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Error Decomposition of Approximations for BWR Core Calculations

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ABSTRACT

In BWR core calculations, various approximations are commonly utilized such as assembly homogenization, use of assembly discontinuity factor (ADF), energy group condensation, and diffusion approximation. We have estimated the errors of these approximations for various cores with different sizes, especially for partially MOX fueled cores because these errors become relatively large. It is seen that for small cores the errors due to the above four approximations are large. The errors decrease with the increase of core size. It is interesting to note that for the any size cores the four errors cancel each others, and the total error is very small. However, the error is significant for cores with control rods. So, we have improved the conventional flux discontinuity factor in addition to the spatial rehomogenization method. The improved method has decreased the error remarkably. The total errors for keff and assembly power are less than 0.1% and 4%, respectively.

1. INTRODUCTION

Usually BWR core calculations(ref.1-4) are performed in a few groups using the assembly homogenized cross sections and flux discontinuity theory. In BWR cores the void fraction varies from 0 to ~70% along the axial direction. So it is necessary to estimate the accuracy of the commonly used calculation method for various void fractions and various core sizes with and without control rods.

The purpose of this paper is to estimate the errors due to the assembly homogenization, use of assembly discontinuity factor (ADF), energy group condensation, and the diffusion approximation, separately for the two-dimensional (XY) core model with various void fractions. From the error decomposition we can understand the applicability of the commonly used method, and the underlying physics.

To get more accurate homogenized cross sections, the rehomogenization method has been introduced[<5>,<6>]. In the method the homogenized cross sections obtained by infinite assembly calculations are modified by the use of the reconstructed flux distribution which is a product of homogeneous core wise flux distribution and the infinite assembly flux distribution.

In this paper this idea has been applied to the flux discontinuity factor. The assembly discontinuity factor is

modified by using the reconstructed flux.

The core model and the method are described in Chap.2. Calculation results are shown in Chap.3, and conclusions are drawn in Chap.4.

2. CALUCULATION METHODS AND MODELS

2.1 Calculation Method

In the conventional diffusion calculations, assembly homogenized cross sections are calculated by using the flux distribution obtained from infinite assembly calculations as a weight.

$$\hat{\Sigma}^{g}_{SA} = \frac{\int \Sigma^{g}_{hetero}(\vec{r}) \cdot \Phi^{g}_{SA}(\vec{r}) d\vec{r}}{\int \Phi^{g}_{SA}(\vec{r}) d\vec{r}} \tag{1}$$

where $\phi_{SA}(r)$ is the assembly flux distribution. The assembly flux discontinuity factor is calculated by

$$ADF^{g}(h) = \frac{\frac{1}{h}\int_{0}^{h} \Phi^{g}_{SA}(s) ds}{\frac{1}{V}\int \Phi^{g}_{SA}(\vec{r}) d\vec{r}} \tag{2}$$

where the numerator is the surface averaged flux, and the denominator is the volume averaged flux.
The improved rehomogenized cross section[<6>] is obtained by

$$\hat{\Sigma}_{rehomo} = \frac{\int \Sigma_{hetero}(\vec{r}) \Phi_{homo}(\vec{r}) \cdot \Phi_{SA}(\vec{r}) d\vec{r}}{\int \Phi_{homo}(\vec{r}) \cdot \Phi_{SA}(\vec{r}) d\vec{r}} \tag{3}$$

$$= \hat{\Sigma}_{SA} \times \frac{\frac{\int \Sigma_{hetero}(\vec{r}) \Phi_{homo}(\vec{r}) \cdot \Phi_{SA}(\vec{r}) d\vec{r}}{\int \Sigma_{hetero}(\vec{r}) \Phi_{SA}(\vec{r}) d\vec{r}}}{\frac{\int \Phi_{homo}(\vec{r}) \cdot \Phi_{SA}(\vec{r}) d\vec{r}}{\int \Phi_{SA}(\vec{r}) d\vec{r}}}$$

On the other hand, to preserve the intra node reaction rate in the core calculation, the following equation is satisfied,

$$\hat{\Sigma}_{rehomo} \int \Phi_{homo}(\vec{r}) d\vec{r} = \int \Sigma_{hetero}(\vec{r}) \Phi_{hetero}(\vec{r}) d\vec{r}$$

$$\cong \int \Sigma_{hetero}(\vec{r}) \Phi_{homo}(\vec{r}) \cdot \Phi_{SA}(\vec{r}) d\vec{r} \tag{4}$$

$$\hat{\Sigma}_{rehomo} = \frac{\int \Sigma_{hetero}(\vec{r}) \Phi_{homo}(\vec{r}) \cdot \Phi_{SA}(\vec{r}) d\vec{r}}{\int \Phi_{homo}(\vec{r}) d\vec{r}} \tag{5}$$

Here, the equation (5) is not consistent with the equation (3). So, we modify the assumption used in the equation (4) to take the consistency between the equations (3) and (5) as follows:

$$\Phi_{\text{hetero}}(\vec{r}) \cong \Phi_{\text{homo}}(\vec{r}) \times \Phi_{\text{SA}}(\vec{r}) \times \frac{\int \Phi_{\text{homo}}(\vec{r}) d\vec{r}}{\int \Phi_{\text{homo}}(\vec{r}) \cdot \Phi_{\text{SA}}(\vec{r}) d\vec{r}} \tag{6}$$

Applying the idea of the equation (6) to the conventional flux discontinuity theory within an assembly, we can introduce the modified flux discontinuity factor (CDF).

$$CDF(h) = ADF(h) \times \frac{\frac{1}{V}\int \Phi_{\text{SA}}(\vec{r}) dr \times \int \Phi_{\text{homo}}(\vec{r}) d\vec{r}}{\int \Phi_{\text{homo}}(\vec{r}) \Phi_{\text{SA}}(\vec{r}) d\vec{r}} \tag{7}$$

When the intra nodal homogeneous flux is flat (i.e. infinite lattice calculation), the modified flux discontinuity factor CDF is equal to the conventional assembly discontinuity factor ADF.

The use of the equations (1) and (2) will be shown satisfactory for BWR cores without control rods as shown in the next section. However, for BWR cores with control rods, there remain significant errors where one uses the equations (1) and (2). However, it will be shown that the equations (3) and (7) lead to satisfactory results.

2.2 Calculation Models

As a reference we have chosen a whole core heterogeneous transport calculation with 16 energy groups (model-1). The group structure is shown in Table1. Fuel rods and surrounding moderator regions are explicitly treated by using fine mesh S16 calculations. The errors due to the assembly homogenization, the use of ADF, the energy group condensation, and the diffusion approximation were calculated as follows (see Table2):

First, homogenized cross sections are calculated from single assembly calculation (AXSs), and they are used in the S16 fine mesh transport calculation with 16 energy groups (model-2), and the difference from the model-1 was defined as the assembly homogenization error. Next, AXSs are used in the fine mesh diffusion calculation with 16 energy groups (model-3), and the difference from the method-2 was defined as the diffusion approximation error. Diffusion coefficients were calculated from $D^g = 1/3\,\Sigma_{tr}^g$ while Σ_{tr}^g is the transport cross sections $\Sigma_{tr}^g = \Sigma_{tot}^g - \mu\Sigma_s^{g\to g}$. Third, both AXSs and ADFs are used in the nodal diffusion calculation with 16 energy groups (model-4), and the difference between the models 3 and 4 was defined as the error of use of ADFs. Last, both AXSs and ADFs are used in the nodal diffusion calculation with 2 energy groups (model-5), and the difference from the model-4 was defined as the energy group condensation error. The 2-group diffusion coefficients are calculated by collapsing the 16-group diffusion coefficients with the neutron spectrum.

These errors were calculated for the four cores including MOX fueled assemblies and high enriched UO2 fueled assemblies and cores with control rods. Fig.1 shows the core and the fuel assembly. The smallest core contains 16 fuel assemblies, and the largest core contains 232 fuel assemblies. ‘H’ denotes the high-enriched UO2 assembly, and ‘M’ denotes the MOX assembly. We considered three patterns of void fractions of 0%, 40%, and 70%.

3. CALCULATION RESULTS

The results of decomposed keff error are shown in Fig.2 and Table 3. The each error is shown separately for individual cores.

In the case of small size core the interaction of fuel assembly and reflector is large, so individual components of the errors are large. The assembly homogenization and group condensation overestimate the eigenvalue, and use of ADF and the diffusion approximation underestimate the eigenvalue. The homogenization error and the error due to the use of ADF are both large for the cores with large void fraction.

As the core size becomes large, errors of each approximation become small and approach certain constant values. It is physically clear that the use of ADF in addition to the assembly homogenization reduces the calculation error caused by the assembly homogenization because the reaction rates are conserved for infinite assembly array. However, it should be noted that the homogenization error becomes negative for the large core, the same sign of the use of ADF. So, although the both errors are small for the large core, the cancellation of the two errors cannot be expected. The diffusion approximation error becomes small for the large core. However, the energy group condensation error does not approach zero.

When we sum up all the approximation errors, namely when we use the commonly used calculation method based on the 2-group diffusion calculation with AXSs and ADFs, the error approaches zero. The total errors for the cores I - IV are -0.09%, -0.06%, -0.05%, and -0.05% for the 0% void fraction core, and -0.31%, -0.14%, -0.02%, and +0.02% for the 70% void fraction core.

The error of assembly power is also decomposed. Figure 3 shows the error of assembly power at the representative positions for the 40% void pattern. The total error is less than 1%.

However this is mainly the cancellation of homogenization error and the use of ADF.

When we apply the conventional Σ and ADF to cores with control rods, the total error becomes large. Figure 4 shows the examples. At the core center, the error is about 5% because the use of ADF leads to large negative corrections. At another control rod inserted position, the error approaches 10%.

Figure 5 shows the error of assembly power calculated by the equations (1),(2) and the equations (3),(7). When one uses the equations (3),(7), the large errors caused by the use of the equations (1),(2) are dramatically decreased, especially at the assemblies with control rods.

CONCLUSIONS

We have numerically decomposed the errors of the approximation used for BWR core calculations. It was found that for keff the assembly homogenization error and the error due to the use of ADF approach a certain negative value with the increase of core size, but the energy group condensation error approaches a certain positive value. Thus the commonly used BWR core calculation method using AXS, ADF, and a few group diffusion approximation gives the relatively accurate results because of the error cancellations. For the cores with control rods, the errors of assembly powers become large for the conventional method. When one applies the improved cross sections and flux discontinuity factor, the errors are decreased.

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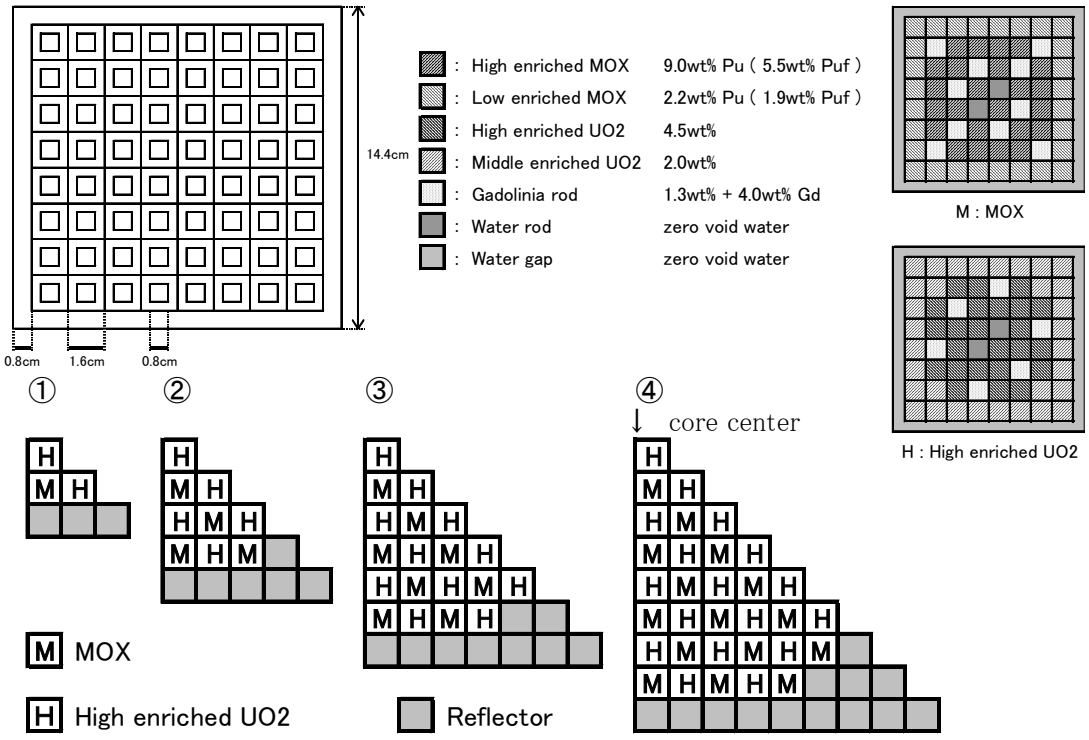

Fig.1 Calculation models

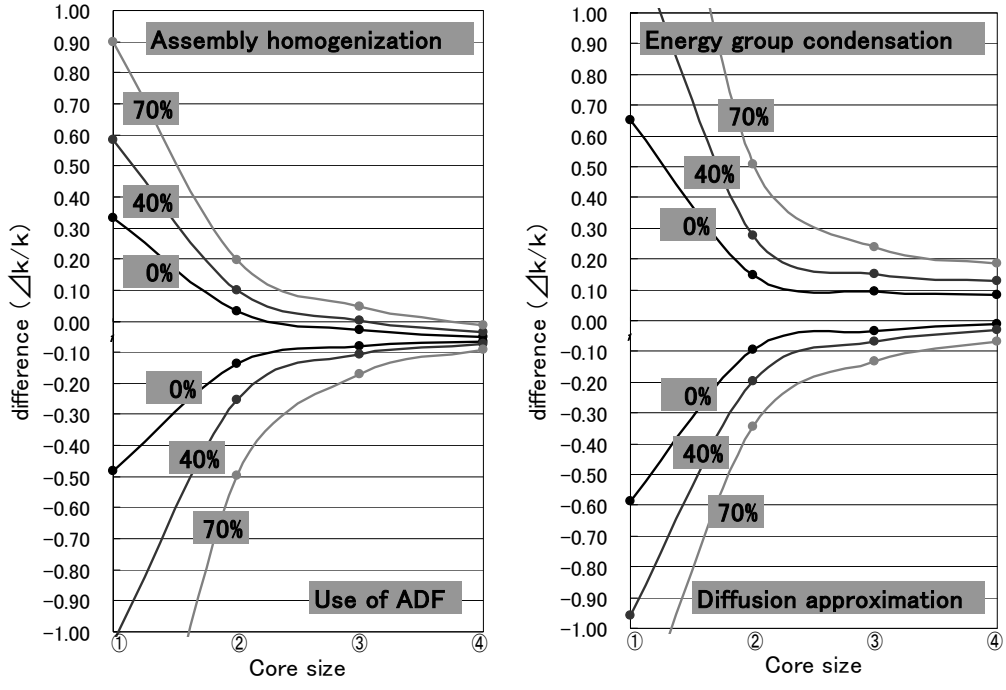

Fig. 2 Decomposition of keff for cases 1~4

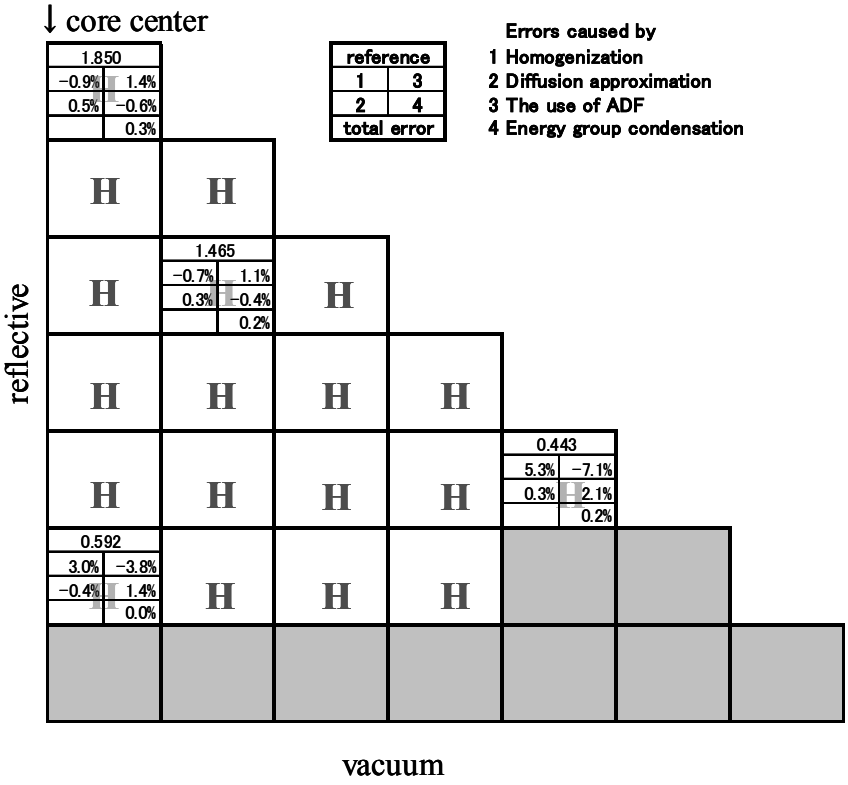

Fig. 3 Errors of Assembly powers for BWR core without control rods by conventional method

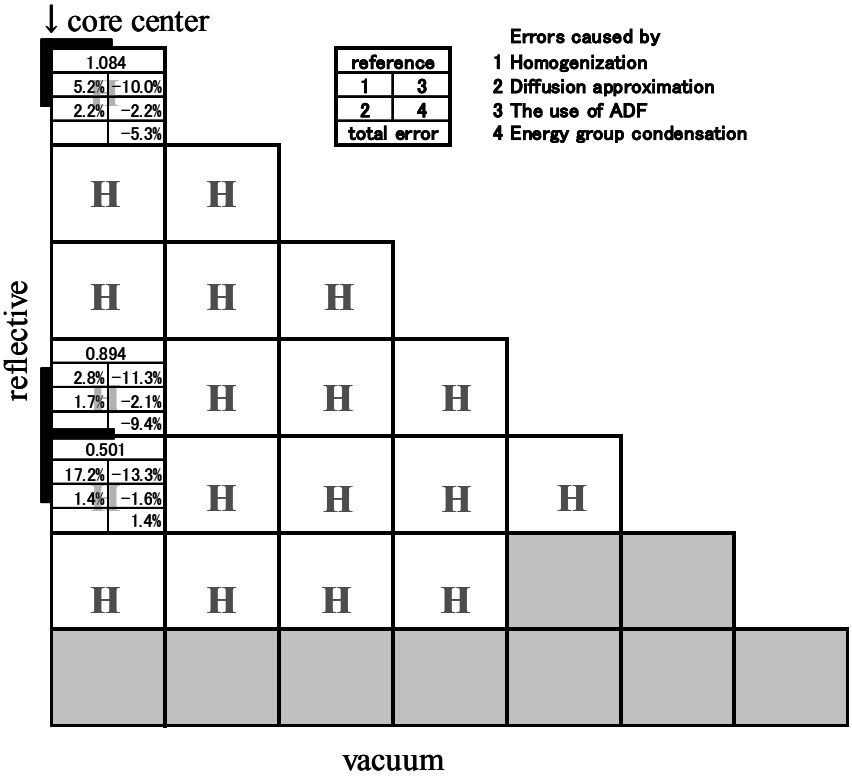

Fig. 4 Errors of Assembly powers for BWR core with control rods by conventional method

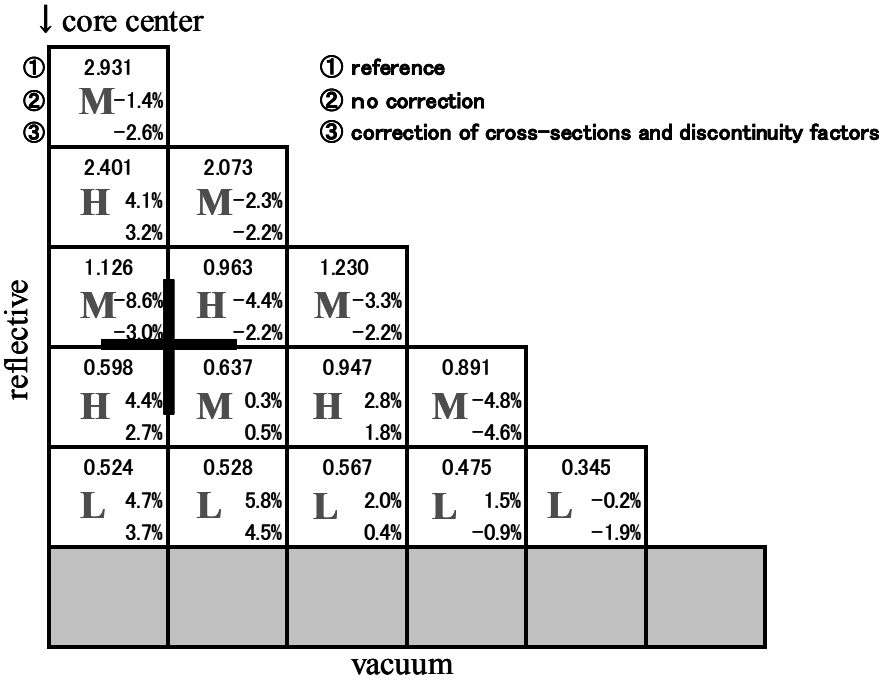

Fig. 5 Errors of Assembly powers for BWR core with control rod by conventional and improved methods

Table 1 Energy Range for 16 Groups

Energy group	Energy range		
1	10.0 (MeV)	~	8.21 (keV)
2	8.21 (keV)	~	5.53 (keV)
3	5.53 (keV)	~	4.000 (eV)
4	4.000 (eV)	~	1.300 (eV)
5	1.300 (eV)	~	1.150 (eV)
6	1.150 (eV)	~	1.097 (eV)
7	1.097 (eV)	~	1.020 (eV)
8	1.020 (eV)	~	0.972 (eV)
9	0.972 (eV)	~	0.850 (eV)
10	0.850 (eV)	~	0.625 (eV)
11	0.625 (eV)	~	0.350 (eV)
12	0.350 (eV)	~	0.280 (eV)
13	0.280 (eV)	~	0.140 (eV)
14	0.140 (eV)	~	0.058 (eV)
15	0.058 (eV)	~	0.030 (eV)
16	0.030 (eV)	~	0.00001 (eV)

Table 2 Calculation Models and Error Decomposition

	Intranode	Theory	Mesh	/	ADF	Energy groups
Model-1	hetero	transport	fine	/	---	16
Model-2	homo	transport	fine	/	---	16
Model-3	homo	diffusion	coarse	/	no	16
Model-4	homo	diffusion	coarse	/	yes	16
Model-5	homo	diffusion	coarse	/	yes	2

Difference			Components of Errors
Model-2	⇔	Model-1	Homogenization
Model-3	⇔	Model-2	Diffusion Approximation
Model-4	⇔	Model-3	Use of ADF
Model-5	⇔	Model-4	Energy Group Condensation
Model-5	⇔	Model-1	(Total Error)

Table 3 Calculation results (Eigenvalue)

Homogeneous Core (UO2)		Core①	Core②	Core③	Core④
Reference		0.91025	1.01514	1.04430	1.05618
errors	Homogenization	0.38%	0.10%	0.04%	0.02%
	Diffusion Approximation	−0.59%	−0.10%	−0.03%	−0.01%
	Use of ADF	−0.33%	−0.06%	−0.02%	−0.01%
	Energy Group Condensation	0.59%	0.07%	0.02%	0.00%
(Total Error)		0.04%	0.01%	0.00%	0.00%

Homogeneous Core (MOX)		Core①	Core②	Core③	Core④
Reference		0.91705	1.01459	1.04187	1.05307
errors	Homogenization	0.39%	0.09%	0.04%	0.02%
	Diffusion Approximation	−0.61%	−0.11%	−0.03%	−0.01%
	Use of ADF	−0.63%	−0.12%	−0.04%	−0.02%
	Energy Group Condensation	0.64%	0.08%	0.02%	0.00%
(Total Error)		−0.22%	−0.06%	−0.02%	−0.02%

Heterogeneous Core (MOX , UO2)		Core①	Core②	Core③	Core④
Reference		0.91609	1.01756	1.04582	1.05753
errors	Homogenization	0.33%	0.03%	−0.03%	−0.05%
	Diffusion Approximation	−0.59%	−0.10%	−0.03%	−0.01%
	Use of ADF	−0.48%	−0.14%	−0.08%	−0.07%
	Energy Group Condensation	0.65%	0.15%	0.09%	0.08%
(Total Error)		−0.09%	−0.06%	−0.05%	−0.05%