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THE COUPLED KINETIC AND THERMAL-HYDRAULIC THREE DIMENSIONAL CODE SYSTEM NLSANMT/COBRA-Ⅳ FOR PWR CORE TRANSIENT ANALYSIS

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ABSTRACT

A three dimensional neutron kinetic nodal code NLSANMT has been developed based on the semi-analytical nodal expansion method and nonlinear iteration strategy, and it has been coupled with the subchannel thermal hydraulic core analysis code COBRA-Ⅳ to form the integrated code system NLSANMT/COBRA-Ⅳ for PWR core transient analysis. This paper describes the features and status of the integrated NLSANMT/COBRA-Ⅳ system. A demonstration application of the NLSANMT/COBRA-Ⅳ code to rod ejection accident analysis is presented using the OECD NEACRP PWR hot zero power full core rod ejection benchmark problem. The NLSANMT/COBRA-Ⅳ solution agrees well with the reference solution, even when one node per assembly and one channel per assembly are used.

1. INTRODUCTION

The accident analysis and safety review of reactor usually require a coupled neutron kinetic and thermal-hydraulic calculation. For severe core accident analysis the three-dimensional kinetic model is necessary to obtain the satisfactory transient results. Based on the semi-analytical nodal expansion method and nonlinear iteration strategy a three-dimensional neutron kinetic code NLSANMT is developed and coupled with the wide used subchannel thermal-hydraulic transient analysis code COBRA-Ⅳ to form the integrated NLSANMT/ COBRA-Ⅳ code system for PWR core transient calculations[1-7].

In this paper, the neutron kinetic and mathematical model of the code NLSANMT, the features and status of the integrated NLSANMT/COBRA-Ⅳ system are described. A demonstration application to rod ejection accident analysis is presented using the OECD NEACRP PWR hot zero power full core rod ejection benchmark problem[8-10]. The NLSANMT/COBRA-Ⅳ solution shows good agreement with the reference solutions, even one node per assembly and one channel per assembly are used.

2. NEUTRON KINETIC CODE NLSANMT

In the NLSANMT, an efficient fully implicit scheme is applied for time discretization of neutron flux equations in combination with directly integration of the delayed neutron precursor equations over the time interval Δt_n. A variable time-step size is used during the numerical procedure for decreasing the total amount of time-steps and computing time. The resulted fixed-source neutron diffusion problem (FSP) is solved by the method based on the semi-analytical nodal expansion method and nonlinear iteration strategy[1-4].

First, the fully implicit scheme is used for time discretization of neutron flux equations as

$$\frac{\partial \Phi_g(\mathbf{r},t)}{\partial t}\Big|_{t_n} = \frac{\Phi_g(\mathbf{r},t_n) - \Phi_g(\mathbf{r},t_{n-1})}{\Delta t_n} \tag{1}$$

where $\Phi(\mathbf{r},t_n)$ is neutron flux at t_n.

And then, the delayed neutron precursor equations are integrated over the time interval $[t_{n-1},t_n]$ assuming a linear dependence of the neutron flux. The result follows as

$$\begin{aligned} C_i(\mathbf{r},t_n) &= C_i(\mathbf{r},t_{n-1})e^{-\lambda_i\Delta t_n} + \frac{\beta_i}{\lambda_i}\Big[\frac{1}{\lambda_i\Delta t_n}(1-e^{-\lambda_i\Delta t_n}) - e^{-\lambda_i\Delta t_n}\Big]\sum_{g'=1}^{G}\nu\Sigma_{fg'}(\mathbf{r},t_{n-1})\Phi_{g'}(\mathbf{r},t_{n-1}) \\ &+ \frac{\beta_i}{\Delta t_n\lambda_i}\Big[\Delta t_n - \frac{1}{\lambda_i}(1-e^{-\lambda_i\Delta t_n})\Big]\sum_{g'=1}^{G}\nu\Sigma_{fg'}(\mathbf{r},t_n)\Phi_{g'}(\mathbf{r},t_n) \end{aligned} \tag{2}$$

where the notations are fairly standard.

Substituting the expression (1) and (2) into the neutron flux equation we obtain the time-discretized neutron flux equations as

$$\begin{aligned} &-\nabla\cdot D_g(\mathbf{r},t_n)\nabla\Phi_g(\mathbf{r},t_n) + \Sigma_{tg}(\mathbf{r},t_n)\Phi_g(\mathbf{r},t_n) = \\ &\sum_{g'=1}^{G}\Sigma_{g'g}(\mathbf{r},t_n)\Phi_{g'}(\mathbf{r},t_n) + \chi_g\sum_{g'=1}^{G}\nu\Sigma_{fg'}(\mathbf{r},t_n)\Phi_{g'}(\mathbf{r},t_n) + S_g(\mathbf{r},t_n) \end{aligned} \tag{3}$$

where

$$\chi_g = (1-\beta)\chi_{pg} + \sum_{i=1}^{I}\beta_i\chi_{dgi}$$

$$
\begin{aligned}
S_g(\mathbf{r},t_n) = {} & \sum_{i=1}^{I}\{\chi_{dgi}\lambda_i\frac{\beta_i}{\Delta t_n\lambda_i}[\Delta t_n-\frac{1}{\lambda_i}(1-e^{-\lambda_i\Delta t_n})]-\beta_i\chi_{dgi}\}\sum_{g'=1}^{G}\nu\Sigma_{fg'}(\mathbf{r},t_n)\Phi_{g'}(\mathbf{r},t_n) \\
& -\frac{1}{v_g\Delta t_n}[\Phi_g(\mathbf{r},t_n)+\Phi_g(\mathbf{r},t_{n-1})]+\sum_{i=1}^{I}\chi_{dgi}\lambda_iC_i(\mathbf{r},t_{n-1})e^{-\lambda_i\Delta t_n} \\
& +\sum_{i=1}^{I}\chi_{dgi}\lambda_i\frac{\beta_i}{\lambda_i}[\frac{1}{\lambda_i\Delta t_n}(1-e^{-\lambda_i\Delta t_n})-e^{-\lambda_i\Delta t_n}]\cdot\sum_{g'=1}^{G}\nu\Sigma_{fg'}(\mathbf{r},t_{n-1})\Phi_{g'}(\mathbf{r},t_{n-1})
\end{aligned}
$$

and all other notations are very fairly standard.

As a result we have the FSP equations similar to the steady-state neutron diffusion equations. The nonlinear iteration semi-analytical nodal expansion method is used for solving the FSP equations. In this method, the surface-average finite-difference neutron current is expressed as follows

$$J^k_{gu+} = -D^{k,FDM}_{gu+}(\overline{\Phi}^{k+1}_g-\overline{\Phi}^k_g)-D^{k,NOD}_{gu+}(\overline{\Phi}^{k+1}_g+\overline{\Phi}^k_g) \tag{4}$$

where $D^{k,FDM}_{gu+}$ denotes the finite-difference coupling coefficient, $D^{k,NOD}_{gu+}$ is the nonlinear nodal coupling coefficient[1-4].

Substituting the expression (4) into the nodal neutron balance equation we can obtain the coarse-mesh finite-difference (CMFD) formulation, which contains the nonlinear nodal coupling coefficients. By solving the two-node problem at the interfaces of two adjacent nodes using the semi-analytical nodal expansion method, the nodal surface-average neutron currents are obtained. Then it is used for determining the nonlinear nodal coupling coefficients by expression (4). By nonlinear iteration strategy the nonlinear nodal coupling coefficients are updated iteratively which forces the CMFD equation to yield the high-order nodal method predicted values.

Solution of the two-node problem is based on the transverse-integrated neutron diffusion equations of FSP. In the semi-analytical nodal expansion method, the transverse-integrated neutron flux is expanded over the node into basic functions as

$$\varphi^k_{gu}(u)=\overline{\Phi}^k_g+\sum_{i=1}^{4}a^k_{gui}p_i(\frac{2u}{\Delta u_k}) \tag{5}$$

where $\overline{\Phi}^k_g$ is the nodal average neutron flux, a^k_{gui} is the expansion coefficient, Δu_k is the nodal width on the u-direction, and $\{p_i(2u/\Delta u_k),\ i=0,1,2,3,4,5\}$ is an orthogonal analytical basic function set as follow

$$
\begin{aligned}
&p_0(t) = 1 \\
&p_1(t) = t \\
&p_2(t) = \frac{1}{2}(3t^2 - 1) \\
&p_3(t) = \frac{\sinh(\alpha_{gu}^k t) - m_{gu1}^k(\sinh)p_1(t)}{\sinh(\alpha_{gu}^k) - m_{gu1}^k(\sinh)} \\
&p_4(t) = \frac{\cosh(\alpha_{gu}^k t) - m_{gu0}^k(\cosh)p_0(t) - m_{gu2}^k(\cosh)p_2(t)}{\cosh(\alpha_{gu}^k) - m_{gu0}^k(\cosh) - m_{gu2}^k(\cosh)}
\end{aligned}
\tag{6}
$$

where

$$
t = 2u/\Delta u_k \qquad , \qquad \alpha_{gu}^k = \sqrt{\frac{\Sigma_{rg}^k}{D_g^k}}\frac{\Delta u_k}{2}
$$

$$
m_{gu1}^k(\sinh) = \frac{1}{N_1}\int_{-1}^{1}\sinh(\alpha_{gu}^k t)p_1(t)dt
$$

$$
m_{gui}^k(\cosh) = \frac{1}{N_i}\int_{-1}^{1}\cosh(\alpha_{gu}^k t)p_i(t)dt \qquad (i = 0,2)
$$

$$
N_i = 2/(2i+1) \qquad (i = 0,1,2)
$$

which results in an efficient algorithm for the solution of the nodal equations for the two-node problem[1-2].

To the two-node problem, 8G unknowns should be calculated, where G is the number of neutron energy groups. In order to calculate the 8G unknowns the follwing equations are used: 2G neutron balance equations, 2G first-order and 2G second-order moment-weighting equations, and 2G equations of the neutron flux and neutron current continuity at the internal interface between two nodes. Using the orthogonality of basic functions set the initial system of 8G nodal equations for two-node problem can be reduced to a set of 1G and 2G equations[1-2].

In NLSANMT the solution of the resulting CMFD equations is performed using traditional outer and inner iterations. The convergence is accelerated by source extrapolation and Wielandt method in combination with coarse mesh rebalancing method. At the exterior of outer iteration a nonlinear iteration is using for updating the nonlinear nodal coupling coefficients.

Verification of the NLSANMT code has been performed against a series of LWR benchmark and test problems[2]. The comparison of numerical results with those of other nodal codes shows good agreement for all considered transient parameters and confirms its validity, stability and high efficiency, even using rather coarse spatial and temporal meshes calculation.

3. SUBCHANNEL THERMAL-HYDRAULIC CODE COBRA-Ⅳ

The subchannel thermal hydraulic steady and transient analysis code COBRA-Ⅳ is a well verified code widely used in nuclear industry, which is the extended version of the COBRA-Ⅲ subchannel analysis code developed at the Pacific Northwest Laboratory[5-7]. The COBRA-Ⅳ code can compute the flow and enthalpy distributions in nuclear fuel rod bundles and core for both steady and transient conditions.

In the COBRA-Ⅳ code the two-phase flow is assumed to be incompressible, homogeneous and thermal-equilibrium, but thermally expandable. The conservation equations for mass, energy, axial and transverse momentum conservation can be solved by two numerical methods, implicit method and explicit method. COBRA-Ⅳ is very flexible for modeling reactor core and fuel assemblies. A flow channel in the COBRA-Ⅳ model can be a subchannel or a lumped channel representing an entire fuel assembly.

In addition, COBRA-Ⅳ also solves heat conduction equations for nuclear fuel rods to compute the rod internal temperature distribution and the rod surface heat fluxes, in which the effects of radial and axial conduction and temperature dependence of thermal conductivity are considered. The method of Weighted Residuals (MWR) using orthogonal collocation is used in radial direction and the finite difference method is used for time derivatives and axial space derivatives.

4. THE COUPLED NLSANMT/COBRA-Ⅳ CODE SYSTEM

NLSANMT and COBRA-Ⅳ are coupled to form the integrated code system NLSANMT/COBRA-Ⅳ for PWR core transient analysis through a module for controlling the data transfer between the two codes. Each one of the two codes is otherwise left intact. The inputs of the NLSANMT and the COBRA-Ⅳ remain unaffected by the integration, and is read in only once at the beginning of a case of coupled analysis[2].

In the NLSANMT/COBRA-Ⅳ code, the NLSANMT reactor core model is very flexible that an assembly can be divided into a node or some nodes in radial direction. Accordingly each node is represented as a coolant channel in COBRA-Ⅳ model, but the axial meshing can differ from NLSANMT. An interface module is used for controlling the data transfer and interpolation for axial direction between the two codes. On the one hand the module calls the COBRA-Ⅳ for coolant density, coolant temperature and fuel temperature feedback calculations during the NLSANMT feedback iterations of a neutronic calculation. On the other hand the power distribution calculated by NLSANMT is transferred to COBRA-Ⅳ for thermal-hydraulic calculation. During each time step an iteration calculation between NLSANMT and COBRA-Ⅳ is performed till power distribution converges and the code interface can use the COBRA-Ⅳ feedback based on the most updated power distribution.

The coupled NLSANMT/COBRA-Ⅳ code system has been used to simulate rod ejection accident. As a benchmark application of the coupled NLSANMT/COBRA-Ⅳ code system, the OECD NEACRP PWR hot zero power full core rod ejection benchmark problem has been analyzed. The results are presented in the next section.

5. OECD NEACRP PWR ROD EJECTION BENCHMARK

There are six PWR rod ejection benchmark problems documented in ref. 8. All of them have been calculated by NLSANMT/COBRA-Ⅳ and the numerical results are in good agreement with the reference solution[2]. Of the six, problem C1, the hot zero power full core case, is most severe and challenging and is chosen for analysis with NLSANMT/COBRA-Ⅳ. The PWR core geometry is of the Westinghouse 3-loop core type, with 157 fuel assemblies. The top and bottom of the active core are covered with 30cm thick axial reflectors. In radial the core is surrounded by one layer of reflector assemblies, which are of the same size as the fuel assembly. The ejected rod is located near the core periphery. The initial core power is 2775W.

This benchmark problem makes some simple assumptions on thermal-hydraulic and fuel temperature modeling. The gap conductivity is assumed constant. The temperature dependence of material properties is provided through given equations. Rod expansion and cross flow effects are not considered. A flat power distribution in the fuel rod is assumed. The clad surface heat transfer model is not specified in ref. 8, but the reference solution assumes Dittus-Boelter correlation for the heat transfer. All these assumptions can be easily implemented in COBRA-Ⅳ.

In our analysis, NLSANMT/COBRA-Ⅳ models the core radially with one node per assembly and one channel per node. In axial NLSANMT models with 16 unequal nodes over the active core height and one node for the top and bottom axial reflectors respectively, but COBRA-Ⅳ models with 80 equal meshes. The time step interval Δt_n used 0.0025 sec time step up to 1 sec, then switched to 0.02 sec time step up to 2 sec and then switched to 0.2 sec time step up to 5 sec.

The reference solution of this problem was calculated by the PANTHER code provided by Nuclear Electric. There are two versions of this reference solution, which differ appreciably from each other. The old reference solution is documented in ref. 9, and the new one is documented in ref. 10. There are a number of refinements for the new solution. The most significant effect is from a change of a steam table named “SIBDYM” to another steam table called “CEGB”. The next more significant effect is from refinements in solving the fuel rod heat conduction equation.

The results of the steady state calculation for this benchmark problem are compared to the old and new reference solutions in Table 1. The results of the transient calculation are compared to the old and new reference solutions in Table 2 and the core average power as a function of time are plotted in Figure 1. The results show that good agreement with the reference solution is obtained, even when one node per assembly and one channel per assembly are used. And from ref. 9 we can know that our numerical results are in the range of results obtained by the other codes.

Table 1. Comparison of Rod Ejected Steady State Calculations

Solution	Boron(ppm)	Rod worth (pcm)	Power peak	Fuel temp (℃)
Old Ref.	1135.29	953.12	2.1870	286.0
New Ref.	1128.29	949.09	2.1867	-
NLSANMT/COBRA-Ⅳ	1101.50	940.08	2.1872	286.0

Table 2. Comparison of Rod Ejected Transient Calculations

Solution	Max power in transient	Max power at time(s)	Core power at 5s	Fuel temp at 5s (℃)	Coolant temp at 5s (℃)
Old Ref.	4.7515	0.2675	0.1458	315.88	291.46
New Ref.	4.4112	0.2712	0.1460	315.91	291.53
NLSANMT/COBRA-Ⅳ	4.5021	0.2700	0.1484	317.58	291.50

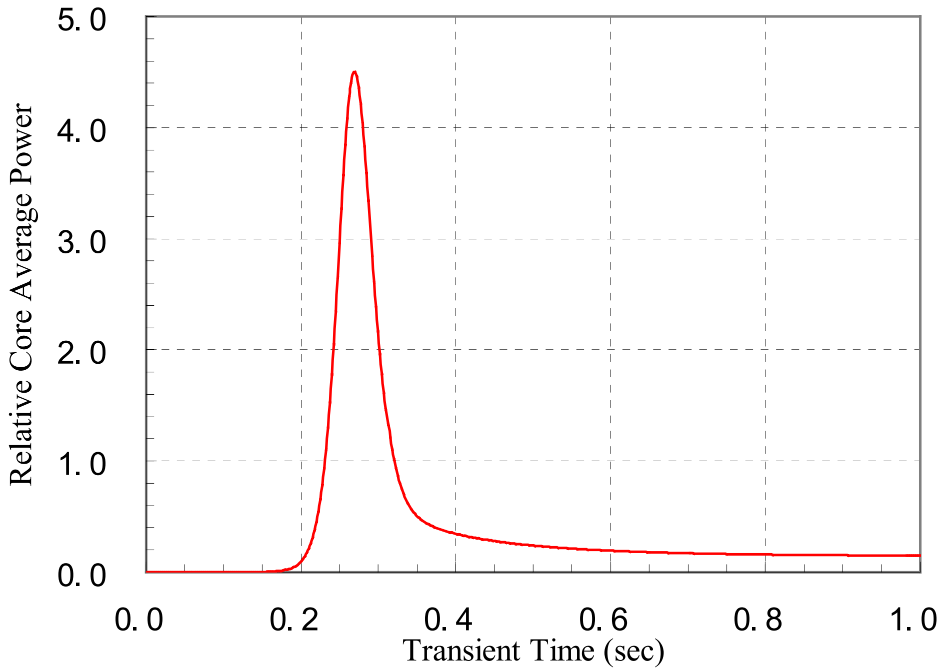

Figure 1. OECD NEACRP PWR Rod Ejection Benchmark Problem C1

CONCLUSIONS

The three dimensional neutron kinetic nodal code NLSANMT based on the nonlinear iteration semi-analytical nodal expansion method has been coupled with the subchannel thermal hydraulic steady and transient analysis code COBRA-Ⅳ to form the integrated code NLSANMT/COBRA-Ⅳ for PWR core transient analysis. An application of the coupled code system to the OECD NEACRP PWR hot zero power full core rod ejection benchmark problem shows that good agreement with the reference solution is obtained, even when one node per assembly and one channel per assembly are used.

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REFERENCES

1 V. G. Zimin, H. Ninokata, "Nodal Neutron Kinetics Model Based on Nonlinear Iteration Procedure for LWR Analysis", *Ann. Nucl. Energy*, **vol. 25,** pp.507-528 (1998).

2 C. K. Liao, "*Study on Numerical Method for Three Dimensional Nodal Space-Time Neutron Kinetic Equations and Coupled Neutronic/Thermal-Hydraulic Core Transient Analysis*", Ph.D. thesis, Xi'an Jiaotong University, China, 2002.

3 P. J. Turinsky, M. K. Al-chalabi, et al., "*NESTLE: A Few-Group Neutron Diffusion Equation Solver Utilizing the Nodal Expansion Method for Eigenvalue, Adjoint, Fixed-Source Steady-state and Transient Problems*", EGG-NRE-11406, June 1994.

4 K. S. Smith, "Nodal Method Strategy Reduction by Nonlinear Iteration", *Tran. Am. Nucl. Soc.*, **vol. 44**, pp.265 (1983).

5 C. W. Stewart, C. A. Mcmonagle, et al, "*Core Thermal Model: COBRA-Ⅳ Development and Applications*", BNWL-2212, NRC-4, Pacific Northwest Laboratories, Jan. 1977.

6 C. W. Stewart, C. A. Mcmonagle, et al, "*COBRA-IV-I: An Interim Version of COBRA for Thermal-Hydraulic Analysis of Rod Bundle Nuclear Fuel Elements and Cores*", BNWL-1962, UC-32, Pacific Northwest Laboratories, Mar. 1976.

7 C. W. Stewart, C. A. Mcmonagle, et al, "*COBRA-IV-I: the Model and the Method*", BNWL-2214, NRC-4, Pacific Northwest Laboratories, July 1977.

8 H. Finnemann, A. Galati, "*NEACRP 3-D LWR Core Transient Benchmark, Final Specification", NEACRP-L-335 (Revision 1)*, October 1991 (January 1992).

9 H. Finnemann, H. Bauer, A. Galati, R. martinelli, "*Results of LWR Core Transient Benchmark, OECD Nuclear Energy Agency Report*", NEA/NSC/DOC(93)25, October 1993.

10 M. P. Knight, P. Bryce, "Derivation of A Refined PANTHER Solution to The NEACRP PWR Rod-ejection Transient", *Proceedings of the Joint International Conf. On Mathematical Methods and Supercomputing For Nuclear Applications*, Vol.1, p302, Saratoga Springs, October 5-9, 1997.