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MULTI-OBJECTIVE IN-CORE FUEL MANAGEMENT OPTIMIZATION FOR PWR

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ABSTRACT

The sensitivity factor was introduced into the Simulated Annealing method (SA) to determine the multi-objective function. It aimed at efficient use of the weighting factor in order to contra pose different objectives as in the case of nuclear fuel management. In this case the probability distribution method was used to calculate the introduced sensitivity factors. The first cycle of the Dayabay Nuclear Plant (DNP) was optimized with the Multi-Objective Simulated Annealing program MOSA written for this paper. Two optimization objectives were chosen as concrete examples: the maximization of discharged fuel with the minimization of the power peaking factor. The results showed that the greatest improvements in multi-objective optimization could be achieved by the use of sensitivity factors.

1. INTRODUCTION

This paper presents the multi-objective optimization problems. Many methods exist for solving multi-objective problems [1][2]. Generally, the methods can be separated into two kinds: the first kind uses a random weight generator, such as the weighting method; and the other kind uses fixed weight, such as the constraint method and the hybrid method. The random weight method can contain a lot of samples to a certain degree. The random weight method needs a much longer calculation time and may not reach Pareto optimal. The fixed weight method generally uses the calculation experience to determine the weighting factor [3]. In general, in fuel management optimization the objective functions present a large tradeoff between objectives, i.e., a large improvement in one objective is usually sacrificed in order to gain relatively little in the other objectives. This, in fact, is unreasonable. This research proposes a new method; the sensitivity factor was introduced to the Simulated Annealing method (SA) to determine the multi-objective function. It aimed at using the weighting factor efficiently in the objective function to contra pose the choice of different objectives, for instance, in the case of nuclear fuel management.

2. SENSITIVITY FACTOR OF OBJECTIVE FUNCTION

2.1 IMPORTANCE OF THE SENSITIVITY FACTOR

First a simple example of what the sensibility factor is. The difficulty of the hybrid method is reverted in the problem of calculating the weight of the objective function. In the case of weights w_1 1 and w_2 0, only the first objective was considered. With w_1 0 and w_2 1, only the second objective was considered. If the weight factors are w_1 0.5 and w_2 0.5 according to the definition of the weight both objectives should be equivalently considered. It is very difficult to reach the goal with the same utility using the two objective functions. The result may be the ignoring of one objective while considering the other objective. For example, the first objective function f_1 is the minimization of the power peaking factor (values taken between 1 and 1.5). The second objective function f_2 is the maximization of reactivity at the end-of-cycle k_{eff} (values taken between 1 and 1.2). If in our calculation we adopt relative values (f_{1ref}=1.4; f_{2ref}=1.15), then the weighting objective function can be written as:

$$F(\vec{x}) = w_1 F_1(\vec{x}) + w_2 F_2(\vec{x}) \tag{1}$$

$$F_1(\vec{x}) = \frac{f_1(\vec{x})}{f_{1ref}(\vec{x})} - 1 \tag{2}$$

$$F_2(\vec{x}) = -(\frac{f_2(\vec{x})}{f_{2ref}(\vec{x})} - 1) \tag{3}$$

The calculation of function F, F_1 and F_2 values for two loading patterns are given by:

1) $f_1 = 1.34 \quad , f_2 = 1.152$

$$F\ (\vec{x}) = 0.5(\frac{1.34}{1.4} - 1) + 0.5(1 - \frac{1.152}{1.15})$$

$$= 0.5 \times (-0.0428) + 0.5 \times (-0.0018)$$

$$= -0.0214 - 0.0009 = -0.0223$$

2) $f_1 = 1.23 \quad , f_2 = 1.155$

$$F(\vec{x}) = 0.5(\frac{1.23}{1.4} - 1) + 0.5(1 - \frac{1.155}{1.15})$$

$$= 0.5 \times (-0.1214) + 0.5 \times (-0.0043)$$

$$= -0.0607 - 0.0021 = -0.0628$$

Originally the two objectives should be equivalently considered but this results in ignoring the second objective, while considering only the first objective. From the example shown before, one can see that using only the simple weighting factor will meet with some difficulty, as this method gives wrong results about the use of the weighting factors. So using the weighting factor in the objective function didn't contra pose the choice of different objectives. Therefore the weighting factor is usually determined through the use at calculation experience. In order to make the weighting factors do their jobs, we propose the introduction of sensitivity factor to adjust the expression of the multi-objective function, with the formula (1) defined as:

$$F(\vec{x}) = w_1 F_1(\vec{x}) + w_2 S_{12} F_2(\vec{x}) \tag{4}$$

Where S_{12} is the sensitivity factor of F_2 relative to F_1 the determination of this factor is described as follows:

If the two objectives have the same utility, then

$$\left|F_1(\vec{x})\right| = S_{12}\left|F_2(\vec{x})\right|$$

Results in

$$S_{12} = \left|\frac{F_1(\vec{x})}{F_2(\vec{x})}\right|$$

Here S_{12} is the sensitivity factor of the maximization of the reactivity at the end-of-cycle k_{eff} relative to the minimization of the power peaking factor. The calculation of the sensitivity factors of two random loading patterns, as mentioned earlier, is given in the following equations

1) $f_1 = 1.34 \quad , f_2 = 1.152$

$$S_{12} = \left|\frac{F_1(\vec{x})}{F_2(\vec{x})}\right| = \frac{0.0428}{0.0018} = 23.8$$

2) $f_1 = 1.23 \quad , f_2 = 1.155$

$$S_{12} = \left|\frac{F_1(\vec{x})}{F_2(\vec{x})}\right| = \frac{0.1214}{0.0043} = 28.2$$

The calculation of the sensitivity factor for the different loading patterns may give different values, but it is usually not needed for each loading pattern. Thus, we use the statistical probability method to calculate only the average values of sensitivity factor (see 4.1).

2.2 ROLE OF SENSITIVITY FACTOR AND WEIGHTING FACTOR

The role of the weight and sensitivity factor can be illustrated in figure1. Space R shows the objective space of optima. Curve N shows the objective space of Pareto frontier. The figure tangents show the optimization results of adopting the different weight and sensibility factors.

Supposing the sensibility factor S_{12} fixed S_{12}=25 the variation of the objective function is considered according to the weight factor. While investigating the effect of weight variation in the objective function when $w_1 = w_2 = 0.5$, the tangent was in B. Moreover when w_2 increased to $w_2^{'}$, it is equivalent to say that the tangent of the straight line decreased, i.e. when $w_1^{'} = 0.04; w_2^{'} = 0.96$, then tangent was in C. When w_2 decreased to $w_2^{''}$, i.e. when $w_1^{''} = 0.96; w_2^{''} = 0.04$, tangent was in A. Therefore we can clearly see the effect of the weight variation.

Now if the sensitivity factor is not introduced, i.e. S_{12}=1, then the corresponding weights to loading patterns A, B and C change. Here, the change of the weight w_1 and w_2 of B loading pattern is:
$w_1 = 0.5$ $w_2 = 0.5 \times 25 = 12.5$
After normalization: $w_1 = 0.04$; $w_2 = 0.96$

When S_{12}=1, as in weighting the objective function, we find loading pattern A but not loading pattern B. The weight of loading pattern A and C are given separately in the table1; the corresponding weight of loading pattern B results in great change, as listed in the table 2. We see that the sensitivity factor is introduced to correct the multi-objective function. The objective function efficiently uses the weighting factor to contra-pose the choice of different objectives.

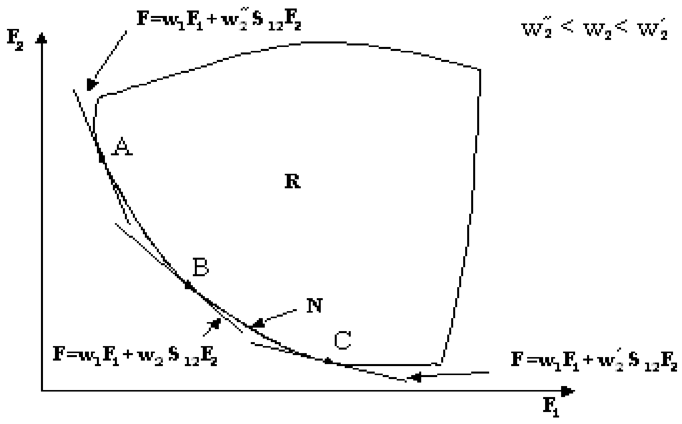

Figure 1.The role of sensitivity factor and weighting factor

Table 1. Loading pattern found by fixed sensitivity and varying weight

Sensitivity S_{12}	Weight w_1:w_2	Loading pattern
25	0.96:0.04	A
25	0.5:0.5	B
25	0.04:0.96	C

Table 2.Loading pattern found without introducing the sensitivity factor

Sensitivity S_{12}	Weight w_1:w_2	Loading pattern
1	0.5:0.5	A
1	0.04:0.96	B
1	0.002:0.998	C

3. MULTI-OBJECTIVE OPTIMIZATION FUNCTION

3.1 MULTI-OBJECTIVE FUNCTION

The in-core fuel management optimization usually adopts three objectives [4] namely: the maximization of the reactivity k_{eff} at the end-of-cycle; the minimization of the assembly relative power peaking factor P_{xy} ; the maximization of the average of Burnup Bu of the fuel region to be discharged. The reactivity k_{eff} at the end-of-cycle the assembly relative power peaking factor and the average of discharged Burnup Bu of the reference are respectively given by k_{ref}, P_{ref} and Bu_{ref}. To construct these three kindS of optimization functions, we can use the relative variation of k_{eff}, P_{xy} and Bu, respectively to k_{ref}, P_{ref} and Bu_{ref}. In general, the change of the sign of the objective function will transform the maximization problem to the minimization problem, Objective function as

$$F(\vec{x}) = \sum_{i=1}^{3} w_i f_i(\vec{x}) \quad (5)$$

$$f_1(\vec{x}) = \tilde{S}_{11}\Delta P_{xy} \quad f_2(\vec{x}) = -\tilde{S}_{12}\Delta k_{eff} \quad f_3(\vec{x}) = -\tilde{S}_{13}\Delta Bu \quad (6)$$

$$\Delta P_{xy} = \frac{P_{xy} - P_{ref}}{P_{ref}} \quad \Delta K_{eff} = \frac{k_{eff} - k_{ref}}{k_{ref}}, \Delta Bu = \frac{Bu - Bu_{ref}}{Bu_{ref}} \quad (7)$$

Where

K_{eff}: Reactivity at the end-of-cycle of the perturbed fuel-loading pattern

K_{ref}: Reactivity of the reference fuel-loading pattern

P_{xy}: assembly relative power peaking factor of the perturbed fuel-loading pattern, it is also chosen as reference objective function

P_{ref}: assembly relative power peaking factor of the reference fuel-loading pattern

Bu: Burnup of the fuel region to be discharged of the perturbed fuel-loading pattern

Bu_{ref}: Burnup of the fuel region to be discharged of the reference fuel-loading pattern

f_1**:** assembly relative power peaking factor minimization throughout the cycle

f_2**:** Maximization of the cycle length (maximizing K_{eff} at the EOC)

f_3**:** Maximization of region average discharged fuel burnup

w_1, w_2 and w_3 are the weighting factors of f_1, f_2 and f_3 .

$\tilde{S}_{12}$ and $\tilde{S}_{13}$ are sensitivity factors of f_2 and f_3 with respect to f_1.

3.2 PENALTY FUNCTION

The in-core fuel management optimization problem, the most widely used of the constraint conditions were the three corresponding respectively to the assembly relative power peaking factor P_{xy} Reactivity k_{eff} and Burnup *Bu*

$$P_{xy} \le P_{\max}; \quad (8)$$

$$k_{\min} \le k_{eff} \le k_{\max}; \quad (9)$$

$$Bu_{\min} \le Bu \le Bu_{\max} \quad (10)$$

where, $P_{\max}$; $k_{\min}$; $k_{\max}$; $Bu_{\min}$; $Bu_{\max}$ are the limit of assembly relative power peaking factor, Reactivity k_{eff} and Burnup *Bu*.

Consider that the constraint C varied in the field [C_{min};C_{max}] then penalty constraint function is:

$$P(\vec{x}) = \max\{0, \frac{C_{\min} - C}{C_{\min}}\} + \max\{0, \frac{C - C_{\max}}{C_{\max}}\} \quad (11)$$

To optimize within constraints, the penalized objective function is given as:

$$\tilde{F}(\vec{x}) = \sum_{i=1}^{3} w_i f_i(\vec{x}) + \lambda \sum_{i=1}^{3} \gamma_i P_i(\vec{x}) \qquad (12)$$

Where λ is penalty factor, γ_i is constrain multipliers. Here $\gamma_i = S_{1i}$;

$$\lambda_{k+1} = \frac{\lambda_k}{0.95} \; ; \; \lambda_1 = 1000 \qquad (13)$$

According to the in-core fuel management optimization problem, we defined the penalty function P_1, P_2, P_3 as:

$$P_1(\vec{x}) = \max\{0, \frac{P_{xy} - P_{\max}}{P_{\max}}\} \qquad (14)$$

$$P_2(\vec{x}) = \max\{0, \frac{k_{\min} - k_{eff}}{k_{\min}}\} + \max\{0, \frac{k_{eff} - k_{\max}}{k_{\max}}\} \qquad (15)$$

$$P_3(\vec{x}) = \max\{0, \frac{Bu_{\min} - Bu}{Bu_{\min}}\} + \max\{0, \frac{Bu - Bu_{\max}}{Bu_{\max}}\} \qquad (16)$$

4. OPTIMIZATION RESULT

Dayabay Nuclear Plant (DNP)[5] is the core for our study. A reference of the in-core fuel assemblies is provided in Fig 7-a. The core used was eighth-core symmetric with 157 fuel assemblies grouped among 3 regions A (1.8wt%), B (2.4wt%), and C (3.1wt%). The burnable poison (BP) rods were restricted to symmetric loadings within fresh fuel assemblies in groupings of 12 and 16 BP rods per assembly. For the assemblies not filled, the symmetric positions are rotationally loaded with equivalent of upper axis.
We used the advanced Non-linear Nodal Green's Function Method (NNGFM)[6] program to handle the neutronic calculation. Through the correction of the reflector in a three-dimensional calculation, the two-dimension model is adopted to navigate the calculation of a multi-step of burnup. The essence of the article is as the following: If sensitivity factor is introduced in a SA algorithm, the multi-objective optimization problem can be solved.

4.1 SENSITIVITY FACTOR CALCULATION

We adopted the statistical probability method to determine the sensitivity factor. Through the random perturbation of assemblies arrangements and burnable poison arrangements, the sampling of 2500 loading pattern, then the assembly relative power peaking factor ΔP_{xy}, the relative reactivity at the end-of-cycle Δk_{eff} and the relative burnup of the fuel region to be discharged ΔBu can be calculated. Formulas are given as follows:

$$\Delta P_{xy} = \frac{P_{xy} - P_{ref}}{P_{ref}} \qquad (17)$$

$$\Delta K_{eff} = -\frac{k_{eff} - k_{ref}}{k_{ref}} \qquad (18)$$

$$\Delta Bu = \frac{Bu - Bu_{ref}}{Bu_{ref}} \qquad (19)$$

Before going into any details of calculation, we have to specify a reference objective function. The role of this function is significant, because in the penalty objective function all sensitivity factors are generated with respect to it. Set S_{11}=1. Here we choose assembly relative power peaking factor as the reference objective function. Then we calculate for each loading pattern the sensitivity factor $S_{12}$of Reactivity relative to the reference objective function:

$$S_{12} = \left| \frac{\Delta P_{xy}}{\Delta k_{eff}} \right| \qquad (20)$$

Figure 2 shows a sensitivity factor S_{12} for the random loading pattern. From the figure we can see that for a great number of loading patterns the sensitivity factor values most fall between 0 and 60.

Figure 2. Sensitivity factor S_{12} of reactivity relative to power peaking factor

In 2500 sampled loading patterns we calculate the probability of the sensitivity factor:

$$P(S_{12}) = \text{(Total numbers of the loading patterns which have sensitivity factor equal to S)/2500}$$
$$S_{12} \le S < S_{12} + 1$$

The probability distribution of the sensitivity factor is given in figure 3. In the optimization process, if we want to be rigorous in determining the sensitivity factor, we should use the probability distribution function method. This method needs to know the fit function of the probability distribution of sensitivity factors. From the figure we can see that a great number of loading patterns have the sensitivity factor values between 0 and 60, moreover the curve presents a peak. Out of this field if the probability was relatively small, then we don't consider it. In this field we take the approximate sensitivity factor as the value in the peak point. We obtain

$$\tilde{S}_{12} = \max(S_{12}) = 25$$

Here, the sensitivity factor reflected the relation between the reactivity and the assembly relative power peaking factor.

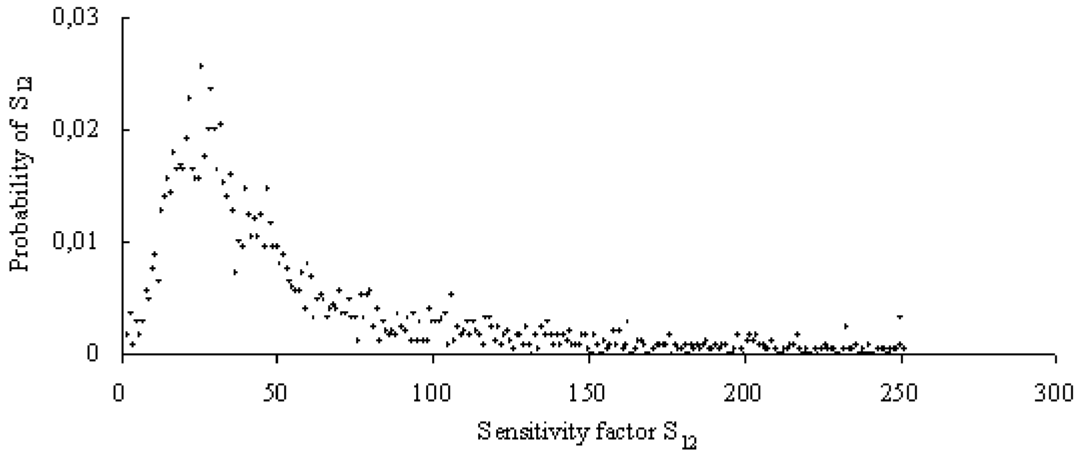

Figure 3. The probability distribution of the sensitivity factor S_{12}

Using the similar method to analyze the sensitivity factor S_{13} the discharged burnup relative to assembly relative power peaking factor. We found

$$S_{13} = \left| \frac{\Delta P_{xy}}{\Delta Bu} \right| \tag{21}$$

Figure 4 shows the sensitivity factor S_{13} of the random loading pattern. The probability distribution of the sensitivity factor $P(S_{13})$ is given in figure 5. In the field between 0 and 10, the peak of the curve is relatively high. At the peak point the probability of the sensitivity factor S_{13} presents a higher value then the probability of the sensitivity factor S_{12}. We take the approximate sensitivity factor as the value in the peak point. We obtain

$$\tilde{S}_{13} = \max(S_{13}) = 3.6$$

Here, the sensitivity factor reflected the relation between the discharged burnup and the assembly relative power peaking factor.

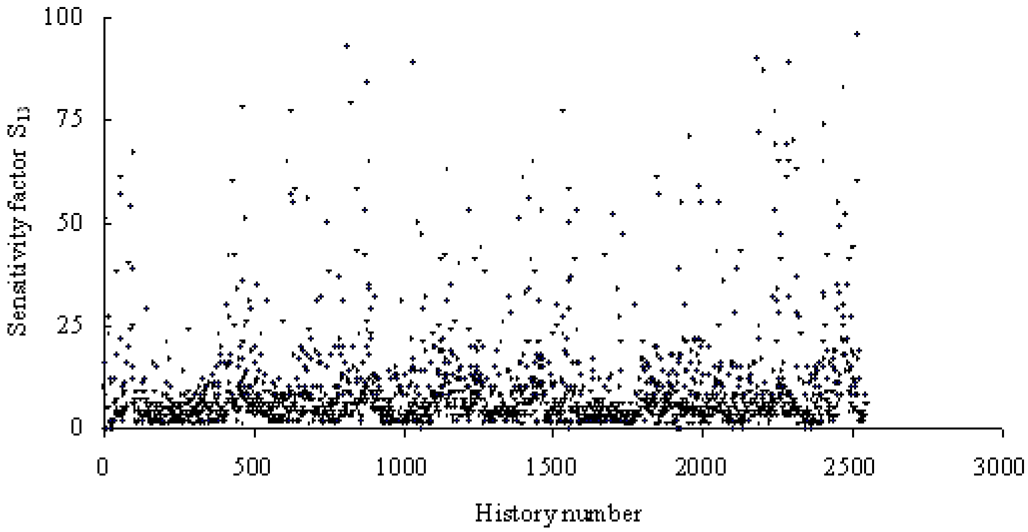

Figure 4. The sensitivity factor S_{13} of discharged burnup relative to power peaking factor

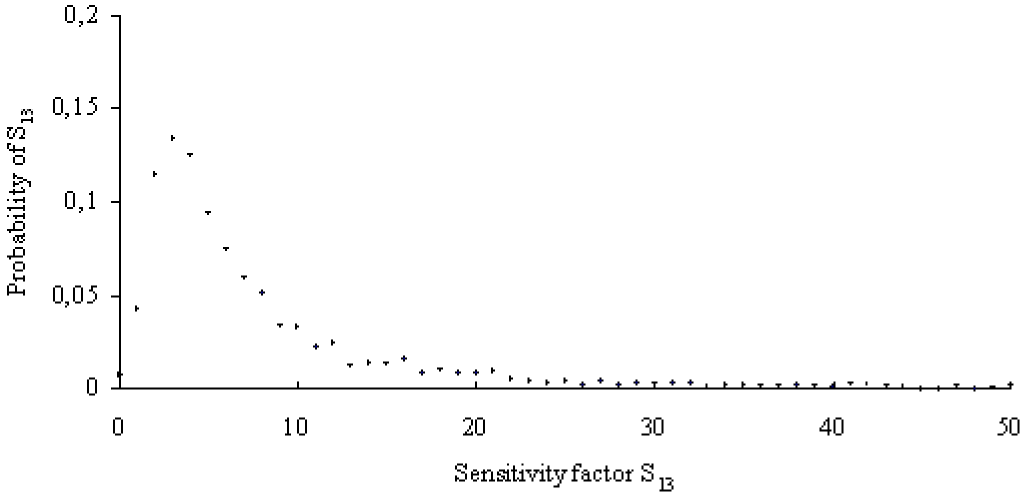

Figure 5.The probability distribution of the sensitivity factors

The use of various weighting factors in the multi-objective optimization has different and varying importance in practical engineering. In the following multi-objective optimization, the maximization of reactivity at EOC and the minimization of power peaking factor are achieved efficiently and simultaneously. Another goal of our study is the maximization of discharged fuel with minimization of power peaking factor. The results are discussed bellow. Note that the Multi-Objective Simulated Annealing program MOSA written for this paper utilizes an adaptive sensitivity control algorithm to estimate the sensitivity factors for the objective functions at the beginning of each optimization problem.

4.2 REFERENCE OPTIMIZATION PARAMETER OF THE FIRST CYCLE

Reference parameters and constraints limit for Simulated annealing algorithm were given in the Table 4.

Table 4.Reference parameters and constraints limit of first cycle of Dayabay core

k_{ref}	P_{ref}	Bu_{ref} (GWd.t^{-1})	$k_{\min}$	$k_{\max}$	$P_{\max}$	$Bu_{\min}$ (GWd.t^{-1})	$Bu_{\max}$ (GWd.t^{-1})
1.10025	1.25	13.178	1.0	1.12	1.26	5.000	50.000

4.3 MAXIMIZATION OF k_{eff} AT EOC AND MINIMIZATION OF POWER PEAKING FACTOR

We optimize the cycle length and the power peaking factor simultaneously. In the case when the sensitivity factor was fixed, in order to test the role of the weighting factor, we used five groups of weights to optimize the multi-objective function. A reference of the in-core fuel assemblies is provided in Figure 6-a. We take w_3 =0 in the formula (12). The weights w_1 and w_2, for example use five representative values. The optimum patterns resulting from optimization are given in figure 6. All other results are shown in the Table 5. The variation of the power peaking factor P_{xy} and the cycle length with weighting factor are given in figure 7.

Table 5.Optimization results

Loading Pattern	Weight w_1:w_2:w_3	*Pxy*	Cycle length EFDP	Discharged fuel ($GWd.t^{-1}$)
Reference		1.254	336.2	13.178
LP 1	1.0:0.0:0.0	1.180	339.5	13.580
LP 2	0.7:0.3:0.0	1.197	344.8	13.654
LP 3	0.5:0.5:0.0	1.209	345.6	13.724
LP 4	0.3:0.7:0.0	1.226	347.2	13.678
LP 5	0.0:1.0:0.0	1.252	348.2	13.727

From Table 5, the optimum loading patterns all increased the cycle length and decreased the power peaking factor, From an engineering judgment viewpoint, they are better then the reference loading pattern. When w_1=1.0, w_2=0.0 and w_3=0.0, the penalty objective function is restricted to power peaking factor as a single objective. As the Power peaking factor decreased by 5.9%, the loading pattern increased the cycle length about 3 EFPD, and the average of the discharged fuel burnup increased about 400 $MWd.t^{-1}$. When w_1=0.0, w_2=1.0 w_3=0.0, the penalty objective function was restricted to cycle length as a single objective, the result of optimization increased the cycle length about 12 EFPD, and the power peaking factor basically didn't change. The average of the discharged burnup increased by 700 $MWd.t^{-1}$ then the reference loading pattern.

A	B	A	B	A	B	A	C
	A	B	A	B	A	C	C
	B	A	B	A	B	C	
	A	B	A	B	C	C	
	B	A	B	A	C		
	A	B	C	C			
	C	C	C				
	C						

	16		16		16		
		12		12		16	
	12		12		12		
		12		12	12		
	12		12				
		12	12				
	16						

a- Reference LP for general optimization

A	B	A	B	A	B	A	C
	A	B	A	B	A	C	C
	B	A	B	A	C	C	
	A	B	A	B	C	B	
	B	A	B	A	C		
	A	C	C	C			
	C	C	B				
	C						

0	12	0	12	0	16	0	0
	0	16	0	8	0	16	0
	16	0	8	0	20	0	
	0	8	0	16	8	0	
	8	0	16	0	0		
	0	20	8	0			
	16	0	0				
	0						

b- w1= 1.0 : w2= 0.0 : w3= 0.0

Figure 6. Optimum patterns of maximization of k_{eff} at the EOC and minimization of P_{xy}

A	B	A	B	A	B	A	C
	A	B	B	B	A	C	B
	B	A	A	A	C	C	
	B	A	A	C	C	B	
	B	A	C	A	C		
	A	C	C	C			
	C	C	B				
	B						

0	8	0	12	0	16	0	0
	8	8	8	8	0	8	0
	8	0	0	0	16	0	
	8	0	0	16	16	0	
	8	0	16	0	0		
	0	16	16	0			
	8	0	0				
	0						

c- w_1= 0.7 : w_2= 0.3 : w_3= 0.0

A	B	A	B	A	C	A	B
	A	B	A	B	A	C	C
	B	A	B	A	C	C	
	A	B	A	C	C	B	
	B	A	C	A	B		
	A	C	C	B			
	C	C	B				
	C						

0	8	0	8	0	20	0	8
	0	12	0	12	0	8	0
	12	0	8	0	16	0	
	0	8	0	16	12	0	
	12	0	16	0	0		
	0	16	12	0			
	8	0	0				
	0						

d- w_1= 0.5 : w_2= 0.5 : w_3= 0.0

A	B	A	B	A	C	A	B
	A	B	B	B	A	C	B
	B	A	A	A	C	C	
	B	A	A	C	C	B	
	B	A	C	A	C		
	A	C	C	C			
	C	C	B				
	B						

0	12	0	20	0	16	0	0
	0	8	0	12	0	8	0
	8	0	8	0	16	0	
	0	8	0	16	8	8	
	12	0	16	0	0		
	0	16	8	0			
	8	0	8				
	0						

e- w_1= 0.3 : w_2= 0.7 : w_3= 0.0

A	B	A	B	A	C	A	B
	A	B	B	B	A	C	C
	B	A	A	A	C	C	
	B	A	A	C	C	B	
	B	A	C	A	B		
	A	C	C	B			
	C	C	B				
	C						

0	16	0	8	0	16	0	0
	0	0	16	16	0	8	0
	0	0	0	0	16	0	
	16	0	0	16	0	8	
	16	0	16	0	0		
	0	16	0	0			
	8	0	8				
	0						

f- w_1= 0.0 : w_2= 1.0 : w_3= 0.0

Figure 6. Optimum patterns of maximization of k_{eff} at the EOC and minimization of P_{xy} (continue)

Figure 7 shows the power peaking factor P_{xy} and the cycle length as functions of weighting factors. From table 5 and figure 7, we find the power peaking factor decreased as the weighting factor w_1 increased; whereas the cycle length increased as the weighting factor w_2 increased. The results show when introducing the sensitivity factor during the optimization process, the power peaking factor objective and cycle length objective almost have the same utility. We can obtain Pareto optimal of interested optimization only by justifying every weighting value.

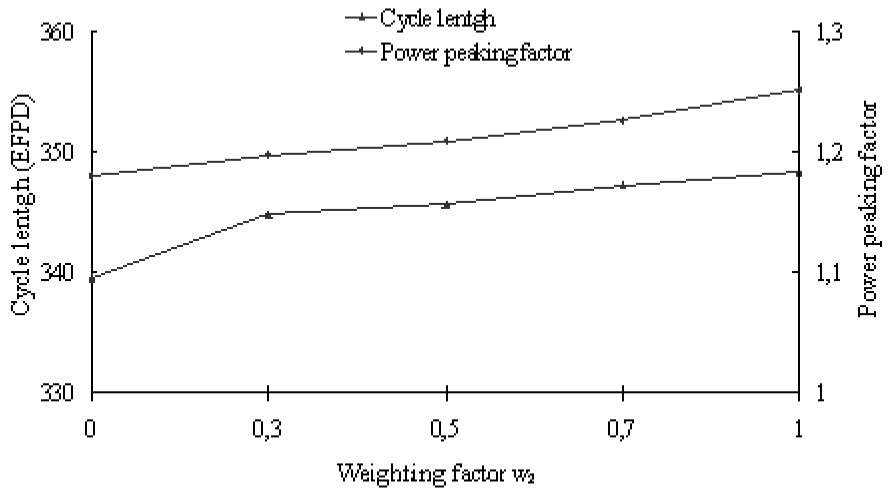

Figure 7.The power peaking factor P_{xy} and the cycle length as a function of the weight

If we do not consider the sensitivity factors, then adopting a random weighting method to generate the multi-objective function, will hold more samples of multi-objective search direction. To a certain degree, it can make up the role of the sensitivity factor, but the random weighting method has a larger calculation time. Another way is using fixed weighting but unconsidered sensitivity factors. In general, the difference between the weights of objective functions is great, in some paper this difference arrived up to the rate of 1000 times [3]. As a matter of fact, weighting factor contain sensitivity factors influences. Thus, a great improvement of one objective is needed in order to gain a little improvement in other objectives. Thus the method is not ideal.

In order to compare the burnup characteristics of every optimization loading patterns results, we gave the variation of the cycle length and power peaking factors with the burnup. Table 6 and 7 listed the results, while figure 8 and figure 9 showed respectively the reactivity k_{eff} and The power peaking factor P_{xy} as a function of cycle length.

Table 6. The reactivity K_{eff} as a function of cycle length

Cycle length EFPD	Reference LP	w_1:w_2:w_3 1.0:0.0:0.0	w_1:w_2:w_3 0.7:0.3:0.0	w_1:w_2:w_3 0.5:0.5:0.0	w_1:w_2:w_3 0.3:0.7:0.0	w_1:w_2:w_3 0.0:1.0:0.0
0	1.100252	1.103	1.111246	1.111702	1.111007	1.115387
50	1.088414	1.090887	1.097417	1.097769	1.097574	1.100057
100	1.077059	1.078835	1.083764	1.083732	1.084143	1.085462
150	1.062155	1.063493	1.067222	1.067102	1.067803	1.068599
200	1.045855	1.047028	1.049928	1.049899	1.050658	1.051247
250	1.028861	1.03005	1.032482	1.032544	1.033233	1.033704
300	1.012161	1.013332	1.015348	1.015477	1.016076	1.016460
350	0.99535	0.996531	0.998317	0.998468	0.998981	0.999246

The figure 8 shows the curves of variation of reactivity k_{eff} with burnup are basically straight lines, and the reactivity k_{eff} at the BOC of each optimal loading pattern is slightly larger then the reference loading pattern. From figure 9, we see that all power peaking factor curves were relatively smooth, and these results were better then reference loading pattern which presented two extremes.

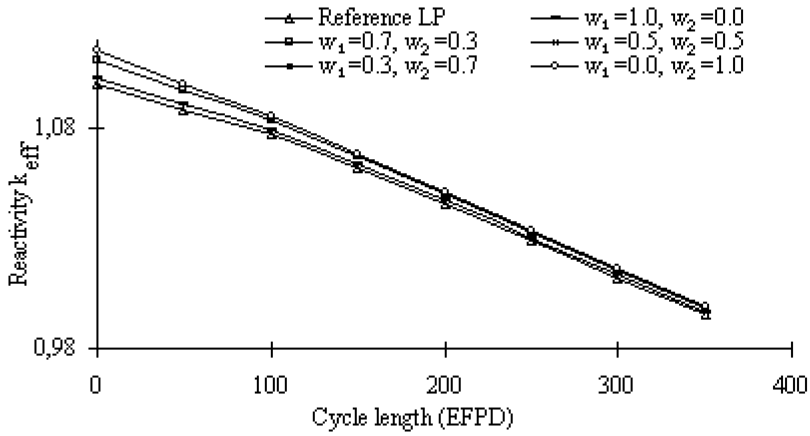

Figure 8.The reactivity k_{eff} as a function of cycle length

Table 7. The power peaking factor P_{xy} as a function of cycle length

Cycle length EFPD	Reference LP	w1:w2:w3 1.0:0.0:0.0	w1:w2:w3 0.7:0.3:0.0	w1:w2:w3 0.5:0.5:0.0	w1:w2:w3 0.3:0.7:0.0	w1:w2:w3 0.0:1.0:0.0
0	1.255	1.182	1.181	1.192	1.227	1.210
50	1.106	1.16	1.165	1.19	1.208	1.245
100	1.163	1.159	1.181	1.176	1.202	1.218
150	1.191	1.171	1.173	1.189	1.205	1.219
200	1.184	1.156	1.183	1.202	1.21	1.232
250	1.152	1.155	1.192	1.207	1.213	1.233
300	1.120	1.162	1.195	1.208	1.212	1.230
350	1.100	1.165	1.195	1.207	1.209	1.224

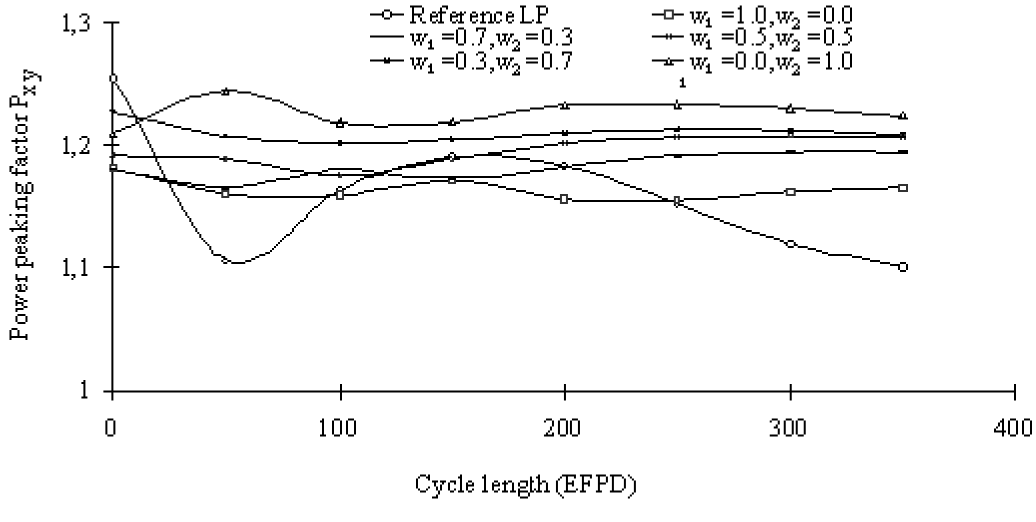

Figure 9. The power peaking factor P_{xy} as a function of cycle length

In addition to the first case examined, the following we will examine another case. The objective of optimization was to maximize the burnup of the discharge fuel with the minimization of the power peaking factor.

4.4 MAXIMIZATION OF DISCHARGED FUEL WITH MINIMIZATION OF P_{xy} FACTOR

In this case, we also used the formula (12), w_2 was taken equal to zero, for w_1 and w_3 we took the same weight, w_1=0.5 and w_3=0.5. The sensitivity factor S_{13} needed in this optimization was calculated by the MOSA program in the beginning of optimization (see 4.1). Loading patterns resulting from optimization are given in figure 10. All other results are shown in the Table 8.

Table 8.Optimization results

Loading Pattern	Weight w_1:w_2:w_3	*Pxy*	Cycle length EFDP	Discharged fuel ($GWd.t^{-1}$)
Reference		1.254	336.2	13.178
LP 1	0.5 : 0.0 : 0.5	1.208	338.5	13.553

A	B	A	B	A	C	A	B
	A	B	A	B	A	C	C
	B	A	B	A	B	C	
	A	B	A	C	C	C	
	B	A	C	A	B		
	A	B	C	B			
	C	C	C				
	C						

0	12	0	8	0	20	0	0
	0	12	0	8	0	8	8
	12	0	8	0	8	0	
	0	8	0	12	12	0	
	8	0	12	0	16		
	0	8	12	16			
	8	0	0				
	8						

w_1= 0.5 : w_2= 0.0 : w_3= 0.5

Figure 10. Optimum pattern of maximization of discharged fuel with minimization of P_{xy} factor

The MOSA program found the optimum loading patterns increased the average of the discharged fuel burnup about 375 $MWd.t^{-1}$ and decreased the power peaking factor decreased by 3.7%. From an engineering judgment viewpoint, the optimum patterns were betters then the reference loading pattern.

CONCLUSIONS

The sensitivity factor was introduced to the Simulated Annealing method (SA) to determine the multi-objective function. The probability distribution method was used to calculate the introduced sensitivity factors. The first cycle of the Dayabay Nuclear Plant (DNP) was optimized with the Multi-Objective Simulated Annealing program MOSA. Two objectives of optimization were chosen as a concrete example: simultaneously maximizing the reactivity at EOC and minimizing the power peaking factor. Another goal of our study was the maximization of discharged fuel with the minimization of power peaking factor. The results showed that the greatest improvements in multi-objective optimization are achieved by using sensitivity factors. It was our aim to efficiently use the weighting factor in the objective function to contra pose the choice of different objectives, for instance in the case of the nuclear fuel management optimization problem.

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