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NODAL PERTURBATION METHODOLOGY FOR A REACTOR OPERATOR-AID SYSTEM

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ABSTRACT

Although the Heuristically based Generalized Perturbation Theory (HGPT) has been conceived for application to linear and non-linear problems in reactor physics at any order, its design implementations have generally been limited to the first order.
Currently, FRAMATOME-ANP is developing an advanced Operator-Aid System, relevant to the PWR control in normal operation (e.g., management of control rod position and/or adjustment of the soluble boron concentration) to respect operating margins, as required by the electrical network load-follow currently adopted in conventional French reactors. It makes an extensive use of perturbative methodologies. Several optimization goals, such as precision, running time and the number of parameters, can be selected.
The option presented here searches for the best accuracy of two main parameters (reactivity and Axial-Offset) within an acceptable running time. It joins Classical Perturbation Theory (CPT) and Heuristic Generalized Perturbation Theory (HGPT) prediction features to fast-running coarse-mesh nodal routines to accelerate convergence of the whole computation process.
The paper describes the implementation of the HGPT-based prediction methodology within the Operator-Aid System and presents a large selection of results.

1. FOREWORD

An advanced electrical network is to be managed in such a way as to adapt the power supply to the grid demand, to achieve competitiveness of exploitation and minimize the costs. According to a nuclear-electric production in France, which accounts for more than 70% of the total supply, EDF (Electricité de France) adopted a daily load-follow to operate a large part of his NPP park.

A suitable operating mode has been elaborated accordingly, to guarantee the respect of safety margins and exploitation conditions during the load-follow driven transients. During these transients, the power level of a conventional French PWR can be dropped from 100% to some 50% or less at a rate of several percents (up to 5%) per minute, then the plant is operated at low power for several hours

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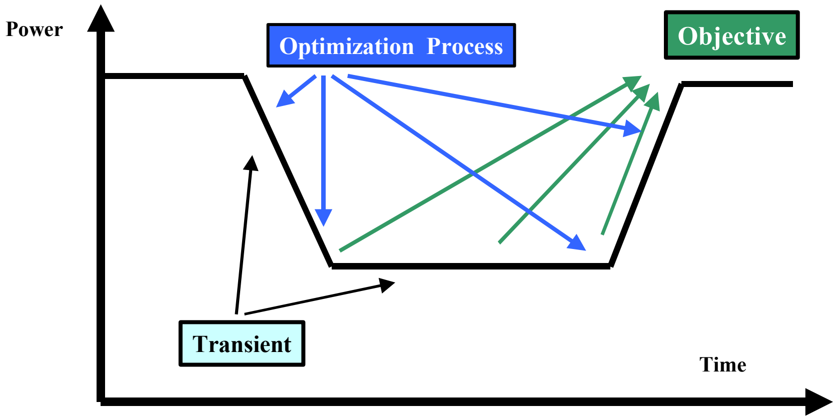

Fig. 1. Typical load-follow transient scheme

and finally, full power is restored roughly in half an hour. This operation is driven by regulating control-groups, reactivity reset being achieved chiefly by soluble boron concentration adjustment. The capacity of the plant to go back to full power at a rate of 5% power change per minute is ensured by the so called "G operating mode".

During the transient, Xe poisoning starts-up oscillations, which are compensated by a fine control-rod adjustment to maintain Axial-Offset close to the nominal value within a maneuver band, the thickness of which depends o the power level. Fig. 1 shows a typical load-follow transient scheme.

At present, some transients demanded by the electrical network could be rejected because of the difficulty in judging with a sufficient precision whether some reactor parameters will remain within the allowed range or not during the whole transient and when back to power. Moreover, all the transients are driven according to a compilation of simple and general rules, which do not take into account the actual status of the system, and are performing fairly, but not always very well.

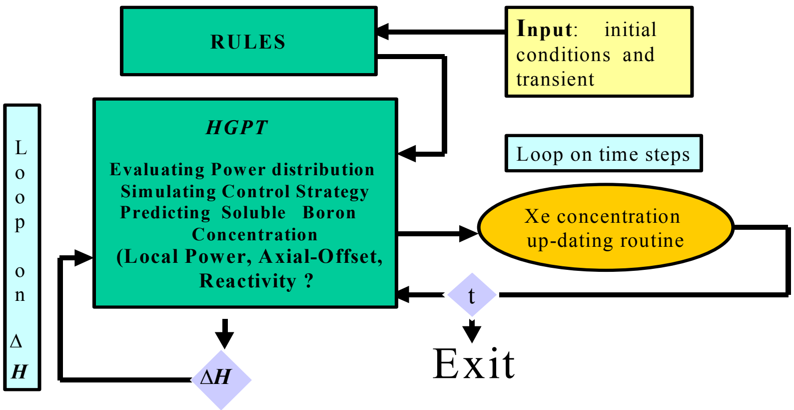

Fig. 2 Operator-Aid System conceptual scheme

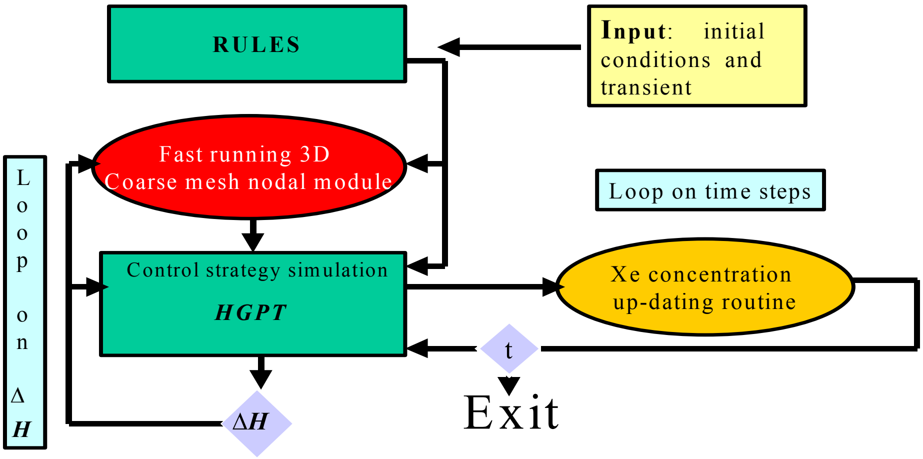

Fig. 3. Simplified scheme of an Operator-Aid System industrial version.

The FRAMATOME-ANP's Operator-Aid System [1] has been devised to help the Operator driving the reactor during the charge transients, maintaining it within the allowed operating range and getting optimal exploitation conditions when back to power. The system conceptual scheme couples CPT and HGPT [2] prediction routines to a suitable spleen function to estimate the 3D power distribution of the whole core after any perturbation, without running any direct iterative calculation, with a limited loss of precision. Knowledge of the actual asymptotic 3D power distribution allows predicting the behavior of the main reactor physical parameters, such as linear power, Axial-Offset, reactivity and shutdown margin. Fig. 2 shows the conceptual scheme of such a system.

When, during normal network load-follow transients strong perturbations occur, they generate huge flux-shape changes. In this case, the HGPT expansion must include several (at least one, the second) higher order terms. In the Operator-Aid system, expansion has been deliberately truncated at the first order to avoid explicit dependence of the reconstruction process on the perturbations themselves. Contribution from higher order terms is accounted for by a suitable weighting coefficient applied to the first order term. This correction mode has been derived heuristically upon the analysis of the alternate converging behavior of the HGPT expansion [3].

This high precision Operator-Aid System is very suitable to PWR control in normal operation. It enables optimizing control rod position and soluble boron concentration during the charge transients. It couples a fast running 3D coarse-mesh nodal module (one mesh per assembly) to CPT / HGPT prediction routines (Fig. 3). The coarse-mesh fast running module reevaluates periodically (at least once per hour, but a higher frequency is allowed) the status of the system, while CPT and HGPT routines predict both reactivity and Axial-Offset at any change of the regulation group position and soluble boron concentration.

2. COMPUTATION ENVIRONMENT

FRAMATOME-ANP has licensed a modern design computation chain, SCIENCE, which couples the transport code APOLLO-2F (which generates cross-sections) to the 3D NEM (Nodal Expansion Method [4 & 5]) core code SMART [6] and to a module devoted to the interpretation of operating experience and its feed-backing to implement chain performance and reduce uncertainty [7].

It is well known that the Boltzmann Operator, as discretized in NEM, is not transposition-invariant, so that the so-called 'physical' and 'mathematical' adjoint fluxes are not congruent functions, unlike the classical finite-difference and finite-element mixed dual schemes, which do enjoy operator invariance against transposition [8]. This problem shows-up as a major threat to the development of Perturbation Theory within NEM computational schemes.

In the SMART code, the problem is even more complicated by the nesting of diffusion and nodal iterations inside the flux convergence loop. That requires computing equivalent diffusion coefficients enabling both modules to converge to the same eigenvalue and flux [6]. This specific computational approach renders the nodal adjoint flux not exploitable, even if discontinuity conditions were imposed on the current instead of on the flux, as stated by Kobayashi [9].

In the framework of implementing HGPT computational features within the SMART code, a wide investigation has been carried out to adapt the original methodology [1] developed for a finite-difference scheme to the NEM computational environment, without introducing any significant change in the perturbation expressions. A suitable, practical solution has been recently found and validated [8]. The first application was made on CPT (Classical Perturbation Theory); it entails the solution of a homogeneous equation to compute the adjoint flux only. Results were very satisfactory indeed and this methodology is now currently adopted for design purposes, e.g. to evaluate the worth of individual control rod cluster groups at reactor start-up.

3. DERIVING HGPT EXPRESSION FOR THE AXIAL-OFFSET

According to the HGPT, after perturbation, the asymptotic value of any observable, which can be written as a ratio of linear integral parameters, as reactivity, Axial-Offset, local power, power per rod, etc…, can be predicted via a limited expansion of order *n*, each term of which, except for the first one, contains suitable derivatives of the flux of order *(n-1)*, called *derivative functions* and an *importance function*, accounting for the first order derivative of the flux [1].

In reference [10], it has been shown that, to avoid fastidious, time-consuming, boring and impractical expansions, higher order contributions can be accounted for by a suitable correction of the first order GPT expansion term [1] quoted as *extrapolation formula.* The HGPT prediction problem can be, that way, reduced to a conventional GPT one.

For the needs of the high precision Operator-Aid System, the only variable of interest for HGPT development is the Axial-Offset, which accounts for the lack in equilibrium of the power between the upper and lower halves of the reactor core. It can be written as follows:

$$AO = \frac{\langle \Sigma_P | \varphi \rangle_h - \langle \Sigma_P | \varphi \rangle_l}{\langle \Sigma_P | \varphi \rangle_h + \langle \Sigma_P | \varphi \rangle_l} \tag{1}$$

where: Σ_P is the averaged cross-section of *power production*, φ is the flux, *the brackets* stand for integration on space and energy and *h* and *l* stand for the upper and the lover halves of the core.

Introducing a simpler quantity linked to the Axial-Offset:

$$R = \frac{\langle \Sigma_P | \varphi \rangle_h}{\langle \Sigma_P | \varphi \rangle_l} \tag{2}$$

and computing its first order variation, we obtain:

$$\frac{R' - R_0}{R_0} = \frac{\delta R}{R_0} = \frac{\langle \delta\Sigma_P | \varphi_0 \rangle_h}{\langle \Sigma_P | \varphi_0 \rangle_h} - \frac{\langle \delta\Sigma_P | \varphi_0 \rangle_l}{\langle \Sigma_P | \varphi_0 \rangle_l} + \frac{\langle \Sigma_P | \delta\varphi \rangle_h}{\langle \Sigma_P | \varphi \rangle_h} - \frac{\langle \Sigma_P | \delta\varphi \rangle_l}{\langle \Sigma_P | \varphi \rangle_l} \tag{3}$$

Defining the *adjoint source* as:

$$\langle S | = \frac{\langle \Sigma_P |}{\langle \Sigma_P | \varphi_0 \rangle_h} - \frac{\langle \Sigma_P |}{\langle \Sigma_P | \varphi_0 \rangle_b} \tag{4}$$

we obtain:

$$\frac{\delta R}{R}^{1order} = \underbrace{\langle \delta S | \varphi_0 \rangle}_{direct} + \underbrace{\langle S | \delta\varphi \rangle}_{spectral} \tag{5}$$

The GPT allows writing, at the first order, the spectral term which accounts for the flux change subsequent to the perturbation, as:

$$\langle S | \delta\varphi \rangle = -\langle \psi^* | \delta H \varphi_0 \rangle \tag{6}$$

where ψ^* is the *importance function* and δH is the variation of the Boltzmann Operator of the reactor system. That way, direct calculation of the perturbed flux is avoided.

Thus, Eq. (5) reads:

$$\frac{\delta R}{R}^{1order} = \langle \delta S^* | \varphi_0 \rangle - \langle \psi^* | \delta H \varphi_0 \rangle \tag{7}$$

Neglecting the direct term, that is very small because the observable does not depend explicitly on the perturbation (control operates via soluble boron concentration adjustment and R group movement), it is straightforward to write the GPT expression for the Axial-Offset starting from Eq. (7):

$$AO'_{(1order)} = AO_0 + \frac{2R}{(R+1)^2}\left(\left\langle \delta S^* \middle| \varphi_0 \right\rangle - \left\langle \Psi^* \middle| \delta H \varphi_0 \right\rangle\right) \tag{8}$$

In the whole development the reactivity reset term is not included, on assumption that the λ reset and the boron reset on reactivity operate in a very similar way [1].

According to the previous statement on *extrapolation formula,* Eq. (8*)* can be immediately extended including HGPT higher order contribution to get:

$$AO'_{(1order)} = AO_0 + \Xi \frac{2R}{(R+1)^2}\left(\left\langle \delta S^* \middle| \varphi_0 \right\rangle - \left\langle \Psi^* \middle| \delta H \varphi_0 \right\rangle\right) \qquad 9)$$

where Ξ is a suitable weighting coefficient [3]

4. AXIAL-OFFSET *IMPORTANCE FUNCTION*

4.1. COMPUTING

The GPT *importance function* is the only solution of the inhomogeneous equation:

$$H^* \psi^* = S^* \tag{10}$$

where H^* is the adjoint Boltzmann Operator and S^* is the *adjoint source* relevant to the Axial-Offset, defined in Eq (4).

The possibility to solve this equation depends on the computational environment. As previously mentioned, the Boltzmann Operator discretized in the NEM scheme is not transposition-invariant, thus the 'physical' and 'mathematical' adjoint fluxes are not congruent and the associated eigenvalues are slightly different. Moreover, in SMART code, the nesting of diffusion and nodal iterations inside the flux convergence loop forces computing an equivalent diffusion coefficient at any nodal iteration. This approach forbids using the nodal adjoint flux within perturbation expansion, even if opportune discontinuity conditions were imposed [9].

It is known that, when the discretization process keeps the symmetry of the matrix relevant to the diffusion term, which is the case for finite differences and mixed dual techniques, 'mathematical' and 'physical' adjoint fluxes are congruent. If the *importance function* is computed in SMART using the converged diffusion scheme obtained at the end of the nodal iteration process [8], it is then possible to use the original HGPT expansion [1] without any significant change. In that way, the original mixed nodal-diffusion equation system is reduced to a unique multigroupe diffusion equation, which possesses the right solutions in flux and current even if the interface diffusion coefficients are modified.

Computation of the GPT *importance function* in SMART was activated and achieved that way.

As said, when large perturbations are involved, expansions must, in principle, include several higher order terms to account for indirect effects, but, in practical applications, expansion is to be limited to the first order, to avoid dependence of the expansion on perturbations. This difficulty must be solved adopting a suitable extrapolation procedure (*extrapolation formula*) [3], which allows obtaining precision that is at least equal to, but generally better than a second order. It reads:

$$\tilde{\psi}^* = F\left(\varphi^*, \psi^*, \tilde{\beta}\right), \tag{11}$$

where $\tilde{\psi}^*$ is the researched solution, that depends on a weighting coefficient $\tilde{\beta}$, which accounts for the average contribution of all the higher modes except for zero φ^* and first ψ^*.

That way, properties of the converged *importance function* computed with SMART satisfy all the conditions required by the extrapolation procedure. That allows extending the approach to any general case.

4.2 ACCOUNTING FOR THE FEEDBACK EFFECT

Eq. (9) depends explicitly on δH, the variation of the Boltzmann Operator, which represents the perturbation. Its change accounts for the effect of the perturbation itself (power scale), the reactivity reset (soluble boron concentration change), the movement of control group and an overall feedback correction due to spatial and spectral effects on cross-section values.

Power scale, reactivity reset and rod movement effects on cross section are known parameters through cross section tables, but the feedback effect cannot be known precisely without a point-wise-evaluation of the power change. This is a major challenge in predicting the behavior of variables during transients, because the Boltzmann Operator change is sometimes known quite poorly.

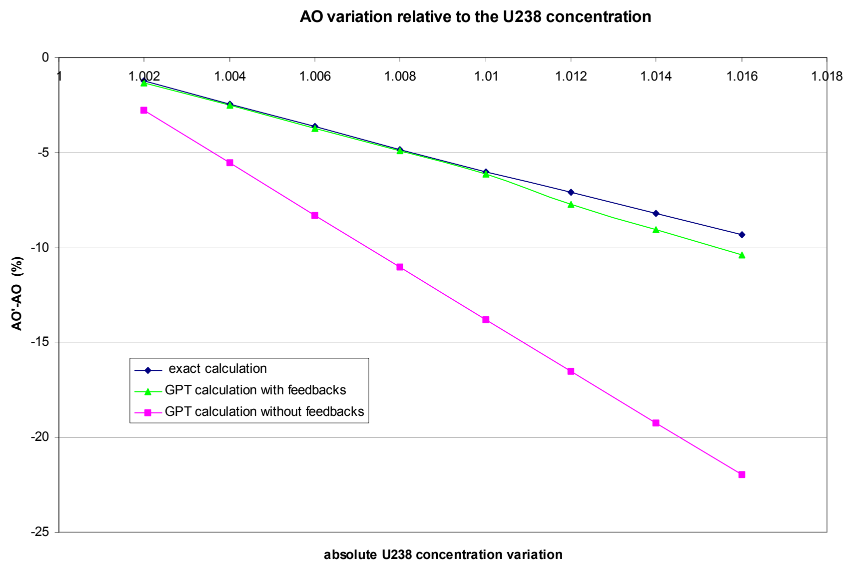

Fig. 4. GPT reconstruction of the Axial-Offset after a local variation of U238 concentration: effect of δH.

Figs.(4 & 5) present the GPT reconstruction of large Axial-Offset changes generated by variation of Xenon and U238 concentrations. It can be seen that, when the variation of the Boltzmann Operator is known exactly, e.g. through the exploitation of a direct calculation, where the feedback effects on cross-sections are evaluated during the converge loop, the results of GPT are excellent. If no feedback corrections at all are made, an error appears which increases with the perturbation magnitude. This error, which is roughly proportional to the perturbation itself, should be quite easily corrected heuristically using a suitable scale factor.

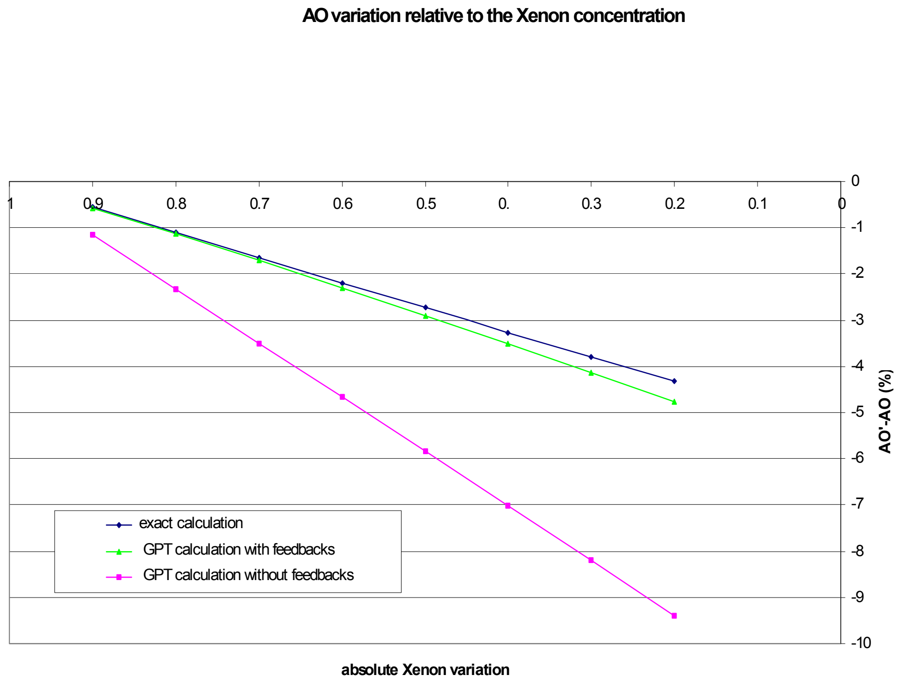

Fig. 5 GPT reconstruction of the Axial-Offset after a change of Xe concentration: effect of δH .

4.3 ACCOUNTING FOR OTHER CORRECTIONS

Movements of control group R generate huge flux changes, which affect the Axial-Offset more than homogenous perturbations considered up here can do. As said in [3], in this case, the HGPT expansion should include several (at least one, the second) higher order terms, but it is deliberately truncated at the first order. Contribution from higher order terms is then to be accounted for by generic weighting coefficients applied to the first order term.

Comparison among Axial-Offset direct-calculations and GPT results, obtained taking and not taking into account feedback, enabled defining a heuristic procedure to evaluate global correction factors accounting for both higher order and feedback effects in a while. Unfortunately, these corrections may have either the same or an opposite sign depending on the control rod movement. Thus, two coefficients, ξ_{ins} and ξ_{extr}, have been defined to account for the differences in the algebraic combination of the corrections. These coefficients depend linearly on the power level, which enables to compute them a priori and generalize the procedure to any power level.

Moreover, the very large meshing adopted in calculations to accelerate their speed, can produce a significant spectral shift between the conditions simulated in the GPT calculations and the actual ones, depending on the position of the regulation group. A suitable fundamental mode correction has been introduced to compensate this shift. It makes use of a coefficient β defined as follows, where *rod* stands for rodded infinite medium configuration, 0 for unrodded, r for removal cross-section and a for absorption cross-section:

$$\beta = \left[\frac{\left(\frac{\Sigma_r}{\Sigma_a} \right)_{rod}}{\left(\frac{\Sigma_r}{\Sigma_a} \right)_0} \right]$$

Figs 6 & 7 here above show the efficiency of the whole reconstruction - prediction process.

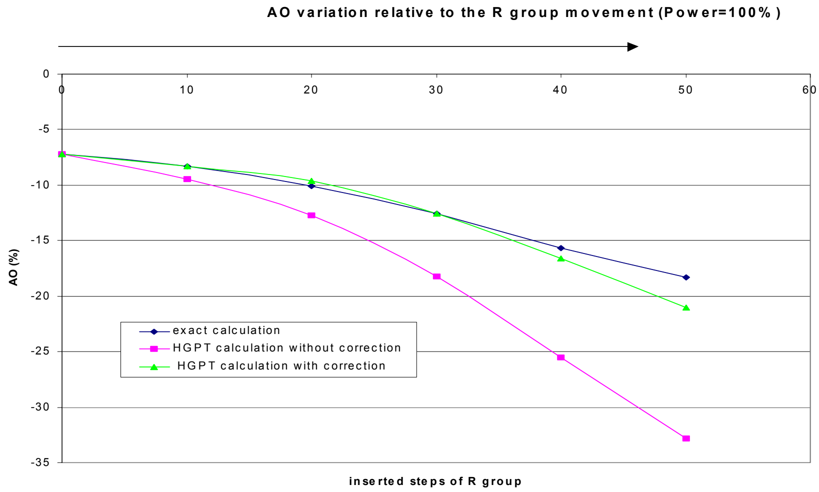

Fig. 6 GPT Axial-Offset reconstruction via HGPT :efficiency of the corrections at full power

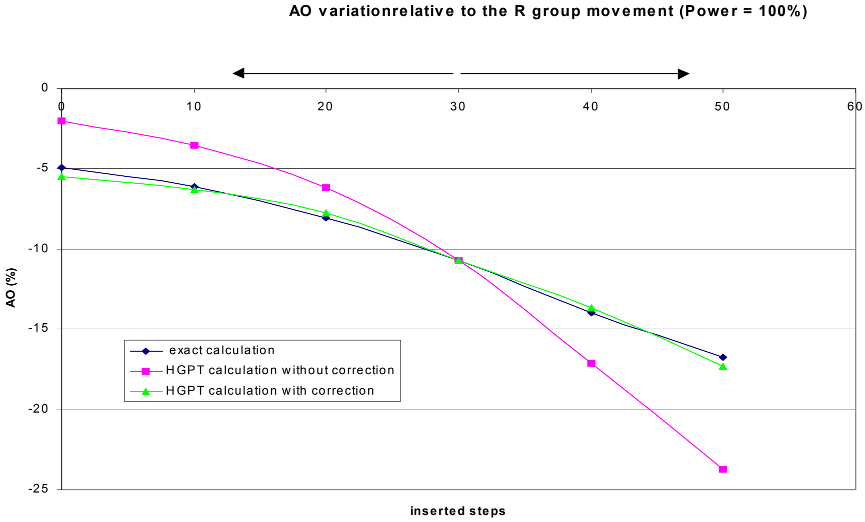

Fig. 7 GPT Axial-Offset reconstruction via HGPT: efficiency of the corrections at full power

4.4 ACCOUNTING FOR THE INITIAL CONDITIONS

As known [1], the GPT *importance function* doesn't depend on perturbations, but it does on the initial conditions. Then, in principle, it would be necessary to recalculate the *importance function* at any new position assumed by the regulation group within the maneuver band. To avoid that fastidious, boring and time-consuming repetition of not useful calculations, an *effective importance function* $\overline{\Psi^*}$, average of the *importance functions* computed at the two extreme positions of the maneuver band, has been defined:

$$\overline{\psi^*} = \frac{\psi_1^* + \psi_2^*}{2} \tag{12}$$

4.5 SETTING-UP A HEURISTIC FORMULA

Summarizing, the original HGPT Axial-Offset definition of Eq. (9), has been modified to take into account the specific needs of an industrial use within the Operator-Aid System. The following effects have been either corrected or accounted for using the coefficients indicated:

- feedback and higher order term contribution to the expansion (coefficients ξ_{ins} and ξ_{extr}),
- meshing (coefficient β),
- dependence on initial conditions (*effective importance function* $\overline{\psi^*}$)

Thus, Eq. (9) now reads:

$$AO' = AO_0 + \frac{2R}{(R+1)^2}\left(-\xi_{insertion} * \left\langle \overline{\psi^*} \middle| \delta H(\beta\varphi_0)\right\rangle\right) \tag{13}$$

in the case of a control group insertion, and

$$AO' = AO_0 + \frac{2R}{(R+1)^2}\left(-\xi_{extraction} * \left\langle \overline{\psi^*} \middle| \delta H(\beta\varphi_0)\right\rangle\right) \tag{14}$$

in the case of an extraction.

5. APPLICATION

The formalism described up-here has been widely tested to simulate and predict Axial-Offset variations after movement of the control group R at several power levels and in a large number of different computational circumstances. Results are very satisfactory indeed.

Fig. 8 shows results at 30% power obtained using ξ coefficients derived from exact calculations. Figs. 9 to 11 show results of a reconstruction of the Axial-Offset at 60% power, made using pre-calculated ξ coefficients. It can be observed that no noticeable differences and no trend appear among results.

Use of the HGPT technique to accelerate the search of the best control strategy within transients allows saving a factor 2 in running time. Coupling a HGPT search on Axial-Offset to a CPT on reactivity reset increases the gain in running time even more (up to roughly a factor 3), without any significant precision loss (less than 0.5% vs. a conventional design-type calculation).

CONCLUSION

Currently FRAMATOME-ANP is developing an advanced Operator-Aid System, relevant to the PWR control in normal operation to help managing control rod position and/or adjusting the soluble boron concentration to respect operating margins, as required by the electrical network load-follow currently adopted in conventional French PWRs

The high precision system couples CPT and HGPT prediction features to fast-running coarse-mesh nodal routines to accelerate convergence of the whole computation process without any significant loss of precision. Implementation of the HGPT-based prediction methodology within the environment of the preliminary version of the system has been achieved adopting an original methodology which allows keeping standard HGPT expansion formulae within NEM computational environment. Use of the HGPT to accelerate the search for the best control strategy combined to a CPT search on reactivity reset allows saving up to a factor 3 in running time, with no significant precision loss.

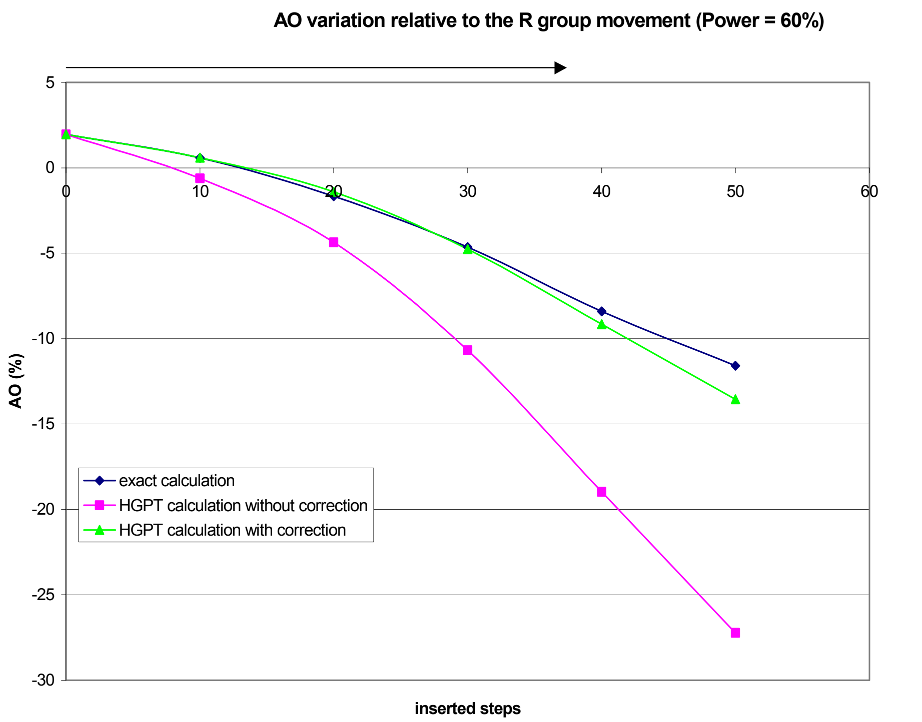

Fig. 8 GPT Axial-Offset reconstruction via HGPT: movement of group R at 30% power

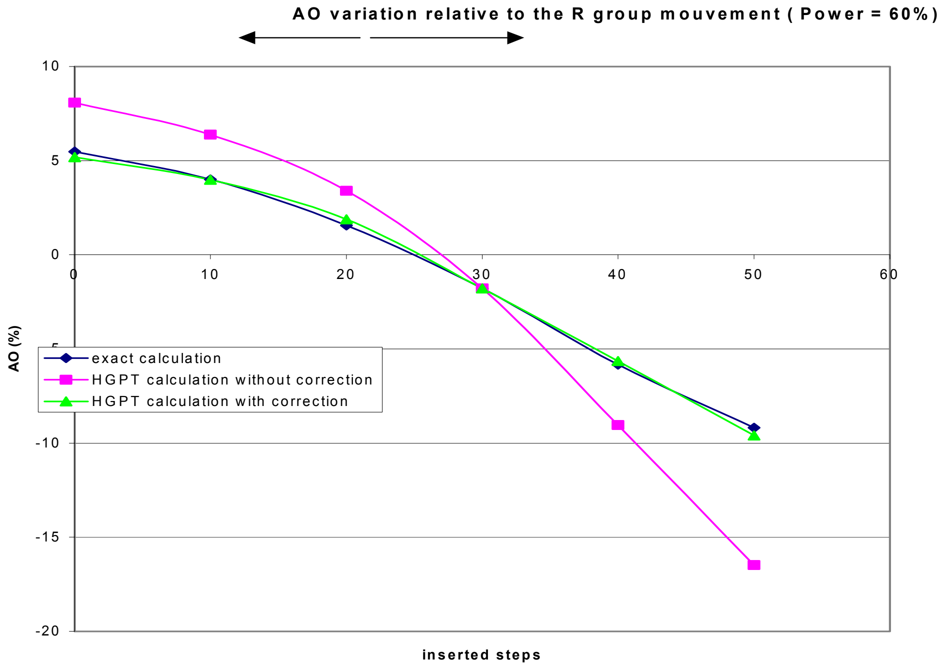

Fig. 9 GPT Axial-Offset reconstruction via HGPT: movement of group R at 60% power

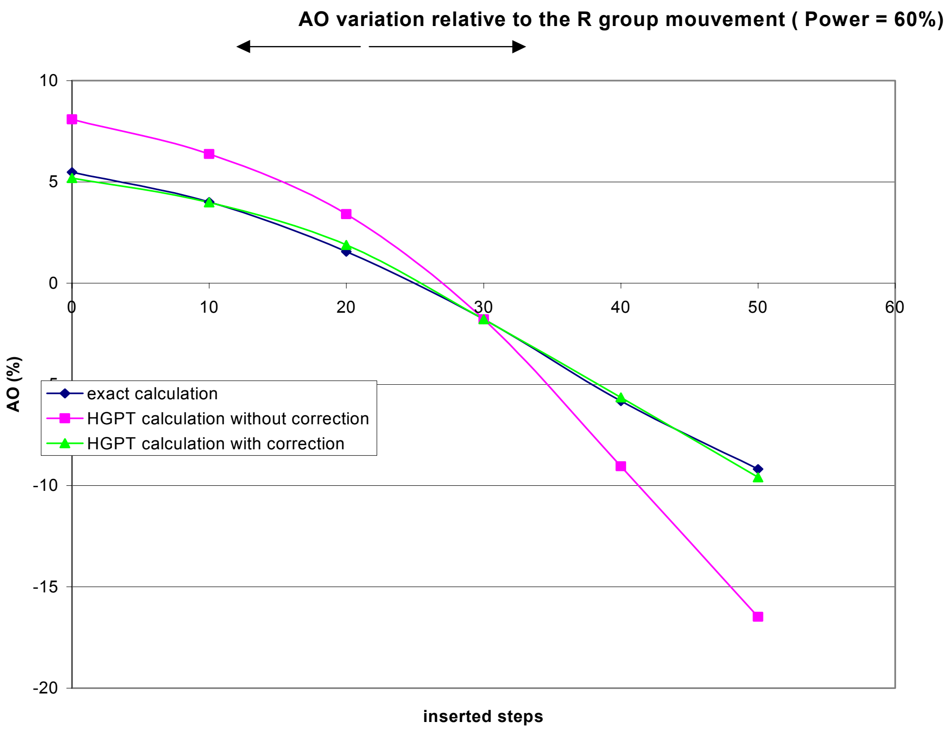

Fig. 10 GPT Axial-Offset reconstruction via HGPT: movement of group R at 60% power

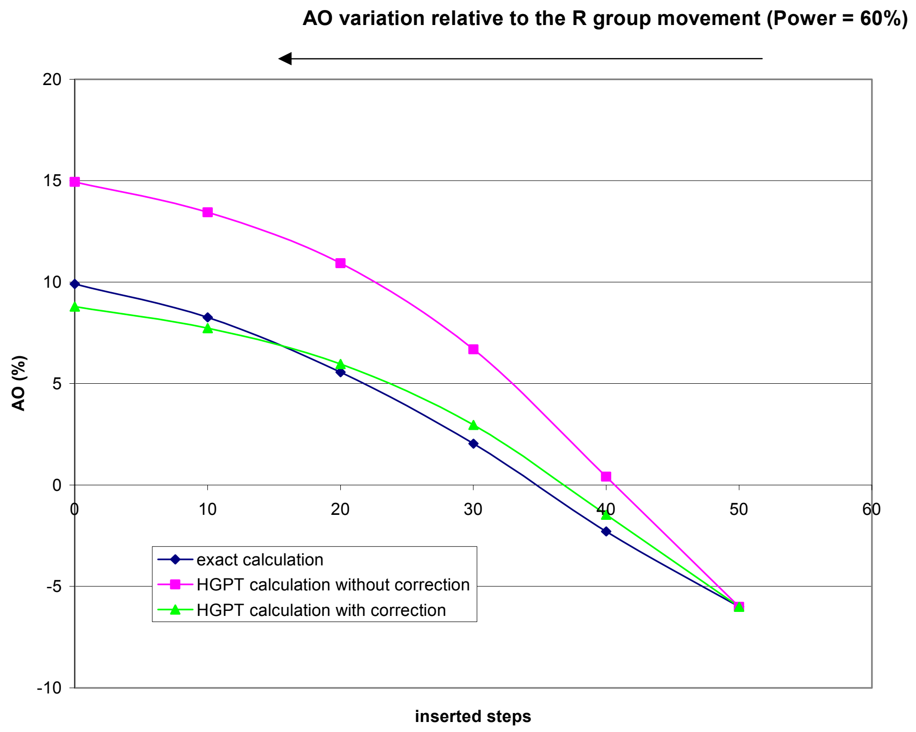

Fig. 11 GPT Axial-Offset reconstruction via HGPT: movement of group R at 60% power

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