COMPARISON OF NODAL DIFFUSION THEORY METHODS FOR ANALYSIS OF MAIN STEAM LINE BREAK ACCIDENTS IN PWR

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ABSTRACT

The unified nodal method (UNM) neutronics code, SUNCARDS, is coupled with the thermal hydraulics code, MARS, into a thermal-hydraulic/reactor kinetics coupled code, MARS/SUNCARDS. The coupled MARS/SUNCARDS code is then applied for the analysis of OECD/NEA pressurized water reactor main steam line break benchmark (MSLB) phase II and III problems. The fact that the UNM can produce several different nodal method solutions to two-group neutron diffusion equations is utilized to examine the effects of nodal computational solution options on the consequences of the MSLB accidents. It is shown that the predictions made by various nodal methods are almost indistinguishable in that the differences in the computational accuracy of different nodal solution methods do not show up in the various benchmark solutions. It is concluded that thermal hydraulic feedback effects may be described by the finer nodalization that may be compatible with nodal neutronics calculations.

1. INTRODUCTION

Recently, we showed[1,2] that the analytical nodal method (ANM)[3] and the analytical function expansion nodal (AFEN) method[4] for the solutions to two-group (2-G) neutron diffusion equations can be integrated into a single unified nodal method (UNM) in which the nodal coupling relations looks exactly like those of the popular nodal expansion method (NEM)[5]. Because the single UNM formulation can produce several different nodal method solutions corresponding to the original ANM, NEM, and the AFEN method formulation according to the nodal computational option, it can provide a good opportunity to compare the neutronics analysis characteristics of the different nodal computational methods on a fair and equal footing. The purpose of this paper is to examine the effect of nodal computational options on the consequence of the main steam line break (MSLB) accident that requires the detailed three-dimensional (3-D) nodal kinetics analysis model. To do so, we combine the UNM solver, SUNCARDS[2], with the MARS code[6] for a thermal-hydraulic/reactor kinetics coupled code MARS/SUNCARDS, to obtain the solutions to OECD/NEA pressurized water reactor (PWR) MSLB Benchmark Phase II and III problems[7], and investigate how the nodal computational options affect the initial steady state and the key transient parameters of the MSLB Benchmark problem solution.

2. UNIFIED NODAL METHOD

The neutronics model for the 3-D kinetics analysis by the UNM is the 2-G diffusion equations.

It is shown in reference 2 that the fully-implicit time integration of the time dependent 2-G diffusion equations leads one to solve the following equations for each spatial node at each time step;

$$-\mathbf{D}^{(n)}\nabla^2\boldsymbol{\phi}^{(n)}(\mathbf{r})+\mathbf{A}^{(n)}\boldsymbol{\phi}^{(n)}(\mathbf{r})=\mathbf{S}^{(n)}(\mathbf{r});\quad \mathbf{D}^{(n)}=\begin{pmatrix} D_1^{(n)} & 0 \\ 0 & D_2^{(n)} \end{pmatrix},$$

$$\mathbf{A}^{(n)}=\begin{pmatrix} \Sigma_{r1}^{(n)}-\dfrac{1}{k_{eff}}\nu\Sigma_{f1}^{(n)} & -\dfrac{1}{k_{eff}}\nu\Sigma_{f2}^{(n)} \\ -\Sigma_s^{(n)} & \Sigma_{r2}^{(n)} \end{pmatrix}. \tag{1}$$

The superscript n denotes the time step. $\mathbf{S}^{(n)}(\mathbf{r})$ is the 2-dimensional source vector the element of which is given by

$$\begin{aligned}\mathbf{S}_g^{(n)}(\mathbf{r})=&\frac{e^{\Omega_g^{(n)}(\mathbf{r})\Delta t}}{v_g\Delta t}\phi_g^{(n-1)}(\mathbf{r})-\frac{\Omega_g^{(n)}(\mathbf{r})\Delta t+1}{v_g\Delta t}\phi_g^{(n)}(\mathbf{r})+\chi_g\sum_{d=1}^{6}\lambda_d e^{-\lambda_d\Delta t}C_d^{(n-1)}(\mathbf{r})\\&+\frac{\chi_g}{k_{eff}}\sum_{d=1}^{6}\beta_d\left[\frac{\lambda_d}{\lambda_d+\omega^{(n)}(\mathbf{r})}\left(1-e^{-\left(\lambda_d+\omega^{(n)}(\mathbf{r})\right)\Delta t}\right)-1\right]\sum_{g'=1}^{2}\nu\Sigma_{fg'}^{(n)}\phi_{g'}^{(n)}(\mathbf{r})\quad ;g=1,2\end{aligned}. \tag{2}$$

The notations are standard and the detailed definitions are available 2. The nodal volume integration over a spatial node m result in the nodal balance relation;

$$\sum_{u=x,y,z}\frac{1}{a_u^m}\left(\mathbf{J}_{ur}^m-\mathbf{J}_{ur}^m\right)+\mathbf{A}\overline{\boldsymbol{\phi}}^m=\overline{\mathbf{S}}^m \tag{3}$$

The superscript n is dropped in Eq. (3) for simplicity of notation.
The UNM derives the flux-current coupling relation from the solution to the transverse integrated one-dimensional (1-D) equation over the given node m;

$$-\frac{d^2}{du^2}\boldsymbol{\phi}_u(u)+\widetilde{\mathbf{A}}\boldsymbol{\phi}_u(u)=-\mathbf{L}_u(u)+\mathbf{S}_u(u)\ ;\ \widetilde{\mathbf{A}}=\mathbf{D}^{-1}\mathbf{A} \tag{4}$$

$\mathbf{DL}_u(u)$ and $\mathbf{DS}_u(u)$ here denote the transverse leakage and transverse integrated source, respectively. $\mathbf{DL}_u(u)$ can be approximated either by the quadratic polynomial transverse leakage (QTL)[5] or by the analytic transverse leakage (ATL)[8] as follows;

$$\mathbf{L}_u(u)=\sum_{i=0}^{2}\mathbf{g}_{iu}\left(\frac{u}{a_u}\right)\mathbf{L}_{iu} \tag{5}$$

The leakage shape functions, $\mathbf{g}_{iu}(\tau)(i=0,1,2)$, are defined differently according to the QTL or ATL, as shown in Table 1. To facilitate the solution to Eq. (4), $\mathbf{S}_u(u)$ is approximated by using the same leakage shape functions;

$$\mathbf{S}_u(u)=\sum_{i=0}^{2}\mathbf{g}_{iu}\left(\frac{u}{a_u}\right)\mathbf{S}_{iu} \tag{6}$$

Like $\mathbf{L}_{iu}\ (i=0,1,2)$, $\mathbf{S}_{iu}\ (i=0,1,2)$ is determined by the node average and the right and left surface values of $\mathbf{S}_u(u)$.

The analytic solution to Eq. (4) is obtained conventionally by a linear combination of two homogeneous solutions and three particular solutions of Eq. (4)[3]. In the UNM, however, it is obtained by assuming an expansion similar to the polynomial expansion in the NEM;

$$\boldsymbol{\phi}_u(u)=\sum_{i=0}^{5}\mathbf{f}_{iu}\left(\frac{u}{a_u}\right)\mathbf{C}_{iu} \tag{7}$$

The explicit form of the five analytic basis functions, $\mathbf{f}_{iu}(\tau)$, depends on the transverse leakage shape functions because they are constructed from the linear combinations of two homogeneous solutions and three particular solutions to Eq. (4). Table 1 shows $\mathbf{f}_{iu}(\tau)$. Note that $\mathbf{f}_{iu}(\tau)$ has the same properties as $h_i(\tau)$, polynomials used in the NEM, in terms of their functional and differential values at the nodal surface, the integral value over the nodal width.
Like the NEM, the five coefficient vectors, $\mathbf{C}_{iu}\ (i=0,1,\cdots,4)$, are determined by the node average flux, $\bar{\boldsymbol{\phi}}=\int_{-\frac{a_u}{2}}^{+\frac{a_u}{2}}\boldsymbol{\phi}_u(u)du$ and two conditions at two nodal surfaces, $\boldsymbol{\phi}_{ur}=\boldsymbol{\phi}_u(+a_u/2)$ and $\boldsymbol{\phi}_{ul}=\boldsymbol{\phi}_u(-a_u/2)$, two weighted residual method equations with odd ($w_1(u)$) and even($w_2(u)$) weighting functions satisfying $\int_{-\frac{a_u}{2}}^{+\frac{a_u}{2}}w_2(u)du=0$;

$$\int_{-\frac{a_u}{2}}^{+\frac{a_u}{2}}w_l(u)\left[-\frac{d^2}{du^2}\boldsymbol{\phi}_u(u)+\tilde{\mathbf{A}}\boldsymbol{\phi}_u(u)+\mathbf{L}_u(u)-\mathbf{S}_u(u)\right]du=\mathbf{0}\ (l=1,2) \tag{8}$$

These conditions yield

$$\mathbf{C}_{0u}=\bar{\boldsymbol{\phi}},\ \mathbf{C}_{1u}=\frac{\boldsymbol{\phi}_{ur}-\boldsymbol{\phi}_{ul}}{2},\ \mathbf{C}_{2u}=\frac{\boldsymbol{\phi}_{ur}+\boldsymbol{\phi}_{ul}}{2}-\bar{\boldsymbol{\phi}},$$

$$\mathbf{C}_{3u}=-\mathbf{M}_{3u}^{-1}\left(\mathbf{M}_{1u}\mathbf{C}_{1u}+\mathbf{DL}_{1u}-\mathbf{DS}_{1u}\right)\quad,\quad \mathbf{C}_{4u}=-\mathbf{M}_{4u}^{-1}\left(\mathbf{M}_{2u}\mathbf{C}_{2u}+\mathbf{DL}_{2u}-\mathbf{DS}_{2u}\right)\quad, \tag{9}$$

where $\mathbf{M}_{1u}=\mathbf{DG}_{1u}^{-1}\mathbf{F}_{1u}$, $\mathbf{M}_{2u}=\mathbf{DG}_{2u}^{-1}\mathbf{F}_{2u}$, $\mathbf{M}_{3u}=\mathbf{DG}_{1u}^{-1}\mathbf{F}_{3u}$, $\mathbf{M}_{4u}=\mathbf{DG}_{2u}^{-1}\mathbf{F}_{4u}$,

$$\mathbf{G}_{iu}=\int_{-\frac{a_u}{2}}^{+\frac{a_u}{2}}w_l(u)\mathbf{g}_{iu}\left(\frac{u}{a_u}\right)du\ \text{ and }\ \mathbf{F}_{iu}=\int_{-\frac{a_u}{2}}^{+\frac{a_u}{2}}w_l(u)\left[\tilde{\mathbf{A}}\mathbf{f}_{iu}\left(\frac{u}{a_u}\right)-\frac{d^2}{du^2}\mathbf{f}_{iu}^{"}\left(\frac{u}{a_u}\right)\right]du$$

($l=1$ for $i=1,3$ and $l=2$ for $i=2,3$).

From Eq.(7) , one can find easily the flux-current relations ;

$$\begin{aligned}\mathbf{j}_{ur}^{+}&=\mathbf{Q}_{0u}\left(6\bar{\boldsymbol{\phi}}-\mathbf{C}_{4u}\right)-\mathbf{Q}_{1u}\mathbf{C}_{3u}-\mathbf{Q}_{2u}\mathbf{j}_{ul}^{+}+\mathbf{Q}_{3u}\mathbf{j}_{ur}^{-}\\ \mathbf{j}_{ul}^{-}&=\mathbf{Q}_{0u}\left(6\bar{\boldsymbol{\phi}}-\mathbf{C}_{4u}\right)+\mathbf{Q}_{1u}\mathbf{C}_{3u}-\mathbf{Q}_{2u}\mathbf{j}_{ur}^{-}+\mathbf{Q}_{3u}\mathbf{j}_{ul}^{+}\end{aligned}. \tag{10}$$

Equation (3) then becomes

$$\bar{\boldsymbol{\phi}} = \left(\mathbf{A} + 12\mathbf{Q}_0\right)^{-1}\mathbf{s} \quad ; \quad \mathbf{s} = \bar{\mathbf{S}}^m + \sum_{u=x,y,z} \frac{1}{a_u}\left[2\mathbf{Q}_{ou}\mathbf{C}_{4u} + \left(\mathbf{I} + \mathbf{Q}_{2u} - \mathbf{Q}_{3u}\right)\left(\mathbf{j}_{ur}^{-} + \mathbf{j}_{ul}^{+}\right)\right] \quad . \tag{11}$$

$\mathbf{Q}_{iu}\left(i = 0,1,2,3\right)$ and $\mathbf{Q}_0$ are defined in reference 1.

Equations (10) and (11) form the basic coupling relations of the UNM to be solved for $\bar{\boldsymbol{\phi}}$ and $\mathbf{j}_{us}^{\pm}$. They resemble exactly those of the NEM, yet they are equivalent to the ANM formulation with either QTL or ATL approximation. Moreover, Eqs.(10) and (11) from the ATL are equivalent to the AFEN method formulation if $\mathbf{L}_{iu}$ is derived from the intranodal flux on which the AFEN method is based[2] $\mathbf{L}_{iu}$ in this case is a function of node average, nodal surface and edge fluxes, which requires additional relations for edge fluxes. Needless to mention, Eqs. (10) and (11) from the QTL represent NEM formulation if one derives them setting $\mathbf{f}_{iu}\left(\tau\right) = h_i\left(\tau\right)\mathbf{I}$ $\left(i = 0,1,\cdots,4\right)$ with the polynomials, $h_i\left(\tau\right)$, used in the NEM. Consequently, Eqs. (10) and (11) represent several different nodal method formulations. The sole difference among them lies in the matrices $\mathbf{M}_{iu}\left(i = 3,4\right)$ required for determining $\mathbf{C}_{3u}$ and $\mathbf{C}_{4u}$. Calculation of $\mathbf{M}_{iu}$ involves integration of matrix functions $\mathbf{f}_{iu}\left(\tau\right)$ and $\mathbf{g}_{iu}\left(\tau\right)$. It is done easily by using the similarity transformation; $\hat{\mathbf{A}} = \mathbf{R}^{-1}\tilde{\mathbf{A}}\mathbf{R} = diag\left(\lambda_1, \lambda_2\right)$ where λ_1 and λ_2 are eigenvalues of $\tilde{\mathbf{A}}$. By use of $\mathbf{R}$, $\mathbf{M}_{iu}$ becomes

$$\mathbf{M}_{iu} = \mathbf{D}\mathbf{R}\hat{\mathbf{M}}_{iu}\mathbf{R}^{-1}\ \left(i = 1,2,3,4\right);\ \hat{\mathbf{M}}_{iu} = \hat{\mathbf{G}}_{lu}^{-1}\hat{\mathbf{F}}_{iu} = diag\left(m_{i1u}, m_{i2u}\right) \tag{12}$$

($l = 1$ for $i = 1,3$ and $l = 2$ for $i = 2,4$).

$\hat{\mathbf{G}}_{iu} = \mathbf{R}^{-1}\mathbf{G}_{iu}\mathbf{R}$ and $\hat{\mathbf{F}}_{iu} = \mathbf{R}^{-1}\mathbf{F}_{iu}\mathbf{R}$ are diagonal matrices. m_{ipu} can then be easily calculated.

3. NUMERICAL RESULTS AND DISCUSSIONS

Tthe UNM solves the same set of nodal coupling equations for both static and kinetics problems, regardless of the nodal computational methods. Therefore, it offers a good opportunity to compare neutronics analysis characteristics of different nodal methods being used in the production codes on a fair and equal footing. As a test problem of the practical interest to do so, the OECD/NEA MSLB benchmark phase II and III problems[7] that require the detailed 3-D neutronics analysis are taken here. Actually these problems have already been analyzed by several different thermal-hydraulic and neutronics coupled codes. We observe that many of these coupled codes utilize the nodal method codes for neutronics analysis. Yet we find it not easy to compare the performance characteristics of the different nodal methods directly from the results of the MSLB problem analyses, because the thermal hydraulics codes as well as the numerical solution method and algorithms used for the nodal neutronics codes are different from one another.

For the analysis of the MSLB benchmark problems, the UNM solver, SUNCARDS, is combined with the MARS code for a thermal-hydraulic/reactor kinetics coupled code

MARS/SUNCARDS. The thermal hydraulics code, MARS, is developed by Korea Atomic Energy Research Institute based on RELAP and COBRA IV[6]. The SUNCARDS has the five different nodal computational options; NEM, ANM/QTL, ANM/ATL, AFEN/CPB, and AFEN/MSS. The ANM/QTL and ANM/ATL refer to the ANM calculations with the QTL and ATL transverse leakage approximations, respectively. The AFEN/CPB and AFEN/MSS refer to the AFEN method calculations in which the former uses the corner point balance (CPB) equation while the latter the method of successive smoothing (MSS) to calculate the unknown corner point or edge fluxes[2]. Figure 1 compares predictions of these five different nodal methods on the transient core power behavior following the initiation of the MSLB accident. Table 2 compares the nodal methods in terms of predictions on the major parameters of the steady state and the transient core such as the initial k_{eff}, radial peak (F_r) and axial power peak (F_z) and the first and second peak powers and the time at the second peak. Most of the neutronics calculations are performed on one-node-per-fuel-assembly (1 N/A) spatial nodalization except for the 4 N/A NEM and 4 N/A ANM/QTL SUNCARDS calculations. Figure 1 and Table 2 show that the five different nodal methods on 1N/A and two nodal methods on 4N/A are almost indistinguishable in prediction of the steady state core neutronics parameters and the transient core power behavior caused by the MSLB accident. For the purpose of comparison with other calculations, MARS/MASTER[9] prediction on the transient core power behavior of phase II problem is added in Figure 1 and some phase II problem participants' results are added in Table 2. The result of phase III problem is also presented in Figure 2. Like the case of phase II, the results of various nodal methods for phase III problem are also almost indistinguishable. The MASTER, PARCS, PANBOX codes in Table 2 are based on the ANM similar to the ANM/QTL. Despite the fact that the thermal hydraulics codes coupled to these neutronics codes are different, the results they obtained for the phase II problem are similar to one another and our MARS/SUNCARDS results.

The current nodal calculations for core neutronics design of PWR are based on the 4 N/A instead of 1 N/A, because 1 N/A nodal calculations do not guarantee the desired computational accuracy. Moreover, the 1 N/A analysis on the NEACRP PWR rod ejection transients has revealed the noticeably different prediction accuracy depending on the nodal methods[2]. The reason why the difference in the prediction accuracy of the nodal methods does not show up in the above analyses of the MSLB benchmark phase II and phase III problems is not clear. But the adoption of the coarser nodalization for thermal hydraulics calculations is supposedly responsible for it because the thermal-hydraulics feedback effects in this case may not be so accurately reflected on the transient core calculations as the difference of the 1 N/A nodal prediction accuracy and 4 N/A can show up.

CONCLUSIONS

The MARS/SUNCARDS analysis indicates that differences of nodal solution methods do not make noticeable differences in predicting the key parameters of the steady state and transient characteristics of the MSLB benchmark phase II and III problems. The reason is not clear. But it is supposedly due to the fact that the coarser nodalization of thermal hydraulic calculation in radial and axial directions cannot take into account sufficiently detailed and accurate thermal hydraulic feedback effects that are compatible with the relatively finer mesh nodal reactor calculations. To assure this, one may require the MSLB analysis with finer nodalization of thermal hydraulic calculations.

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Table 1. Functions $g_{ipu}(\tau)$ and $f_{ipu}(\tau)$.

UNM Options	$\mathbf{g}_{iu}(\tau)$	$\mathbf{f}_{iu}(\tau)$,
ANM / QTL	$\mathbf{g}_{0u}(\tau) = h_0(\tau)\mathbf{I}$ $\mathbf{g}_{1u}(\tau) = h_1(\tau)\mathbf{I}$ $\mathbf{g}_{2u}(\tau) = h_2(\tau)\mathbf{I}$	$\mathbf{f}_{0u}(\tau) = h_0(\tau)\mathbf{I}$ $\mathbf{f}_{1u}(\tau) = h_1(\tau)\mathbf{I}$ $\mathbf{f}_{2u}(\tau) = h_2(\tau)\mathbf{I}$ $\mathbf{f}_{3u}(\tau) = \mathbf{a}_3 \sinh\left(a_u\tau\sqrt{\widetilde{\mathbf{A}}}\right) + \tau\mathbf{b}_3$ $\mathbf{f}_{4u}(\tau) = \mathbf{a}_4 \cosh\left(a_u\tau\sqrt{\widetilde{\mathbf{A}}}\right) + \tau^2\mathbf{b}_4 + \mathbf{c}_4$
ANM / ATL or AFEN	$\mathbf{g}_{0u}(\tau) = \mathbf{I}$ $\mathbf{g}_{1u}(\tau) = \mathbf{a}_1 \sinh\left(\frac{a_u\tau}{\sqrt{2}}\sqrt{\widetilde{\mathbf{A}}}\right)$ $\mathbf{g}_{2u}(\tau) = \mathbf{a}_2 \sinh\left(\frac{a_u\tau}{\sqrt{2}}\sqrt{\widetilde{\mathbf{A}}}\right) + \mathbf{b}_2$	$\mathbf{f}_{0u}(\tau) = \mathbf{I}$ $\mathbf{f}_{1u}(\tau) = \mathbf{a}_1 \sinh\left(a_u\tau\sqrt{\widetilde{\mathbf{A}}}\right) + \mathbf{b}_1 \sinh\left(\frac{a_u\tau}{\sqrt{2}}\sqrt{\widetilde{\mathbf{A}}}\right)$ $\mathbf{f}_{2u}(\tau) = \mathbf{a}_2 \cosh\left(a_u\tau\sqrt{\widetilde{\mathbf{A}}}\right) + \mathbf{b}_2 \cosh\left(\frac{a_u\tau}{\sqrt{2}}\sqrt{\widetilde{\mathbf{A}}}\right) + \mathbf{c}_2$ $\mathbf{f}_{3u}(\tau) = \mathbf{a}_3 \sinh\left(a_u\tau\sqrt{\widetilde{\mathbf{A}}}\right) + \mathbf{b}_3 \sinh\left(\frac{a_u\tau}{\sqrt{2}}\sqrt{\widetilde{\mathbf{A}}}\right)$ $\mathbf{f}_{4u}(\tau) = \mathbf{a}_4 \cosh\left(a_u\tau\sqrt{\widetilde{\mathbf{A}}}\right) + \mathbf{b}_4 \cosh\left(\frac{a_u\tau}{\sqrt{2}}\sqrt{\widetilde{\mathbf{A}}}\right) + \mathbf{c}_4$

Table 2. Comparison of Nodal Method Predictions on Key Core Neutronics Parameters of the MSLB Benchmark Exercise II Problem

CODE	Initial Steady State			Transient State		
	keff	Fr	Fz	1st peak power (W)	2nd peak power (W)	2nd peak time (sec)
MARS/SUNCARDS (1N/A NEM)	1.0066	1.339	1.100	3.28×10^9	9.12×10^8	57.7
MARS/SUNCARDS (1N/A ANM/QTL)	1.0071	1.337	1.100	3.28×10^9	9.41×10^8	57.7
MARS/SUNCARDS (1N/A ANM/ATL)	1.0068	1.335	1.102	3.28×10^9	9.08×10^8	57.7
MARS/SUNCARDS (1N/A AFEN/MSS)	1.0066	1.337	1.097	3.31×10^9	9.24×10^8	57.8
MARS/SUNCARDS (1N/A AFEN/CPB)	1.0067	1.337	1.094	3.31×10^9	9.23×10^8	57.5
MARS/SUNCARDS (4N/A NEM)	1.0068	1.339	1.100	3.26×10^9	9.02×10^8	57.8
MARS/SUNCARDS (4N/A ANM/QTL)	1.0070	1.338	1.101	3.27×10^9	9.09×10^8	57.6
TRAC/NEM*	1.0061	1.331	1.058	3.22×10^9	1.00×10^9	65.2
RESTRAN3D-MOD2*	1.0067	1.337	1.052	3.34×10^9	8.65×10^8	57.9
MARS/MASTER*	1.0071	1.339	1.105	3.24×10^9	8.34×10^8	57.2
TRAC/PARCS*	1.0061	1.372	1.107	3.31×10^9	8.37×10^8	57.7
RELAP5/PANBOX*	1.0058	1.349	1.095	3.30×10^9	8.59×10^8	57.6

* Ref. : B. Tayler and K. Ivanov, "Summary of the Results for 2nd Exercise," *Ad-hoc Meeting on OECD/NRC MSLB Benchmark*, Madrid, Spain, September, 1999.

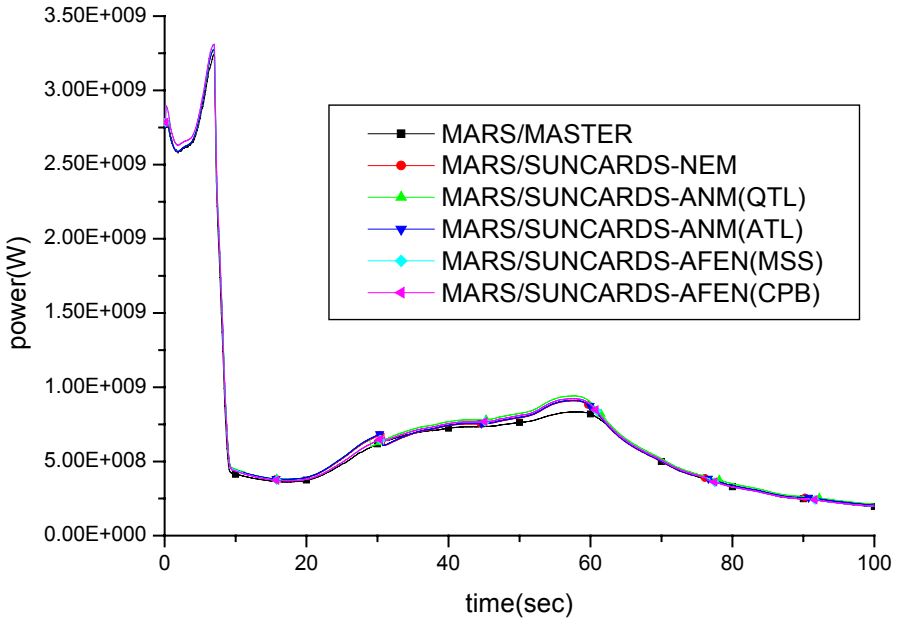

Figure 1. Comparison of Nodal Method Predictions on Transient Core Power Behavior in MSLB Benchmark Exercise II Problem

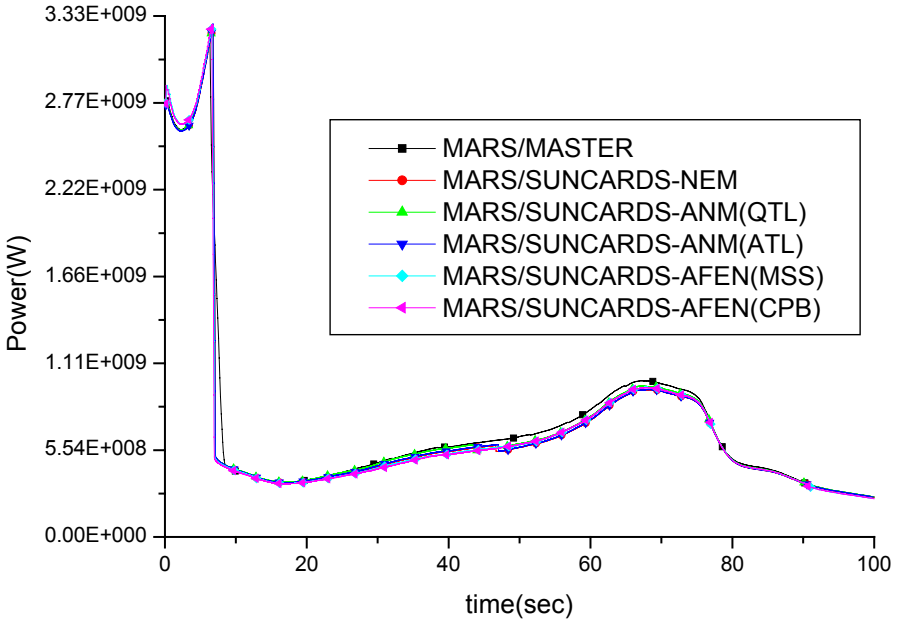

Figure 2. Comparison of Nodal Method Predictions on Transient Core Power Behavior in MSLB Benchmark Exercise III Problem