COMPARATIVE STUDY OF NODAL CROSS SECTION MODELS APPLIED TO MTR CORE ANALYSIS

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ABSTRACT

Although modern nodal diffusion methods are well established and validated for PWR and BWR calculations, this is not the case for Material Testing Reactors (MTR's), where fine-mesh diffusion, or even transport, calculations are normally used. However, this means that routine "core follow" calculations are not practical, due to excessive computing times. The small cores of MTR's (and the resultant high leakage) as well as the heterogeneous nature of core loadings, are usually regarded as being unsuitable to be calculated with nodal diffusion methods. On the other hand, new developments in especially homogenization theory were designed to address these issues.

This paper briefly describes some of the important cross section and feedback methods and presents comparative results for the SAFARI-1 pool type research reactor. The availability of the multi-group analytic nodal method is shown to be essential for such small cores with high leakage. Compared to the reference 8-group calculation differences of nearly 2% in k-eff and over 5% in fuel element power were seen when only two energy groups were used. The effect of the rehomogenization moments method, used to correct the homogenized node cross-sections for changing environmental conditions, was also shown to be important. Differences of up to 4% in the fuel element powers were found, in particular in the control and peripheral elements. The effects of the intranodal cross-section variation model, which take the non-uniform distribution of the depletion and other state parameters inside the element into account, were much smaller with changes in fuel element powers of 1% and less than 0.3% in k-eff. The improvements in the nodal diffusion method were shown to be computationally efficient and are used in practical calculations.

1. INTRODUCTION

When the SAFARI-1 reactor at NECSA in South Africa started the commercial production of radioisotopes and the provision of other irradiation services, it became necessary to accurately predict the flux levels in the reactor core. A direct effect of the commercialization effort was that the reactor could no longer be operated in an equilibrium mode, and that cycle lengths and power levels were now directly defined by the product schedules. The core configuration was also changed to accommodate additional irradiation positions. In this new dynamic environment, and in support of the commercial program, an accurate calculational system had to be developed. The OSCAR-3 system [1,2] has been successfully used for reload optimization and irradiation rig design [3] for the SAFARI-1 reactor.

The OSCAR-3 system is also being used, and in the process of being validated, for core follow calculations of the 2MW HOR pool type research reactor at the Interfaculty Reactor Institute (IRI) of the Delft University of Technology. The reactor is in a process of converting from high-enriched uranium (HEU) fuel to low enriched (LEU) fuel and also in the process to go to a compact core [4]. In a mixed core (HEU and LEU) one would expect additional calculational difficulties due to spectral (HEU vs LEU) effects which would be worsened by the compact core, with its increased leakage. Although the process of validation is on going, very promising results have been reported so far [5,6].

MGRAC (MultiGroup Reactor Analysis Code) is part of the three-dimensional reactor calculational system OSCAR-3 that is used for cycle depletion analysis of MTR and PWR cores under steady-state conditions. MGRAC is an advanced transverse-integration nodal diffusion code based on the Multigroup Analytic Nodal Method [7]. The transverse leakage is represented by quadratic polynomials while the standard Generalized Equivalence Theory (GET) is used in the calculation of the homogenized assembly parameters. The availability of the multi group method, a feature that distinguishes MGRAC from many of the other commercially available nodal codes, will be shown to be the most important feature required to accurately model MTRs. The code can also perform criticality searches on control rod positions, soluble boron concentration, core power levels and coolant conditions. End-of cycle (EOC) searches can be performed based on a target k-eff or zero soluble boron concentration. An analytic microscopic depletion model is incorporated into the code.

Several of the more modern homogenization and cross section feedback techniques have been incorporated in the code system. In this paper the effects of some of the calculational models incorporated in the MGRAC code are quantified for an MTR reactor.

2. HOMOGENIZATION AND CROSS SECTION FEEDBACK MODELS

2.1 CROSS SECTION MODELS

The cross-section model is based on reference ("base") cross-sections and "correction" terms. Lattice depletion calculations are performed at reference conditions to determine the base cross-sections, while perturbations to the reference state at each exposure point are used to calculate correction terms. Both base cross-sections and correction terms are parameterized with respect to the node-averaged exposures and node-averaged state parameters (fuel temperature, moderator temperature, moderator density, and boron concentration).

The effect of the off-reference ("off-base") depletion conditions (the "history" effect) is taken into account in MGRAC by explicitly following the important isotopes. The rate equations for number densities are solved analytically in the micro-depletion module. The importance of the history effect on the microscopic cross sections still has to be determined.

2.2 REHOMOGENIZATION MOMENTS

Changes in the environmental conditions in the core nodal calculations are taken into account by correcting the homogenized cross-sections using the rehomogenization moments method [8]. The homogenized nodal cross-sections are calculated in the lattice calculation with reflective assembly boundary conditions. Normalized one-dimensional Legendre cross-section moments are then calculated and parameterized in the same way as the other assembly homogenization parameters. The environmental effect due to the changing conditions during reactor operation is taken into

account in the core calculation by applying the rehomogenization correction, using the parameterized cross section moments. The rehomogenization correction is calculated by assuming separable quadratic Legendre polynomial intranodal flux and cross-section distributions. The advantage of the Legendre polynomial representation is that the number of coefficients is reduced from 6 to 4 per coordinate direction. Special precaution should be taken not to apply the rehomogenization correction too soon. In the MGRAC code it is applied only after the second feedback iteration.

2.3 INTRANODAL CROSS SECTION VARIATION MODEL

Non-uniform distribution of the depletion and other state parameters inside the assembly is taken into account through an intranodal cross-section variation model based on the non-linear extension of Wagner [9]. Both the intranode flux and cross-sections are approximated by separable quadratic polynomials:

$$\Phi(\vec{u}) = \sum_{u=1}^{M} (a_0^u + a_1^u u + a_2^u u^2) - (M-1)\overline{\Phi}, \tag{1}$$

$$\Sigma(\vec{u}) = \sum_{u=1}^{M} (c_0^u + c_1^u u + c_2^u u^2) - (M-1)\overline{\Sigma} \tag{2}$$

where $\vec{u}$ is a dimensionless local variable, M is the number of spatial directions, $\overline{\Phi}$ is the node volume-averaged flux and $\overline{\Sigma}$ is the volume-averaged cross section. The polynomial coefficients in Eqs (1) and (2) are expressed in terms of the volume-averaged and surface-averaged fluxes and cross sections respectively. The volume-averaged and surface-averaged fluxes are available directly from the nodal solution. Cross-sections are obtained from the library at both average and surface conditions. The surface-averaged power density is calculated as the sum over energy groups of products of the surface-averaged energy-production cross-sections and the surface-averaged fluxes.

The surface-averaged exposures and number densities can be calculated in two ways. With approximate surface depletion the surface exposure is obtained by multiplying the node averaged exposure increment with the ratio of the surface averaged to the node averaged power density:

$$\Delta E_s = \frac{P_s}{\overline{P}} \Delta \overline{E} \tag{3}$$

The surface number densities are obtained by multiplying the node averaged number densities $\overline{N}$ with the ratio of the number densities calculated in the lattice calculation for the surface and average exposure respectively:

$$N(E_s) = \overline{N} \frac{N_a(E_s)}{N_a(\overline{E})} \tag{4}$$

In explicit surface depletion, the rate equations for number densities are solved using the surface conditions.

3. THE SAFARI-1 REACTOR

The SAFARI-1 reactor is a 20MW tank-in-pool type materials testing reactor owned and operated by NECSA at its Pelindaba site near Pretoria, South Africa. An 8 × 9 grid houses 29 fuel elements, 5 control rods, 1 regulating rod, in-core irradiation facilities and reflector elements (see Figure 1 below). The core is fuelled with flat-plate MTR type fuel elements.

A number of in-core and ex-core irradiation facilities are available. The most important irradiation facilities are situated in columns 3 and 8 as indicated in the figure. Hydraulic and pneumatic rabbit facilities in column 9 provide for the irradiation of various material samples. These eight positions are defined as the "in-core irradiation rigs" in the results presented later. A large ex-core pool side facility, directly adjacent to the fuel elements in row H allows bulk irradiations to be performed in relatively high neutron fluxes.

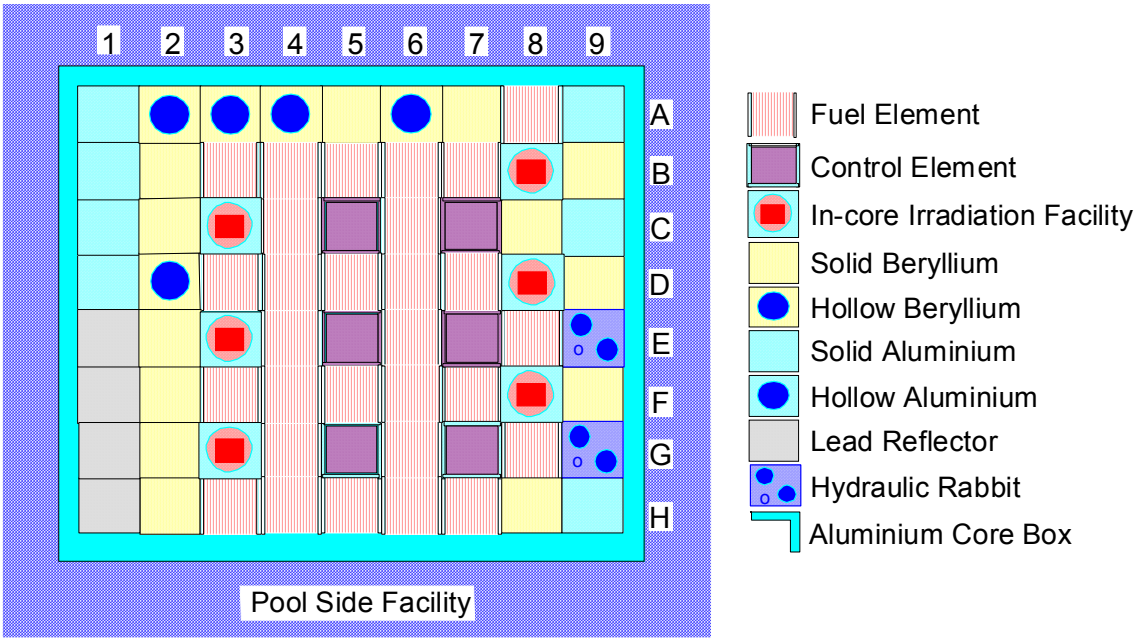

Figure 1. SAFARI-1 core layout for cycle 9907/1.

Core follow calculations for nine cycles up to cycle 9907/1 were performed, which are equivalent to about six months of operation. The typical reload pattern will move fuel elements from the periphery to the centre of the core. Note that during the time of operation considered, a test fuel element was introduced in core position A8. Although this is in a low flux position, large effects were often noticed in this element due to the steep flux gradients.

4. NUMERICAL RESULTS

The energy group effects and the effect of the two cross section models were quantified making use of the SAFARI-1 MTR reactor. The multi-cycle calculations were in each case performed with the model of interest activated. This implies that the total element history was calculated with this model active. All the results presented in this section are for the end of the 9th cycle (9907/1). The results for cross section libraries with differing numbers of neutron energy groups as well as results for rehomogenization and intra-node shape corrected cross sections are presented.

4.1 GROUP EFFECTS

Due to the small heterogeneous cores, and resultant high leakage, found in MTR's, it is necessary to use several energy groups in the nodal calculations. This is shown in Table 1 where the reactivity and power distributions are compared for representative energy group structures. As expected the error, relative to the 8-group reference, consistently increases with decreasing number of energy groups selected. The large errors in the averaged fluxes of in-core irradiation positions in the 4 and 2-group cases are unacceptable from the viewpoint of isotope production calculations.

The variation in the relative power distribution for the different cases is quite large. Errors of over 5% are seen for two groups while the maximum fuel element power is also under predicted. Even in the 5-group case a noticeable effect can be seen. It is interesting to note that in the 2, 4 and 5-group cases the power in the test fuel element in position A8 was consistently under-estimated during the period considered.

The variation in the U-235 masses are, however, considerably smaller than the relative power variations since the cycle length are relatively short compared to the element's lifetime in the core (2 to 3 fresh elements are introduced per cycle). The element will therefore occupy several positions (moving inwards) during its lifetime which will have an averaging effect. Despite of this the maximum absolute differences in the element U-235 masses amount to about 1 gram in both the 4 and 2-group cases. This can be considered unacceptable for safeguard purposes, which normally require MTR operators to report inventories to an accuracy of 0.1 g. It has been found in the practical application of MGRAC in the routine core follow calculations for the SAFARI-1 reactor (since 1995) that the use of six energy groups still gives reasonable turnaround times. It should be noted that the number of energy groups is not only determined by accuracy requirements, but is also important in the prediction of the production rate of isotopes, especially those that depend on reactions with a distinct cut-off energy. The effects of isotope production rigs on the core will also be better taken into account.

Table I. Effect of number of energy groups on MTR multi-cycle analyses.

Case: 8 group (reference)	**6 groups**	**5 groups**	**4 groups**	**2 groups**
k-eff: 0.99743	Negligible	+191 pcm	+838 pcm	+1907 pcm
Max assembly power: 1.297	Negligible	-0.5%	-0.1%	-1.3%
Min assembly power: 0.559	Negligible	+1.7%	-1.2%	-1.1%
Relative power variation	Negligible	-0.6% to +1.7%	-1.8% to +2.3%	-2.6% to +5.7%
U-235 mass distribution	Negligible	-0.1% to +0.2%	-0.4% to +0.5%	-0.8% to +0.6%
In-core Irradiation Rigs: Thermal flux variation	Negligible	+0.4% to +2.4%	-2.6% to +1.4%	-4.5% to +2.2%
In-core Irradiation Rigs: Fast flux variation	Negligible	-0.3% to +0.4%	-3.8% to -0.8%	-16.7% to -4.2%
Relative CPU time: 100	53	45	27	13

4.2 REHOMOGENIZATION MOMENTS EFFECTS

The small fuel element of an MTR means that the environment has a relatively large effect on assembly-averaged cross-sections. This effect can approximately be accounted for by using cross-section moments. The importance of this correction is illustrated in Table 2 where the multi-cycle calculations were performed in 6 groups. The most important effect is on the relative assembly powers (3-4% differences). A comparative relative power map with percentage differences is given in Figure 2.

Table II. Effect of re-homogenization applied to multi-cycle MTR calculation

Value	**Re-homogenization**		**Difference**
	OFF	**ON**	
k-eff	0.99854	0.99966	+ 113 pcm
Maximum assembly power	1.297	1.295	- 0.2%
Minimum assembly power	0.580	0.560	- 3.4%
Relative power variation			- 3.4% (periphery) to + 3.5 % (control)
U-235 mass distribution			- 0.4% to + 0.3%
Pu-239 mass distribution			- 1.1% to + 0.4%

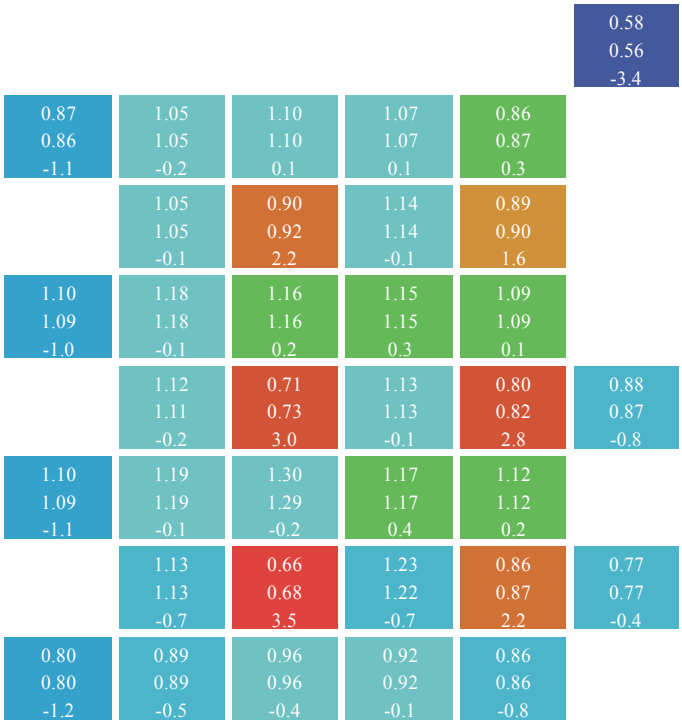

Figure 2. Relative power distribution and differences with re-homogenization applied EOC 9907/1.

From the map it is clear that the largest effects were seen for the control elements and peripheral fuel elements, as could be expected since these are in an environment that differs appreciably from the infinite medium environment of the lattice calculation. The effect for the test fuel element are once again of particular interest and is even larger (and in the opposite direction) than the effect for this element due to the 2-group case. Once again, the absolute errors in U-235 masses amount to about 1 gram. The CPU time penalty incurred by applying this correction is between 15 and 20%.

4.3 INTRANODAL CROSS SECTION VARIATION EFFECTS

In MTR cores, fairly large power, and consequently exposure gradients can occur across fuel elements. This can lead to spatially averaged cross-sections that differ substantially from those that correspond to the node-averaged values of exposure and power. Furthermore, if such an element (for example, from the periphery) is moved towards the centre of the core significant local effects may occur. The effect of using the intra-node shape correction for exposure and power is illustrated in Table 3 for the 6-group case. Note that the equilibrium xenon density option had to be activated in order to be able to use the approximate surface depletion model.

For the MTR calculations no thermal hydraulic feedback was active and the power shape feedback therefore only includes the variation in the xenon concentration within a node. However, even without temperature feedback the effect is smaller than expected and will be investigated as part of future work. The intra-node exposure shape feedback on cross sections does, however, present a noticeable effect. Activating this feedback correction increases CPU time by about 5%. The results presented are for the surface exposures determined in the approximate way. The option that calculates the surface exposures explicitly gives similar results indicating small spectral variations across this core and with a time penalty of an additional 3%.

Table III. Effect of intra-node shape feedback on multi-cycle MTR calculations

	Feedback option		
Value	**None**	**Power**	**Exposure**
k-eff	0.99854	- 9 pcm	- 251 pcm
Max assembly power	1.297	Negligible	+ 0.2 %
Min assembly power	0.580	Negligible	- 0.3 %
Relative power variation	-	Negligible	- 1.0 % to + 0.5 %
U-235 mass distribution	-	Negligible	- 0.2 % to 0.0 %
Pu-239 mass distribution	-	Negligible	- 0.4 % to + 0.6 %

With two independent effects, the question of cancelation of errors naturally arises. As can be deduced from Tables 2 and 3, the difference in reactivity is indeed reduced to –132 pcm when both rehomogenization moments and exposure shape feedback are employed. However, the differences in the assembly power distribution seen in Table 2 are not much affected.

An important safety parameter to be determined is the control rods worths. The total worth of all six control rods was calculated at the beginning and end of the cycle considered here. With the use of rehomogenization moments, the predicted worth is about 179 pcm less than the case with no feedback. When exposure feedback is also activated, the discrepancy is increased to –239 pcm (nearly 1%). At the beginning of the cycle, the difference in worth is slightly less at –188 pcm.

CONCLUSIONS

The OSCAR-3 calculational system has been used very successfully for the core reload planning and support calculations of the SAFARI-1 MTR for many years. Results in this paper show that this was only due to the multi-group implementation of the nodal diffusion method available in MGRAC. Six groups were shown to give adequate accuracy while the efficiency of the MGRAC code makes it practical to perform full three-dimensional six group core follow calculations on a routine basis. A further advantage is the availability of the multigroup fluxes that are often needed to predict radioisotope production rates.

The accuracy of results can further be improved by the use of more advanced homogenization methods. The use of rehomogenization corrections, which take into account the effect of a changing environment on homogenized assembly parameters, can be regarded as essential. In particular, large effects were seen for the peripheral and control-fuel elements. The intra nodal cross section and flux shape feedback method, which assumes a smooth quadratic variation within a node, has smaller effects on the power shape but did however had a noticeable effect on k-eff.

An interesting observation is that the influence of both the rehomogenization moments and the intra-node shape feedback are less in MTR calculations than for a typical PWR power reactor. The reason for this still has to be determined.

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