# REDUCING ROD CUSPING EFFECT IN NODAL EXPANSION METHOD CALCULATIONS

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## ABSTRACT

In neutronics calculations control rods are modeled incrementing the cross sections by an amount interpolated in parametric tables. This amount depends on the average physical conditions of the node. For nodes containing the tip of a rod, a weighting factor must be applied in order to keep the correct reaction rate. Several methods exist to compute this factor. The simplest one is to take the volume fraction occupied by the tip of the control rod in the node. This method produces what is known as rod cusping: a strong discontinuity appears in the control rod worth versus rod insertion curve. The method developed here, based on a physical equivalence, allows a significant reduction of this effect.

## 1. INTRODUCTION

Modeling a control rod within the Nodal Expansion Method presents a difficulty: the position of the tip of the rod does not always match node boundaries. The nodal equations consider uniform cross sections for the whole node and a weighting function must be used to homogenize the control rod contribution inside the node. With a simple volume weighting the $k_{eff}$ behavior versus control rod insertion shows discontinuities (see Fig. 1). According to this curve, a discontinuity appears in the first derivative, at each boundary between two nodes. This numerical effect is known as rod cusping. Rod cusping effect can cause some problem in evaluating differential rod worth curves for control rods or in analyzing transients involving control rod movements.

To reduce the rod cusping effect, more sophisticated methods, based on a flux weighting technique, have been introduced. One of the first is the work of H. S. Joo in 1984 [1] under the supervision of professor A. F. Henry. A selection of these methods is outlined in section 2. Other methods consist in interpolate the weighting factors, as a function of the fractional amount of control rod insertion in the node, in tables generated by previous parametric calculations [2], or to use a parabolic relation with user's supplied coefficients [3].

The idea that lies behind the method developed here is to try to fit the reaction rates near the rod tip as if the node was divided into two nodes, the upper controlled by the rod and the lower not filled. To achieve our task an equivalence approach is done. The rod cross section weighting factors are computed by forcing the reaction rates of the classical nodal expansion (the expansion achieved if the node contained only one region) to match the reaction rates of the reference nodal expansion (the expansion achieved if the node was divided into two regions). This condition is not sufficient because it guarantees only the equivalence of the average nodal equations. To complete the equivalence a correction to the weighted residual nodal equations is also computed.

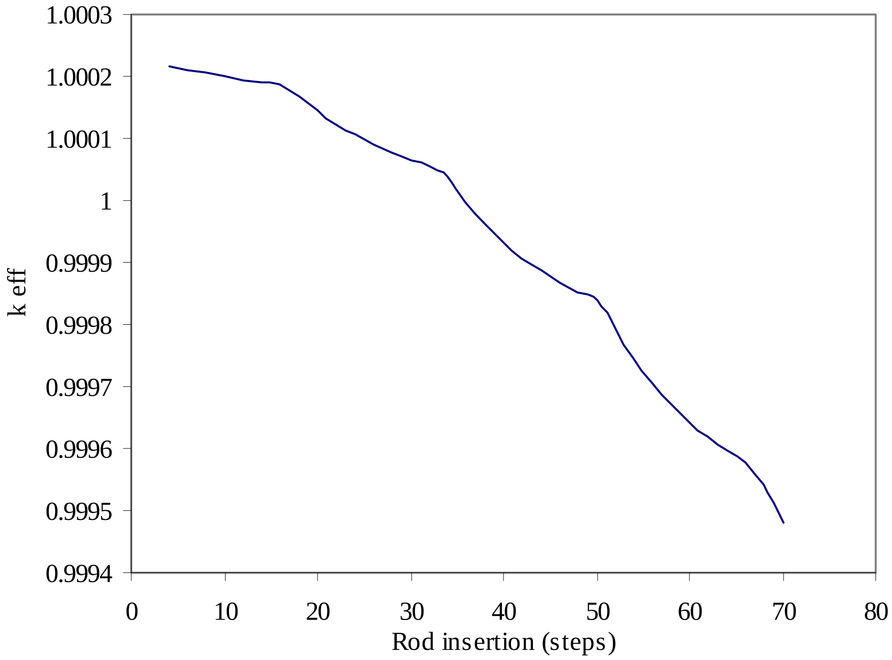


Fig. 1. $k_{eff}$ versus control rod insertion with volume weighting

## 2. RELATED WORK

Many works have been done on the subject of rod cusping. We report in this section some of them in order to permit a comparison between the different approaches and the new one presented in this paper.

### 2.1 APPROXIMATE FLUX WEIGHTING METHOD

This method has been proposed by J. C. Gehin [4] in 1992 to avoid discontinuities of the flux versus time behavior in kinetics transients involving control rod movements. He introduced a simple correction model. In absence of an average flux in the unrodded (NR) and rodded (R) fractions of the node he approximated them by the following relations:

$$\Phi_{NR,g} = \frac{\Delta z_{k-1}\Phi_{k-1,g} + (1 - f_{ins})\Delta z_k \Phi_{k,g}}{\Delta z_{k-1} + (1 - f_{ins})\Delta z_k}$$

$$\Phi_{R,g} = \frac{\Delta z_{k+1}\Phi_{k+1,g} + f_{ins}\Delta z_k \Phi_{k,g}}{\Delta z_{k+1} + f_{ins}\Delta z_k}$$

where $\Phi_{k,g}$ is the neutron flux in node $k$, $\Delta z_k$ is the size of the node in the vertical $z$ direction and $f_{ins}$ is the fraction of insertion of the rod in the node. With these definitions, he computed the average cross sections of type $X$ ($X$ = absorption, neutron-production, removal from group1 to 2) by the relation:

$$\bar{\Sigma}_{X,g} = \frac{(1-f_{ins})\Sigma_{NR,X,g}\Phi_{NR,g} + f_{ins}(\Sigma_{NR,X,g} + \Delta\Sigma_{X,g})\Phi_{R,g}}{(1-f_{ins})\Phi_{NR,g} + f_{ins}\Phi_{R,g}}$$

## 2.2 ANALYTICAL FLUX WEIGHTING WITH AXIAL DISCONTINUITY FACTORS METHOD

This method has been proposed by K. S. Smith et al. [5] in 1992. In this method, the two-group flux distribution in the vicinity of a control rod tip is computed by solving analytically a one-dimensional two-region problem (like the one shown in Fig. 2) using the neutron current resulting from the general nodal solution as boundary conditions. The intranodal flux shapes are integrated analytically over the volume of the node to obtain flux-volume-weighted cross sections. Additionally, the axial discontinuity factors for the top and bottom of the partially rodded node are computed by taking the ratios of the heterogeneous to homogeneous boundary fluxes (found by solving the two-group diffusion equations with flux-volume weighted cross sections and fixed current boundary conditions at the top and bottom of the node).

## 2.3 BILINEAR WEIGHTING METHOD

This method has been proposed by Y. H. Kim and N. Z. Cho [6,7] in 1990. According to this method, for a partially rodded node, the equivalent homogenized cross sections of type $X$ for group $g$ are calculated by the relation:

$$\bar{\Sigma}_{X,g} = \frac{\int_0^{\Delta z_k} \Phi_g \Sigma_{X,g} \Phi_g^+ dz}{\int_0^{\Delta z_k} \Phi_g \Phi_g^+ dz}$$

and the equivalent homogenized diffusion coefficient is calculated by:

$$\bar{D}_g = \frac{\int_0^{\Delta z_k} \Phi_g \Phi_g^+ dz}{\int_0^{\Delta z_k} \Phi_g \frac{1}{D_g} \Phi_g^+ dz}$$

where the integration is performed over the node $k$, and $\Phi_g$ and $\Phi^+_g$ are the heterogeneous forward and adjoint fluxes inside the node.

# 3. EQUIVALENT-NODE METHOD

As mentioned before, the method developed here is based on the equivalence between two configurations. The former, considered as the reference, is a two-node configuration where the upper one is fully controlled by the rod tip. The total size of these two nodes is equal to the calculation mesh size $\Delta z$ in the core model. The second is the calculation node where the tip of the rod is homogenized.

A scheme of the node is shown in Fig. 2.

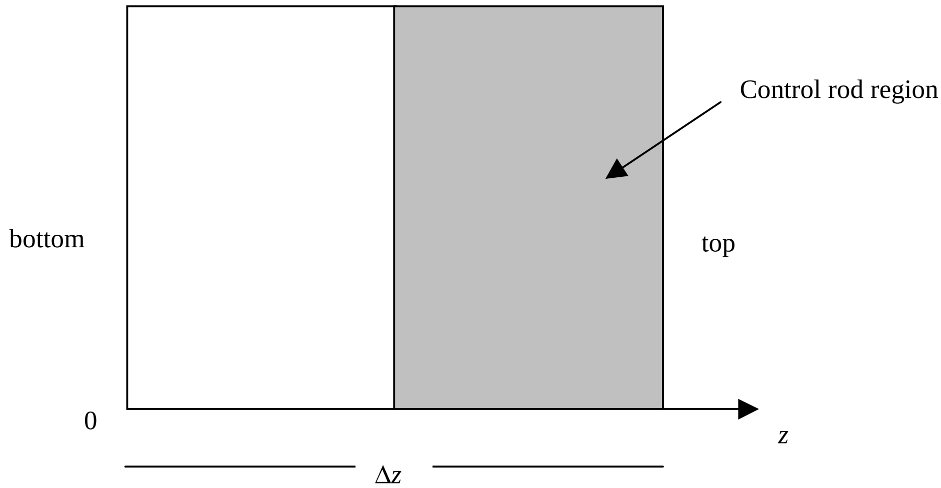


Fig. 2. A node containing the tip of a control rod.

The purpose of the equivalence is to obtain parameters that will force the nodal flux calculation to give results close to the ones that would be obtained by splitting the rodded nodes into two regions. This is done by imposing that the average two-group neutron balance and its first and second moment, computed in the two configurations, be the same. From the condition of equality of the average two-group neutron balance we obtain the rod cross section weighting factors. From the condition of equality of the first and second moment of the two-group neutron balance we obtain corrections to the right hand sides of the first and second moment nodal equations.

## 3.1 BASIC DEFINITIONS

To compute the rod cross section weighting factors and the first and second moment corrections we set equivalence between the two configurations described above.

The diffusion equation, for the two energy groups is:

- fast group:

$$divJ_1 + (\Sigma_{a,1} + \Sigma_r)\Phi_1 - \frac{1}{k_{eff}}\sum_{g=1}^{2}\nu\Sigma_{f,g}\Phi_g = 0 \tag{1}$$

- thermal group:

$$-\Sigma_r\Phi_1 + divJ_2 + \Sigma_{a,2}\Phi_2 = 0 \tag{2}$$

where the neutron current is given by the Fick's law:

$$J_g = -D_g\, grad\Phi_g \tag{3}$$

According to the Nodal Expansion Method, the one-dimensional problem (along the vertical *z* direction) is solved integrating the above equations on the transverse directions *x* and *y*. Using the following definition:

$$L_g(z) = \frac{1}{\Delta x}\int_0^{\Delta x} J_{y,g}\,dx + \frac{1}{\Delta y}\int_0^{\Delta y} J_{x,g}\,dy$$

Eqs. (1), (2) and (3) become:

$$\frac{\partial J_1(z)}{\partial z} + (\Sigma_{a,1}(z) + \Sigma_r(z))\Phi_1(z) - \frac{1}{k_{eff}}\sum_{g=1}^{2} \nu\Sigma_{f,g}(z)\Phi_g(z) = -L_1(z) \qquad \text{(1bis)}$$

$$-\Sigma_r(z)\Phi_1(z) + \frac{\partial J_2(z)}{\partial z} + \Sigma_{a,2}(z)\Phi_2(z) = -L_2(z) \qquad \text{(2bis)}$$

$$J_g(z) = -D_g(z)\frac{\partial \Phi_g(z)}{\partial z} \qquad \text{(3bis)}$$

Then, we consider the flux as an expansion on a set of orthogonal polynomials, limited to order two:

$$\Phi_g(z) = \sum_{i=0}^{2} a_{i,g} P_i(u) \qquad (4)$$

The polynomials are defined by the following relations:

$$\begin{aligned} P_0(u) &= 1 \\ P_1(u) &= u \\ P_2(u) &= u^2 - \frac{1}{12} \end{aligned} \qquad (5)$$

The variable $u$ is the reduced coordinate inside the node defined as:

$$u = \frac{z}{\Delta z} - \frac{1}{2} \qquad (6)$$

To take into account this definition, the derivative operator in the reduced coordinate system must be defined as follows:

$$\frac{\partial}{\partial z} = \frac{1}{\Delta z}\frac{\partial}{\partial u} \qquad (7)$$

With the polynomials defined by Eq. (5) the coefficients of this expansion can be related directly to the surface flux $\Phi_{zp,g}$ in the positive $z$ direction, $\Phi_{zm,g}$ in the negative $z$ direction and to the average flux $\Phi_{a,g}$ by the relations:

$$
\begin{aligned}
a_{0,g} &= \Phi_{a,g} \\
a_{1,g} &= \Phi_{zp,g} - \Phi_{zm,g} \\
a_{2,g} &= 3(\Phi_{zp,g} + \Phi_{zm,g} - 2\Phi_{a,g})
\end{aligned}
\tag{8}
$$

The average and surface fluxes come from the nodal flux calculation.

The above flux formulation is the same for the two node-configurations. As far as the cross sections are concerned we distinguish each of them.

For the two-region configuration the cross sections inside the node are uniform in each region. The cross sections in the upper region (the region with the rod) are equal to the ones of the lower region with an additional delta. Introducing the term $f_c$ representing the position of the control rod tip in the node in the reduced coordinate system, related to the fraction of insertion $f_{ins}$ of the control rod by the following relation:

$$
f_c = \frac{1}{2} - f_{ins}
\tag{9}
$$

the cross section of type $X$ and the diffusion coefficient are defined by:

$$
\begin{aligned}
\Sigma_{X,g}(u) &= \begin{cases} \Sigma_{X,g}, & -\frac{1}{2} \le u < f_c \\ \Sigma_{X,g} + \Delta\Sigma_{X,g}, & f_c \le u \le +\frac{1}{2} \end{cases} \\
D_g(u) &= \begin{cases} D_g, & -\frac{1}{2} \le u < f_c \\ D_g + \Delta D_g, & f_c \le u \le +\frac{1}{2} \end{cases}
\end{aligned}
\tag{10}
$$

For the equivalent node with only one region, the relations defining the cross sections from the values corresponding to each of the two regions are:

$$
\begin{aligned}
\Sigma_{X,g}(u) &= \Sigma_{X,g} + f_{W,g}\Delta\Sigma_{X,g}, & -\frac{1}{2} \le u \le +\frac{1}{2} \\
D_g(u) &= D_g + f_{W,g}\Delta D_g, & -\frac{1}{2} \le u \le +\frac{1}{2}
\end{aligned}
\tag{11}
$$

## 3.2 ROD CROSS SECTION WEIGHTING FACTORS DETERMINATION

The diffusion equation expressed by Eqs. (1bis) and (2bis) are valid for the mesh in both two and one region configurations. Our aim is to obtain reaction rates for the one-region configuration as we were solving the problem in the two-region configuration. The equations allowing us to compute the weighting factors are obtained imposing the equality between the integrals of the left hand sides (LHS) of Eqs. (1bis) and (2bis) in the two configurations. This means that the transverse leakage term $L_g(z)$ in Eqs. (1bis) and (2bis) is assumed as the same in the two configurations.

Let us begin with the fast diffusion equation expressed by Eq. (1bis) after substitution of Eqs. (4), (5), (6) and (7). To integrate it in the two-region configuration we must define the cross sections conforming to Eq. (10). To integrate it in the one-region configuration we must use Eq. (11). To insure the equality between the global reaction rates we force the equality between the two integrals.

$$\begin{aligned}&\int_{-1/2}^{+1/2} LHS[Eq.(1bis)]du \,|_{\mathrm{Using}[Eq.(10)]}\\ &= \int_{-1/2}^{+1/2} LHS[Eq.(1bis)]du \,|_{\mathrm{Using}[Eq.(11)]}\end{aligned}$$

This leads to:

$$\begin{aligned}&-24(k_{eff}\Delta z^2\Delta\Sigma_r a_{0,1} - 2k_{eff}\Delta D_1 a_{2,1} + \Delta z^2 a_{0,1}(k_{eff}\Delta\Sigma_{a,1} - \Delta\nu\Sigma_{f,1}))f_{W,1} + 24\Delta z^2 a_{0,2}\Delta\nu\Sigma_{f,2} f_{W,2} =\\ &(2f_c-1)(-24k_{eff}\Delta D_1 a_{2,1} + k_{eff}\Delta z^2\Delta\Sigma_r(12a_{0,1} + (1+2f_c)(3a_{1,1} + 2f_c a_{2,1})) -\\ &\Delta z^2(2f_c(1+2f_c)a_{2,1}\Delta\nu\Sigma_{f,1} + (12a_{0,2} + (1+2f_c)(3a_{1,2} + 2f_c a_{2,2}))\Delta\nu\Sigma_{f,2} +\\ &12a_{0,1}(\Delta\nu\Sigma_{f,1} - k_{eff}\Delta\Sigma_{a,1}) + (1+2f_c)(-2k_{eff} f_c a_{2,1}\Delta\Sigma_{a,1} + 3a_{1,1}(\Delta\nu\Sigma_{f,1} - k_{eff}\Delta\Sigma_{a,1}))))\end{aligned} \tag{12}$$

In the same way, for the thermal diffusion equation expressed by Eq. (2bis), we obtain:

$$\begin{aligned}&24\Delta z^2\Delta\Sigma_r a_{0,1} f_{W,1} + (48\Delta D_2 a_{2,2} - 24\Delta z^2 a_{0,2}\Delta\Sigma_{a,2}) f_{W,2} =\\ &(2f_c-1)(\Delta z^2\Delta\Sigma_r(12a_{0,1} + (1+2f_c)(3a_{1,1} + 2f_c a_{2,1})) +\\ &24\Delta D_2 a_{2,2} - \Delta z^2(12a_{0,2} + (1+2f_c)(3a_{1,2} + 2f_c a_{2,2}))\Delta\Sigma_{a,2})\end{aligned} \tag{13}$$

The solution of the system composed by Eqs. (12) and (13) in the unknowns $f_{W,1}$ and $f_{W,2}$ provides the weighting factors.

One can notice that in the case of a null variation $\Delta D_g$ of the diffusion coefficient, the expression of the weighting factors reduces to the following:

$$f_{W,g} = \frac{\int_{f_c}^{1/2}\Phi_g du}{\int_{-1/2}^{1/2}\Phi_g du} = \frac{(2f_c-1)(12a_{0,g} + (1+2f_c)(3a_{1,g} + 2f_c a_{2,g}))}{24a_{0,g}}$$

i.e. the weighting factor is the ratio between the integrals of the flux in the rodded zone and in the full node.

## 3.3 MOMENT CORRECTIONS DETERMINATION

The derivation of the equations corresponding to the first and second order moment corrections is similar to the one described in section 3.2. The equations permitting to compute these corrections are obtained by imposing the equality, in the two configurations, between the integrals of the left hand sides of Eqs. (1bis) and (2bis), multiplied by the first order or second order polynomial defined in Eq. (5). In this equivalence, for the one-node configuration, a correction $c_{i,g}$ ($i$=1,2, $g$=1,2) is added to the right hand side of the $i$-th order residual equation corresponding to energy group $g$. For the fast neutrons equation and order 1 this is expressed by:

$$\begin{aligned}&\int_{-1/2}^{+1/2} P_1(u) LHS[Eq.(1bis)]du\,|_{\text{Using}[Eq.(10)]}\\&= \int_{-1/2}^{+1/2} P_1(u) LHS[Eq.(1bis)]du\,|_{\text{Using}[Eq.(11)]} - c_{1,1}\end{aligned}$$

This leads to:

$$\begin{aligned}c_{1,1} = &-\frac{1}{192 k_{eff}\Delta z^2}(48 k_{eff}(4 f_c^{\,2} - 1)\Delta D_1 a_{2,1} + \\&k_{eff}\Delta z^2 \Delta\Sigma_r(-24(4 f_c^{\,2} - 1)a_{0,1} + (1 + 8 f_c^{\,2} - 48 f_c^{\,4})a_{2,1} - 8 a_{1,1}(8 f_c^{\,3} + 2 f_{W,1} - 1)) + \\&\Delta z^2((-1 - 8 f_c^{\,2} + 48 f_c^{\,4})a_{2,1}\Delta\nu\Sigma_{f,1} + \\&(24(4 f_c^{\,2} - 1)a_{0,2} - (1 + 8 f_c^{\,2} - 48 f_c^{\,4})a_{2,2} + 8 a_{1,2}(8 f_c^{\,3} + 2 f_{W,2} - 1))\Delta\nu\Sigma_{f,2} + \\&k_{eff}(1 + 8 f_c^{\,2} - 48 f_c^{\,4})a_{2,1}\Delta\Sigma_{a,1} + 24(4 f_c^{\,2} - 1)a_{0,1}(\Delta\nu\Sigma_{f,1} - k_{eff}\Delta\Sigma_{a,1}) + \\&8 a_{1,1}(8 f_c^{\,3} + 2 f_{W,1} - 1)(\Delta\nu\Sigma_{f,1} - k_{eff}\Delta\Sigma_{a,1})))\end{aligned} \tag{14}$$

In the same way, for the thermal diffusion equation we obtain:

$$\begin{aligned}c_{1,2} = &-\frac{1}{192\Delta z^2}(48(4 f_c^{\,2} - 1)\Delta D_2 a_{2,2} + \\&\Delta z^2 \Delta\Sigma_r(24(4 f_c^{\,2} - 1)a_{0,1} - (1 + 8 f_c^{\,2} - 48 f_c^{\,4})a_{2,1} + 8 a_{1,1}(8 f_c^{\,3} + 2 f_{W,1} - 1)) + \\&\Delta z^2(-24(4 f_c^{\,2} - 1)a_{0,2} + (1 + 8 f_c^{\,2} - 48 f_c^{\,4})a_{2,2} - 8 a_{1,2}(8 f_c^{\,3} + 2 f_{W,2} - 1))\Delta\Sigma_{a,2})\end{aligned} \tag{15}$$

Applying the same transformations to the order 2 moment equations we obtain, for the fast group:

$$\begin{aligned}c_{2,1} = &\frac{1}{2880}(\frac{480 f_c(1 - 4 f_c^{\,2})\Delta D_1 a_{2,1}}{\Delta z^2} + \Delta\Sigma_r(15(-1 - 8 f_c^{\,2} + 48 f_c^{\,4})a_{1,1} + \\&4(40 f_c^{\,3}(6 a_{0,1} - a_{2,1}) + 144 f_c^{\,5} a_{2,1} + 5 f_c(-12 a_{0,1} + a_{2,1}) + 2 a_{2,1}(2 f_{W,1} - 1))) + \\&\sum_{g=1}^{2}(15(1 + 8 f_c^{\,2} - 48 f_c^{\,4})a_{1,g} - 160 f_c^{\,3}(6 a_{0,g} - a_{2,g}) + \\&20 f_c(12 a_{0,g} - a_{2,g}) - 576 f_c^{\,5} a_{2,g} + 8 a_{2,g}(1 - 2 f_{W,g}))\frac{\Delta\nu\Sigma_{f,g}}{k_{eff}} + \\&(-15(1 + 8 f_c^{\,2} - 48 f_c^{\,4})a_{1,1} + \\&4(40 f_c^{\,3}(6 a_{0,1} - a_{2,1}) + 144 f_c^{\,5} a_{2,1} + 5 f_c(-12 a_{0,1} + a_{2,1}) + 2 a_{2,1}(2 f_{W,1} - 1)))\Delta\Sigma_{a,1})\end{aligned} \tag{16}$$

and, for the thermal group:

$$
\begin{aligned}
c_{2,2} = &-\frac{1}{2880\Delta z^2}(-20 f_c(\Delta z^2 \Delta\Sigma_r(12a_{0,1} - a_{2,1}) + 24\Delta D_2 a_{2,2} + \Delta z^2(-12a_{0,2} + a_{2,2})\Delta\Sigma_{a,2}) + \\
&160 f_c^{\,3}(\Delta z^2 \Delta\Sigma_r(6a_{0,1} - a_{2,1}) + 12\Delta D_2 a_{2,2} + \Delta z^2(-6a_{0,2} + a_{2,2})\Delta\Sigma_{a,2}) + \\
&24\Delta z^2 f_c^{\,2}(\Delta\Sigma_r(5(6 f_c^{\,2} - 1)a_{1,1} + 24 f_c^{\,3} a_{2,1}) + ((5 - 30 f_c^{\,2})a_{1,2} - 24 f_c^{\,3} a_{2,2})\Delta\Sigma_{a,2}) + \\
&\Delta z^2(\Delta\Sigma_r(-15a_{1,1} + 8a_{2,1}(2 f_{W,1} - 1)) + (15a_{1,2} + 8a_{2,2}(1 - 2 f_{W,2}))\Delta\Sigma_{a,2}))
\end{aligned}
\tag{17}
$$

## 4. RESULTS

The method to compute the control rod weighting factors and the moment corrections (equivalent node method) has been implemented in the cross section module of the SMART system [8]. It has been tested in several configurations and each set of results showed that the rod cusping was reduced. The results of this method have been compared to those obtained with the other two methods implemented in SMART: the volume weighting and the approximate flux method (section 2.1).

We will follow the following terminology:

- *Vol*. Volume weighting method,
- *Flux* Approximate flux weighting method,
- *Equiv*. Equivalent-node method.

Fig. 3 shows the complete response to a control group (group R) in terms of $k_{eff}$ for a reactor of 429 cm height. The core is axially modeled by 16 equally spaced nodes. The configuration corresponds to an average core exposure of 4000 MWD/T in the 4th cycle. The group R response has been analyzed with a calculation each 2 steps from its full withdrawal to its full insertion.

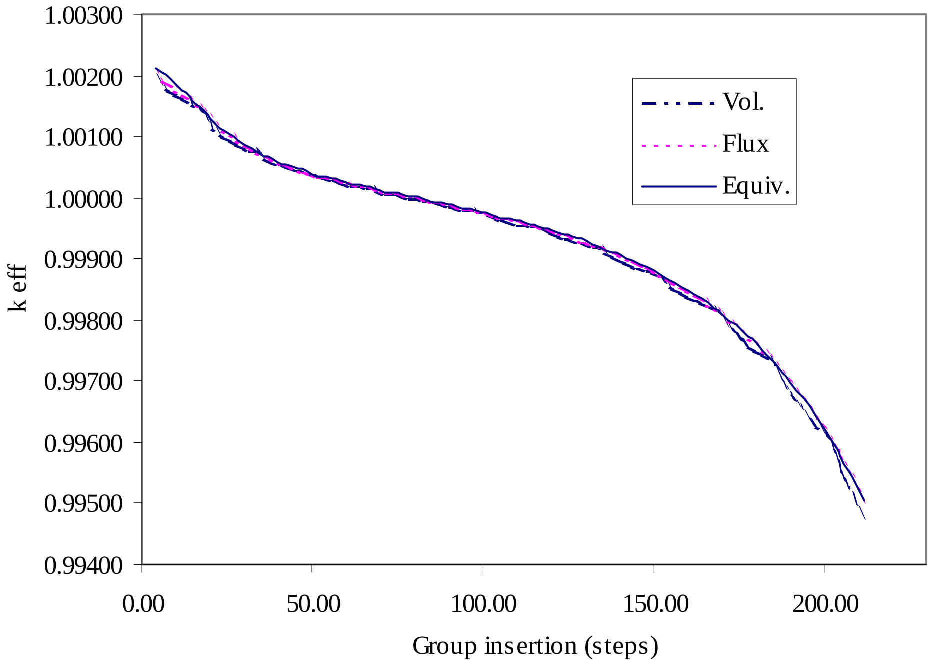


Fig. 3. Group R response for a 429 cm height reactor.

A detail is presented in Fig. 4. We can see that both the methods (*Flux* and *Equiv.* curves) reduce the rod cusping effect. However, in the first node, the *Equiv* curve has a better behavior than *Flux* curve.

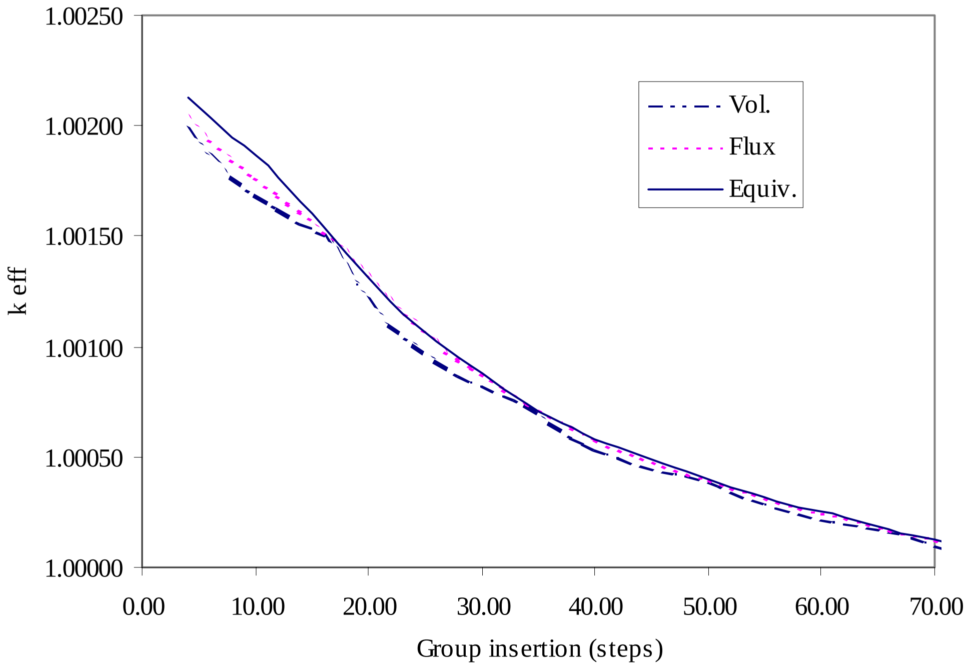


Fig. 4. Group R response for a 429 cm height reactor (detail).

Fig. 5 shows a comparison between the responses to group R in terms of differential worth for a core of 368 cm height, obtained with the volume weighting (*Vol.* curve), the approximate flux weighting method (*Flux* curve) and the equivalent-node method (*Equiv.* curve). The core is axially modeled by

16 equally spaced nodes. The configuration corresponds to the beginning of cycle 1. The group R response has been analyzed with a calculation each 2 steps from its full withdrawal to its full insertion (i.e. 225 steps). A critical boron search has been done at each group position. A reference calculation without rod cusping effect has been performed (*Ref.* curve). The mesh grid of this reference is fine enough to permit to the rod tip to be always fully inserted in a node. The size of each node is equal to the rod step. It can be seen that the error introduced by our method is less than 0.3 pcm/step, substantially lower than that introduced with volume weighting and slightly lower than that introduced by the approximate flux weighting. It is interesting to notice that the response in the top of the core is more realistically evaluated with the equivalent-node method. It predicts an increasing behavior of the rod worth where the other ones predict a decreasing.

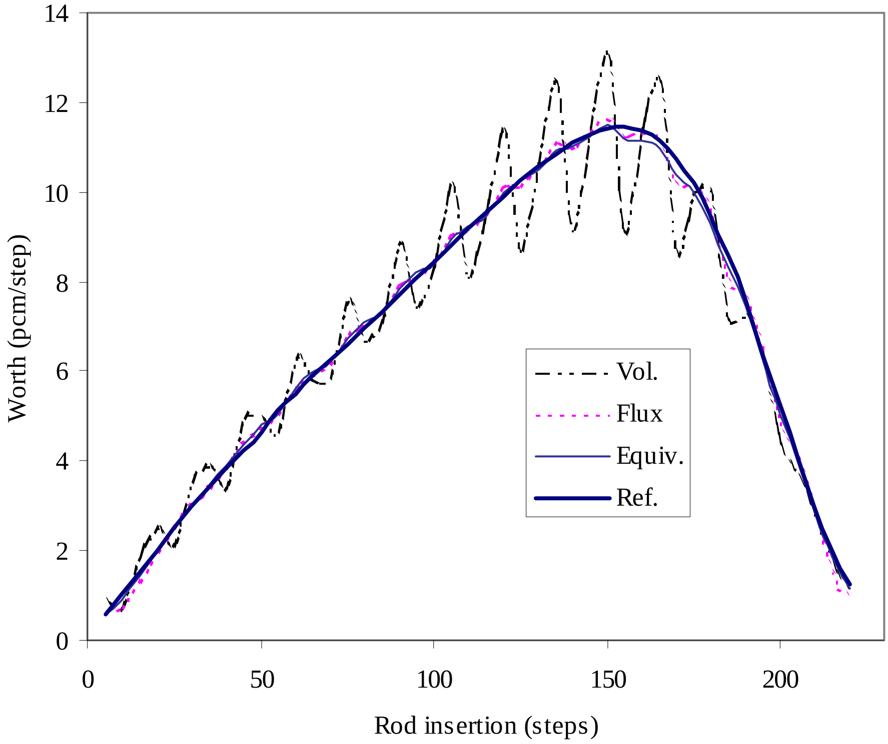


Fig. 5. Group R response in terms of differential worth for a 368 cm height reactor.

In order to verify the axial flux distribution some comparisons with measurements have been done. These calculations have been performed on a 429 cm height reactor at the end of cycle 2 (Cycle exposure: 9285 MWD/T). The core configuration corresponds to group R inserted at 8 steps, i.e. at 3 % of fuel height. The core is axially modeled by 16 equally spaced nodes. Group R controls a fraction 0.48 of the upper fuel node (12.88 cm). The results obtained with the three rod weighting options are shown in Fig. 6. This figure compares the computed axial activity with the instrument readings in a channel controlled by group R for the upper fuel zone. As the result of the nodal calculations is an intranodal flux shape, an axial activity shape has been estimated on the basis of order 2 polynomials. We can see that the equivalent node method matches the measurements better than the other ones.

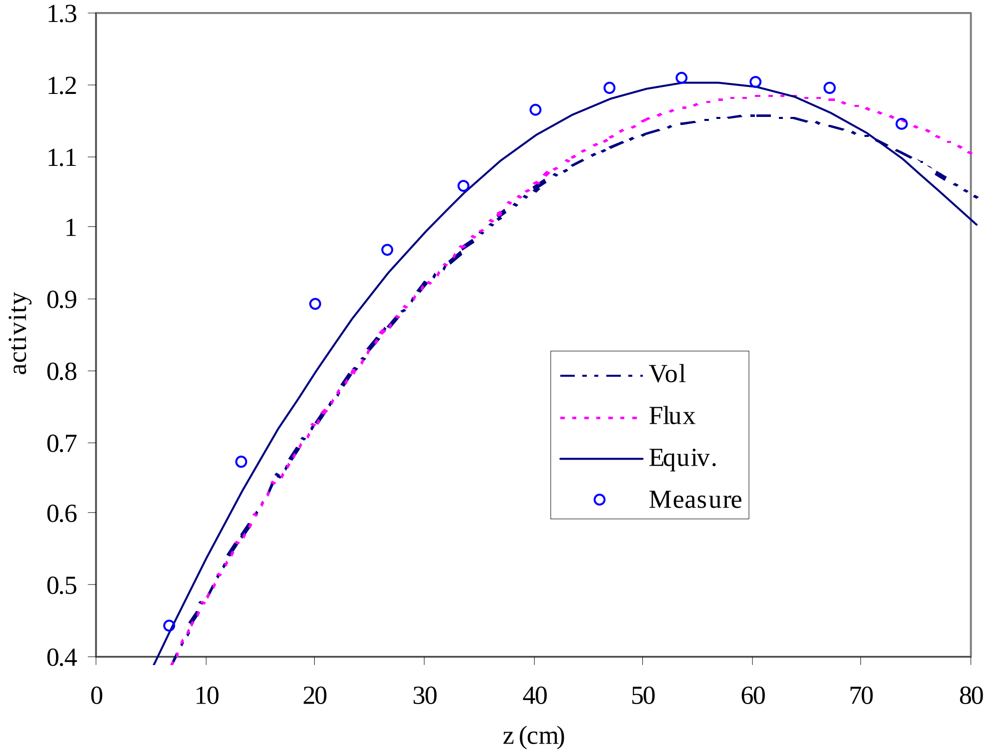


Fig. 6. Detector activity versus axial position from the top in a channel controlled by group R for a 429 cm height reactor.

## CONCLUSIONS

The results presented in section 4 show a good improvement of our method (equivalent node) compared to the volume weighting method and also to the approximate flux weighting method. This is due to the fact that the equivalent node method applies a physical approach consistent with the nodal expansion method.

The results have shown that the equivalent node method in the top part of the fuel zone is more realistic than the volume weighting method. This is an important feature in on-line calculations in power plants, where controls groups are usually slightly inserted in the core.

Our weighting method has also the advantage to make easier the computation of critical control rod axial position. The smooth behavior of the rod response computed by the equivalent node method permits to reduce instabilities in the search algorithm.

As far as the computer run time is concerned, the calculation of the weighting factors with the equivalent node method does not increase it. This calculation is limited to the nodes containing the tip of the control groups. Their number is small compared to the total number of nodes in the model.

## ACKNOWLEDGEMENTS

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