CONSISTENT GROUP COLLAPSING SCHEME FOR MULTI-GROUP MOC CALCULATION

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ABSTRACT

A consistent group collapsed macro-group MOC calculation scheme for multi-group MOC calculations is established. To conserve the average angular reaction rate at each track, an equivalence parameter, named angular flux discontinuity factor (AFDF) is defined. The equivalent macro-group MOC calculation is applied either to accelerate the multi-group (>=35) MOC calculation or to obtain an approximate solution. The performance of the equivalent macro-group MOC calculation is examined for assembly level and small PWR core problems using 7 or 35 energy group cross sections. In the aspect of acceleration, the equivalent macro-group calculation is turned out to have no merits because it requires more energy group sweeps and computing time than the normal calculation. However in obtaining an approximate solution, the macro-group MOC calculation reduces the computing time by a maximum of 50% in 35-group small PWR problem without any significant loss of accuracy. The macro-group calculation is thus effective in generating sufficiently accurate approximate solutions.

1. INTRODUCTION

Recently, equivalent diffusion accelerated MOC transport calculation scheme[1,2,3] was introduced to obtain a MOC transport solution efficiently. In an early work[2], the coarse mesh finite difference(CMFD) acceleration in the MOC transport calculations showed about 10 to 50 speedups depending on the problem size. However, when applied to multi-group (>=35) calculations that are general in lattice calculation, the MOC multi-group calculation still requires long computing time because it increases linearly with the number of energy groups. Therefore some early studies focused on obtaining an approximate solution instead of multi-group transport solution by condensing multi-group constants to few-group constants using approximate spectra[4]. This few-group MOC calculation was good for reducing the computing time, but it has a drawback in that the use of approximate spectra cannot guarantee the solution accuracy.

In the work here, an equivalent macro-energy group MOC equation is derived by integrating multi-group MOC transport equations, the equivalence parameter named angular flux discontinuity factor(AFDF) is introduced to conserve the incoming and outgoing angular fluxes at the end of all the tracks, and finally to conserve the regional reaction rates. This equivalent macro-group MOC calculation overcomes the drawback of the early work in that it reproduces the original multi-group MOC solution. In Section 2, the AFDF is derived from the multi-group MOC equation by integrating all the MOC multi-group equations defined in a given region. The equivalent few-group MOC calculation with this AFDF can be used either to accelerate the multi-group MOC calculation or to

obtain an approximate solution. Section 3 describes the applications of equivalent macro-group MOC calculations with AFDF. In Section 4, the equivalent macro-group MOC calculation scheme is implemented to DeCART[2] code which is a CMFD accelerated MOC ray tracing code and solves the heterogeneous whole reactor core problems. The performance of the equivalent macro-group calculation is then examined in the aspects of both acceleration and obtaining an approximate solution.

2. EQUIVALENT GROUP CONDENSATION

To retrieve the multi-group MOC solution from a macro-group MOC calculation with a reduced number of groups, equivalence parameters to conserve the regional reaction rate and net neutron leakage are essentially required. In this paper, the angular flux discontinuity factor(AFDF) is introduced to conserve the outgoing angular flux with the regionwise group condensed cross sections. This section is to derive the equivalence parameters and to verify the equivalence between the multi-group and macro-group MOC solutions.

The average angular reaction rate in a MOC track can be expressed in terms of incoming and outgoing angular fluxes as:

$$\Sigma_{tr}^{k,g}\overline{\psi}_{m,t}^{k,g} = \sin\theta\frac{\psi_{i,m,t}^{k,g}-\psi_{o,m,t}^{k,g}}{l_{m,t}^{k}}+Q^{k,g}, \tag{1}$$

where $\psi_{i,m,t}^{k,g}$, $\psi_{o,m,t}^{k,g}$ are incoming and outgoing angular fluxes of Track t in direction Ω_m and Region k, and $l_{m,t}^{k}$ is the track length.

The total regional reaction rate can be obtained as the weighted sum of all the angular reaction rates as:

$$V^{k}\Sigma_{tr}^{k,g}\phi^{k,g} = \sum_{t} w_m \Delta A_m \sin\theta\left(\psi_{i,m,t}^{k,g}-\psi_{o,m,t}^{k,g}\right)+V^{k}Q^{k,g}, \tag{2}$$

where $\phi^{k,g}$ is the scalar flux for a Region k and Group g, and defined as:

$$\phi^{k,g} = \frac{1}{V^{k}}\sum_{m} w_m l_{m,t}^{k}\Delta A_m \overline{\psi}_{m,t}^{k,g}.$$

Here, V^{k}, w_m and ΔA_m are Region k volume, Angle m weight and the ray spacing that is defined for each angle.

The reaction rate for a macro-energy group can be derived by integrating the Eq. (2) over all the energy groups belonging to a macro-group G as:

$$V^{k}\Sigma_{tr}^{k,G}\phi^{k,G} = \sum_{m} w_m \Delta A_m \sin\theta\left(\psi_{i,m,t}^{k,G}-\psi_{o,m,t}^{k,G}\right)+V^{k}Q^{k,G}, \tag{3}$$

where $\Sigma_{tr}^{k,G}$, $\phi^{k,G}$ and $Q^{k,G}$ are the transport cross-section, scalar flux and neutron sources, and $\psi_{i,m,t}^{k,G}$, $\psi_{o,m,t}^{k,G}$ are the incoming and outgoing angular fluxes defined in macro- group *G* which are obtained using the multi-group solution by:

$$\Sigma_{tr}^{k,G} = \frac{1}{\phi^{k,G}} \sum_{g \in G} \Sigma_{tr}^{k,g} \phi^{k,g},$$

$$\phi^{k,G} = \sum_{g \in G} \phi^{k,g},$$

$$Q^{k,G} = \sum_{g \in G} Q^{k,g}, \tag{4}$$

$$\psi_{i,m,t}^{k,G} = \sum_{g \in G} \psi_{i,m,t}^{k,g} \quad \text{and}$$

$$\psi_{o,m,t}^{k,G} = \sum_{g \in G} \psi_{o,m,t}^{k,g}.$$

In Eq. (3), with the given condition of group condensed transport cross section, neutron source and incoming angular flux, the macro-group reaction rate can be conserved if and only if macro-group MOC calculation produces the same *outgoing* angular flux as the group condensed macro-group value of the multi-group solution. However, there is no guarantee that the macro-group MOC calculation generates the same outgoing angular flux as the group condensed value. Therefore, the conservation of the outgoing flux is forced here by the equivalent group condensation parameter named angular flux discontinuity factor(AFDF) which is defined as;

$$f_{m,t}^{k,G} = \frac{\psi_{o,m,t}^{k,G}}{\hat{\psi}_{o,m,t}^{k,G}}, \tag{5}$$

where $\hat{\psi}_{o,m,t}^{k,G}$ is the outgoing angular flux obtained from the macro group MOC calculation using the group condensed transport cross section, incoming angular flux and neutron source from multi-group MOC calculation. By multiplying AFDF to the macro-group outgoing angular flux of a region boundary, a correct macro-group incoming angular flux to the neighboring region can be obtained so that the reaction rates can be conserved for the entire tracks. The above definition of the AFDF requires that the AFDF are determined at all the tracks and all the macro-groups. Therefore the equivalent macro-group MOC calculation results in a large memory requirement to store all the AFDF's.

3. APPLICATION OF EQUIVALENCE PARAMETERS

The macro-group MOC calculation using the equivalent group condensation parameter of eq.(5) can be used either to accelerate the multi-group MOC calculation or to obtain an approximate solution. In this section, these two applications are explained using the flow diagrams.

3.1 ACCELERATION OF MULTI-GROUP MOC CALCULATION

To use the macro-group MOC calculation as an acceleration scheme, some modules for estimation of AFDF, calculation of macro-group MOC and update of the multi-group MOC solutions from the macro-group solution are required. Figure1 shows how these functional modules work with the CMFD and multi-group MOC modules in the DeCART code. When the program starts, the multi-level CMFD modules solve the whole problem with the homogenized cross sections which are generated using the initial flux guess at first iteration or using the multi-group MOC solutions at the previous iteration step. If the error reduction criterion of the CMFD solution is satisfied, the multi-group regional flux and boundary angular flux that are multi-group MOC solutions are updated using the CMFD cell-wise flux and current. The macro-group MOC module first performs the group condensation to generate the macro-group regional constants, scalar flux and boundary angular flux as defined in eq.(4). And then using the AFDF's that are initially guessed or generated at the previous iteration step, the macro-group MOC module determines the macro-group MOC solutions. When the macro-group MOC calculation is finished, the macro-group MOC module finally updates the multi-group MOC solutions from macro-group MOC solutions. The multi-group MOC module performs the multi-group MOC calculation with the fixed number of inner iterations and up-scattering iterations. During the multi-group MOC calculation, the multi-group MOC module generates not only the outgoing partial current for the current correction coefficient for the CMFD module, but also the group condensed outgoing angular fluxes and neutron sources for the AFDF calculation that is performed at the subsequent module. The AFDF calculation module first calculates the regional macro-group cross sections and then performs the ray sweeping using the group condensed solutions which are generated during the multi-group MOC calculation. During the macro-group ray sweeping, the group condensed outgoing angular flux from multi-group MOC calculation is used both for the AFDF calculation and for incident angular flux at the next track. The generated AFDF is stored for the next macro-group MOC calculation. The iterative calculation between the CMFD accelerated multi-group MOC calculation and the equivalent macro-group MOC calculation continues until convergence.

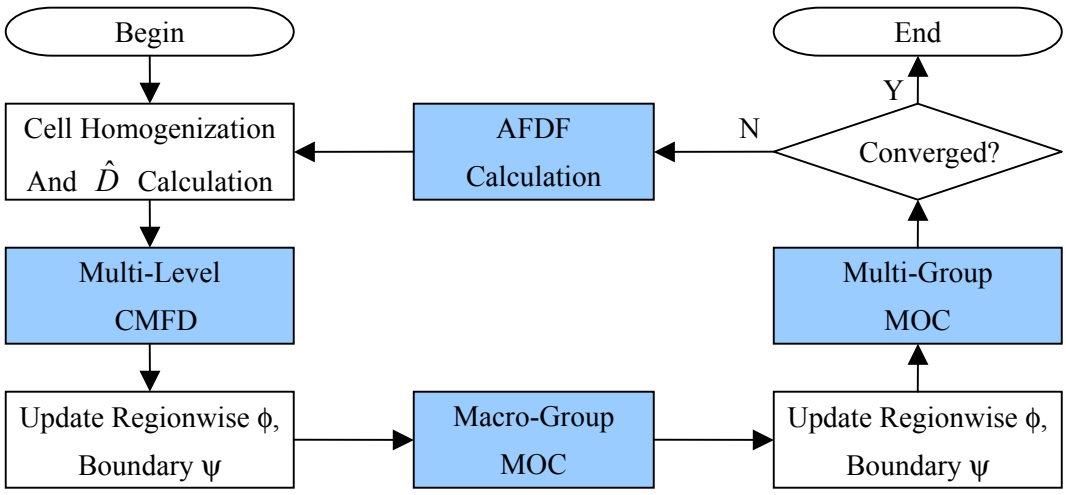


Figure 1. Iterative Algorithm for Macro-Group MOC Acceleration

3.2 APPROXIMATE SOLUTION OF MULTI-GROUP MOC CALCULATION

Figure 2 shows the calculation flow diagram to obtain an approximate solution of multi-group MOC calculation using the macro-group MOC calculation. To obtain an approximate solution from the macro-group MOC calculation, accurate AFDF's are essential. Therefore at the beginning of execution, the normal iterative calculations between multi-level CMFD and multi-group MOC calculation are performed until the solution converges to switch on condition for the macro-group MOC calculation. If the condition for macro-group MOC calculation is satisfied, the multi-group MOC ray sweeping for the group condensed angular flux and neutron source and then the subsequent AFDF calculation are followed. Using these approximate AFDF, the macro-group MOC calculation with the macro-group CMFD acceleration is continued to obtain a converged solution. The iterative calculation between the macro-group CMFD and MOC calculations is very similar with the normal iterative calculation that is performed before the condition for macro-group MOC calculation is satisfied.

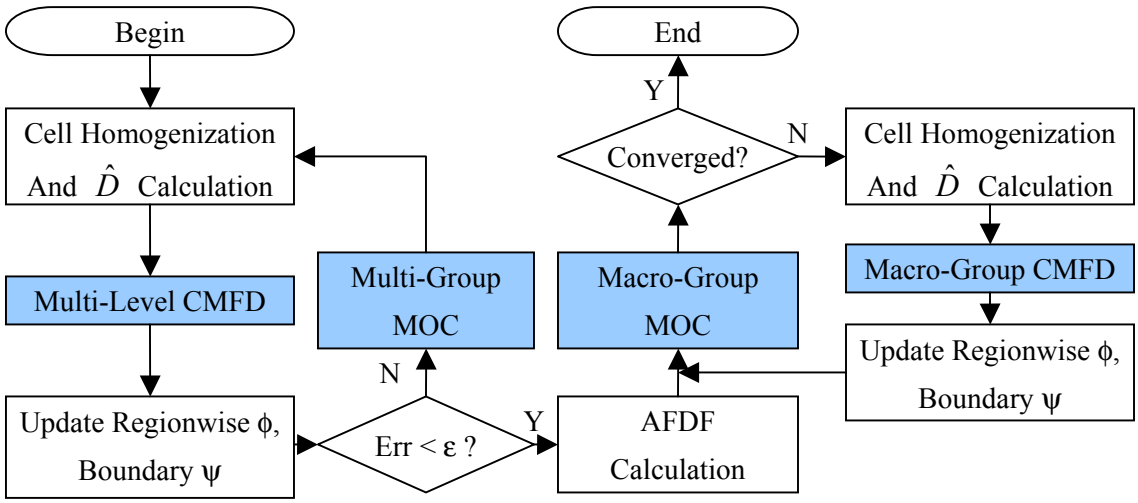
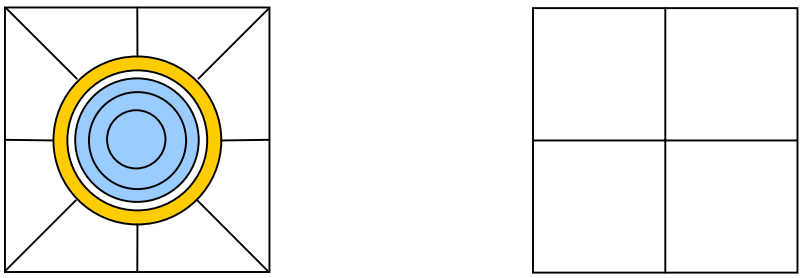


Figure 2. Iterative Algorithm for Approximate Solution from Macro-Group MOC Calculation

4. COMPUTATIONAL RESULTS

The macro-group MOC ray tracing scheme is implemented to the DeCART code and the performances are examined by solving two different level problems; assembly level and small PWR core problems. In assembly level problems, 5 different assemblies, 3 MOX fuel and 2 UO_2 fuel assemblies, are solved with reflective boundary condition. The assembly configurations are the same as those in KAIST benchmark core problems[5]. The small PWR core problem is the KAIST benchmark core problem without the dummy reflector region as shown in Figure 3. These two different level problems are solved in two different cross section libraries, i.e. 7-group of KAIST benchmark problem and 35-group cross section libraries. The 35-energy group cross section data are generated from the DENT-2D code[6] that employs 35-group library.

The performance of the equivalent macro-group MOC calculation is first examined in the aspect of acceleration by comparing the convergence behavior and computing time with the results of no equivalent macro-group MOC calculation. In this case, the reference solutions are first obtained with tight convergence criterion, and then the convergence of the fission source is checked using this reference. The second aspect of the AFDF performance is the approximate solution using the approximate AFDF. In the second aspect, the performance analysis is focused on the accuracy of the approximate solution and computing time. For this performance analysis of macro-group calculations, most of the cell types are divided into 13 flat flux regions except for baffle and reflector cells that are divided into 4 flat flux regions as in Figure 4, and 2 macro-groups are applied. For the simplicity of the macro-group MOC calculation, the first macro-group is scheduled to cover no up-scattering micro groups. In input values, 2 polar angles and weights that are suggested to be the optimal values, 16 azimuthal angles in 360 degree and 0.05 cm ray spacing are used. The up-scattering iteration in multi-group MOC calculation is required to obtain a converged solution effectively. Therefore in this paper, one up-scattering iteration is introduced except for the no macro-group calculation case using 35-group library that employs two up-scattering iterations.



(a) Fuel, Guide Tube, Absorber and Control Rod Cells

(b) Baffle and Reflector Cells

Figure 4. Cell Divisions for MOC Transport Calculation

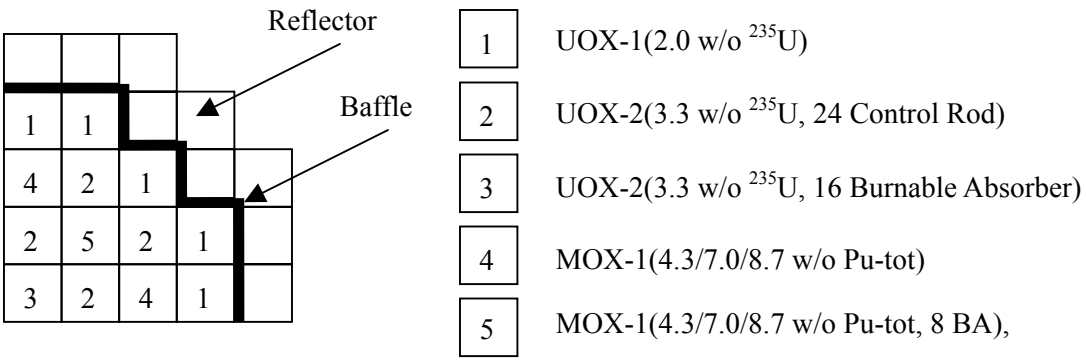


Figure 3. Core Configuration of Small PWR Core Problem

4.1 ACCELERATION ASPECT

Table I shows the computational results of the equivalent macro-group accelerated multi-group MOC calculation compared with the no macro-group calculation. In 7-group problems, the macro-group accelerated calculation reduces the number of multi-group MOC updates only for assembly types 1, 2, and small PWR problems. However, the macro-group accelerated calculation requires the ray sweeping for AFDF generation and macro-group calculation. Including these ray sweepings that are not necessary in the normal CMFD accelerated calculation, the macro-group accelerated calculation need more ray sweeps to obtain a converged solution. In 35-group problems, the macro-group accelerated calculation reduces the total number of energy group sweeps at the assembly level problems of types 1, 2 and small PWR problems. However, the macro-group accelerated calculation cannot reduce the computing time because this scheme needs more computational overhead in handling a large memory for AFDF storage. Therefore the macro-group calculation has no merit in the acceleration aspect.

4.2 ASPECT TO OBTAIN AN APPROXIMATE SOLUTION

Table II shows the computational results when the macro-group calculation is used to obtain an approximate solution. This scheme requires a few multi-group MOC updates to obtain approximate spectra that are used for the subsequent AFDF calculation. The total number of energy group sweeps is also reduced about 30 ~ 50 %, resulting in the computing time saving of maximum 50 % in 35-group small PWR problem. The approximate macro-group calculation is more effective in 35-group problems showing about twice computing time saving than in 7-group problems. In the accuracy of the approximate macro-group calculation, there is no loss of accuracy in 5 pcm of reactivity errors and 0.1 % power errors. Therefore the macro-group calculation can be successfully used in obtaining sufficiently accurate approximate solution.

5. CONCLUSIONS

A consistent group collapsed macro-group MOC calculation scheme for multi-group MOC calculation has been established and angular flux discontinuity factor (AFDF) was derived as an equivalence parameter. In the acceleration aspect, the equivalent macro-group MOC calculation is concluded to have no merits. However in obtaining an approximate solution, the macro-group MOC calculation reduced the computing time by a maximum of 50% in 35-group small PWR problem without any significant loss of accuracy. Therefore, the macro-group MOC calculation is effective in generating sufficiently accurate approximate solutions.

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Table I. Computational Results in Acceleration Aspect of Macro-Group Calculation

Energy Groups	Problems	Types	No Macro-Group Acceleration				Macro-Group Acceleration			
			N_{it} [1]	N_s [2]	T_{RS} [3]	T_{total} [4]	N_{it} [1]	N_s [2]+N_m [5]	T_{RS} [3]	T_{total} [4]
7	Assembly Level Problems	1	5	55	5.9	6.3	4	44+16	9.3	10.0
		2	6	66	7.1	7.7	5	55+20	11.6	12.4
		3	5	55	6.0	6.5	5	55+20	11.9	12.6
		4	5	55	6.0	6.5	5	55+20	11.8	12.6
		5	5	55	6.1	6.6	5	55+20	11.9	12.6
	Small PWR Problem		7	77	146	168	6	66+24	256	280
35	Assembly Level Problems	1	5	395	43	46	5	285+60	48	52
		2	5	395	43	46	6	342+72	57	60
		3	6	474	52	56	6	342+72	58	62
		4	5	395	43	47	6	342+72	58	62
		5	5	395	44	47	6	342+72	58	62
	Small PWR Problem		7	553	1061	1217	7	399+84	1364	1520

1) The total number of iterations between multi-level CMFD and multi-group MOC module
2) The total number of energy group sweeps in multi-group MOC module
3) CPU time used in macro- and multi-group MOC module, seconds on 1 GHz PENTIUM III PC
4) Total CPU time, seconds on 1 GHz PENTIUM III PC
5) The total number of energy group sweeps required in macro-group MOC calculation

Table II. Computational Results in Approximate Solution of Macro-Group Calculation

Energy Groups	Problems	Types	N_{it} [1]	N_s [2]+N_m [5]	T_{RS} [3]	T_{total} [4]	Saving [6], % T_{RS}/T_{total}	Errors	
								k_{eff}, pcm	Power, %
7	Assembly Level Problems	1	3	33+6	5.1	5.8	13.6/7.9	-0.2	0.1 [7]
		2	3	33+6	5.1	5.8	28.2/24.7	-1.0	0.1 [7]
		3	3	33+6	5.1	5.8	15.0/10.8	-1.2	0.0 [7]
		4	3	33+8	5.4	6.1	10.0/6.2	0.0	0.0 [7]
		5	3	33+8	5.4	6.1	11.7/7.6	-1.0	0.1 [7]
	Small PWR Problem		3	33+6	89	110	39.0/34.5	-0.8	0.1 [8]
35	Assembly Level Problems	1	3	193+14	25	28	41.9/39.1	+0.4	0.1 [7]
		2	3	193+10	24	27	44.2/41.3	-1.6	0.1 [7]
		3	3	193+14	25	28	51.9/50.0	+4.3	0.1 [7]
		4	3	193+14	25	28	41.9/40.4	+1.3	0.1 [7]
		5	3	193+14	25	28	43.2/40.4	-1.9	0.1 [7]
	Small PWR Problem		3	193+14	470	592	55.7/51.4	+2.4	0.1 [8]

6) $(T_1-T_0)/T_0*100$, T_0: Time of no macro-group calculation, T_1: Time used for approximate solution
7) Maximum pin power error
8) Maximum assembly power error