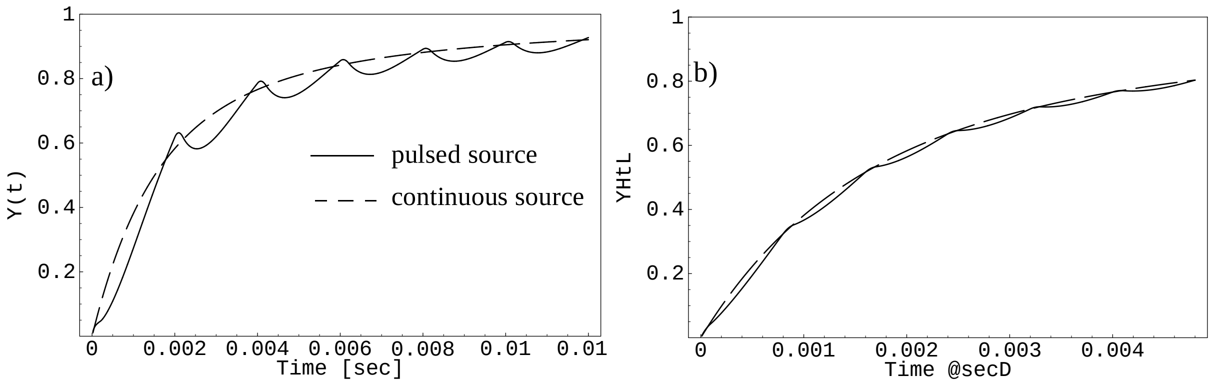

Figure 3. The Feynman Y function (scaled to the asymptotic value) for a deterministically pulsed source and for a stationary source, for two different pulse repetition times and pulse widths. a) T_0=2x10^{-3}s, W=0.2x10^{-3}s; b) T_0=0.8x10^{-3}s, W=0.1x10^{-3}

in the same system is shown in both Figures with a dashed line. It is seen that the curve for the deterministically pulsed system shows periodic oscillations around the curve corresponding to the continuous source-driven case. The fluctuations are though not too large, and with some intuition, the Y-curve of the system, as measured with a continuous source, can be estimated from the pulsed measurement. Fig. 3a corresponds roughly to the case of the MUSE experiments what regards the quantitative relationship between pulse repetition time and prompt neutron time constant. Several relatively large oscillations of the pulsed curve can be observed. To fit a theoretical curve to such a pulsed measurement is not practical, in view of the fact that the solution contains the prompt neutron time constant α in a very implicit way. One possibility, as mentioned also in [5], is to train a neural network with input data consisting of Y curves corresponding to different reactivities, calculated from (24)-(27), to identify the reactivity of an unknown system by using the measured Y-curve as input. Fig. 3b, on the other hand, shows a case when the pulse repetition time is much shorter than $1/\alpha$. In that case several small amplitude fluctuations of the pulsed Y curve around the one corresponding to the continuous source drive system can be observed. In fact the pulsed curve follows the continuous one so closely that the prompt neutron time constant can be inferred by fitting the Y curve of a continuous source to the measurement. Such a high repetition frequency can, however, not be achieved in the current MUSE experiments.

3.2 STOCHASTIC PULSING

The solution for the case of stochastic pulsing is even more complicated than for the variance of the stochastic source itself. However, the same solution method can be applied; the corresponding nested multiple integrals can be cast in the form of convolutions, then Laplace transform methods and inversion with complex function theory methods and residue calculus yield the solution in the form of a very fast converging harmonic series. Details of the solution are given in [6] and [7], here again we only quote the final result for the variance-to-mean as

$$\frac{\sigma_Z^2(T)}{\langle Z(T)\rangle} = 1 + \frac{2\lambda_d S T_0^5 \alpha}{T\pi^4 W}\sum_{n=1}^{\infty}\frac{1}{(4n^6\pi^2 + n^4\alpha^2 T_0^2)}\left(\sin\frac{n\pi T}{T_0}\right)^2\left(\sin\frac{n\pi W}{T_0}\right)^2 + \frac{\lambda_d\lambda_f}{\alpha^2}\langle\nu(\nu-1)\rangle\left(1 - \frac{1-e^{-\alpha T}}{\alpha T}\right) \quad (28)$$

Here the last term corresponds to the Feynman Y-curve with a continuous (constant) source, whereas the second term on the r.h.s. constitutes an oscillating deviation from it. This deviation is non-negative, and thus the Y curve of the continuous source constitutes a lower envelope of the stochastically pulsed system. Further, the solution for the shochastically pulsed case can be given by a relatively simple formula, since the series on the r.h.s. of (28) converges rather fast, so that only one or two terms need to be kept.

However, as predicted in the foregoing, the deviations from the case of the continuous-source driven case are much larger for the stochastic pulsing than for the deterministic pulsing. This is illustrated quantitatively in Figs. 4a and b. It is seen that the oscillating deviations from the case of the constant

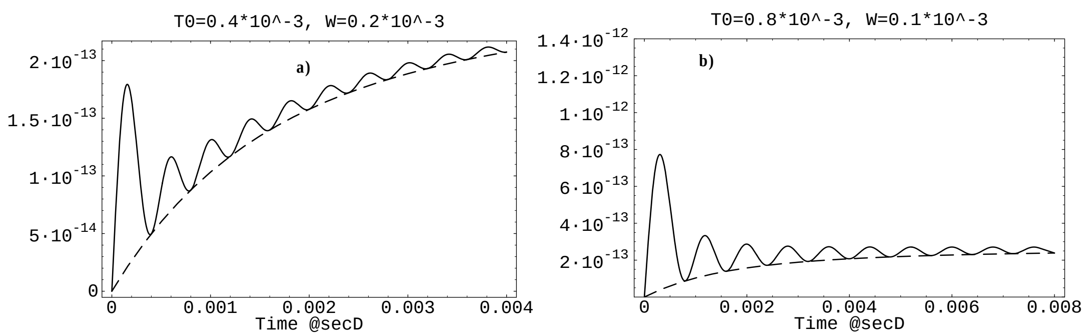

Figure 4. The Feynman Y functions for a stochastically pulsed source and for a stationary source, for two different pulse repetition times and pulse widths.
a) T_0=0.4x10^{-3}s, W=0.2x10^{-3}s; b) T_0=0.8x10^{-3}s, W=0.1x10^{-3}

source are much larger than in the case of the deterministic pulsing. There is a huge overshoot of the pulsed curve at short measurement times T. These large oscillations can be traced back to the positive temporal correlations of the source neutrons for the stochastically pulsed case. Especially for the case of relatively large pulse repetition times T_0, in comparison with $1/\alpha$, the deviations are too large for the applicability of the Feynman technique. The Figure also illustrates the fact that the Feynman curve corresponding to the constant source is obtained as the lower envelope of the stochastically pulsed one. However, due to the large oscillations, this fact does not help much in the practical application of the method. It appears that the deterministically pulsed measurement is superior to the stochastically pulsed one, and thus it should be preferred in the measurements. This requires synchronisation between the pulsing and the start of the measurement blocks.

3.3 COMPARISON WITH MEASUREMENTS

A few measurements have been performed by two different groups in the MUSE project so far and have been communicated to us ([11] and [12]). These measurements were made under circumstances that correspond to the case of stochastic pulsing, since no synchronisation between pulsing and the time gating of the measurement was made. These measurements show up the trends predicted by the theoretical calculations in this paper, as illustrated in Figure 4. These results are, however, very preliminary, and will not be show here. A quantitative comparison between measurements and theory will be shown at the time of the conference.

4. CONCLUSIONS

It was shown that the Feynman-alpha formula in the case of the stochastically pulsed system leads to a simpler analytic expression, and to a solution whose lower envelop equals the Feynman curve corresponding to a system driven with a continuous (constant) source. The solution in the case of the deterministically pulsed case is more complicated and the relationship between the pulsed and continuous case is not so explicit. Despite of all these facts, the case of deterministic pulsing (i.e. synchronising between the pulses and the measurement) is more advantageous, because the oscillating deviations from the smooth Feynman curve, known from the case of the constant source, are much smaller in that case. The evaluation can be performed by fitting a curve, corresponding to the continuous source, to the measurements, or by resorting to more advanced signal analysis methods such as neural networks. The irregularities of the pulsed Feynman curve are smaller the larger the pulse repetition frequency is, hence one should use the highest available pulsing frequency in the measurements.

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