TRANSIENTS MODAL ANALYSIS USING TRAC/BF1-MODKIN

G. Verdú and R. Miró
Departamento de Ingeniería Química y Nuclear
Universidad Politécnica de Valencia
Camino de Vera, 14
46022 Valencia, Spain.
gverdu@iqn.upv.es; rmirohe@iqn.upv.es

D. Ginestar
Departamento de Matemática Aplicada
Universidad Politécnica de Valencia
Camino de Vera, 14
46022 Valencia, Spain.
dginesta@mat.upv.es

V. Vidal
Departamento de Sistemas Informáticos y Computación
Universidad Politécnica de Valencia
Camino de Vera, 14
46022 Valencia, Spain.
vvidal@dsic.upv.es

ABSTRACT

TRAC/BF1-MODKIN code is a time domain analysis code to study transients in a BWR reactor. This new code uses the best estimate code TRAC/BF1 to give account of the heat transfer and thermal-hydraulic processes and a 3D neutronics module called MODKIN. This module uses a modal method to integrate the neutron diffusion equation in the approximation of two energy groups, which is based on the assumption that the neutronic flux can be approximately expanded in terms of the dominant lambda modes associated with a static configuration of the reactor core. A modes time updating strategy has been developed for this method, and this allows to consider the modal method as a quasistatic method to integrate the neutron diffusion equation. To check the performance of TRAC/BF1-MODKIN code, the Peach Bottom turbine trip transient has been simulated, since this transient is a dynamically complex event where neutron kinetics is coupled with thermal-hydraulics in the reactor primary system, and reactor variables change vary rapidly.

1. INTRODUCTION

A Nuclear reactor simulator consists mainly of two different blocks, which solve the models used for the basic physical phenomena taking place in the reactor. In this way, there is a neutronic module which simulates the neutron balance in the reactor core, and a thermal-hydraulics module, which simulates the heat transfer in the fuel, the heat transfer from the fuel to the water, and the different condensation and evaporation processes which take place in the reactor core and in the condenser systems.

As neutronic module we have used a nodal modal method, called MODKIN [1]. This code solves the neutron diffusion equation in the approximation of two energy groups, and it is based on the expansion of the neutronic flux in terms of the dominant Lambda modes of a given configuration of the reactor core.

As thermal-hydraulic module we have used the best estimate code TRAC/BF1 [2], which uses a two fluids, six-equation model to give account of the thermal-hydraulic phenomena. The coupled code has been called TRAC/BF1-MODKIN and it can be used to simulate transients considering the neutronic phenomena in 3D geometry and the thermal-hydraulic processes in multiple-channel 1D geometry.

Since there is a physical interpretation for the amplitudes evolution of the different neutronic modes, especially in the analysis of instability events, this combination of codes can become very useful to analyze this kind of transients. To test the performance of the coupled code in a transient where the coupling between core phenomena and system dynamics play an important role, we have studied the exercise proposed in the Peach Bottom Turbine Trip benchmark [3]. This transient is a dynamically complex event where neutron kinetics is coupled with thermal-hydraulics in the reactor primary system, and reactor variables change vary rapidly. Thus, this transient constitutes a good benchmark problem to test a coupled thermal-hydraulics-3D neutronics code. The rest of the paper is organized as follows, in section 2, we present a brief description of MODKIN and TRAC/BF1 codes and the coupling between the two modules. Section 3 is devoted to the numerical results obtained with TRAC/BF1-MODKIN in the study of Peach Bottom turbine trip benchmark problem. The main conclusions of the paper are presented in section 4.

2. TRAC/BF1-MODKIN code

2.1 MODKIN module

MODKIN module is based on a nodal modal method to integrate the time dependent neutron diffusion equation in the approximation of two energy groups [1]. To describe the nodal modal method, we start from the neutron diffusion equation

$$\left[v^{-1}\right]\frac{\partial \phi}{\partial t} + L\phi = (1-\beta)M\phi + \sum_{k=1}^{K}\lambda_k C_k \chi \,, \tag{1}$$

$$\frac{\partial C_k}{\partial t} = \beta_k \left[\nu\Sigma_{f1} \;\; \nu\Sigma_{f2}\right]\phi - \lambda_k C_k \,, \quad k = 1,\ldots,K, \tag{2}$$

where, K is the number of delayed neutron precursors considered,

$$L = \begin{bmatrix} -\nabla\cdot(D_1\nabla)+\Sigma_{a1}+\Sigma_{12} & 0 \\ -\Sigma_{12} & -\nabla\cdot(D_2\nabla)+\Sigma_{a2} \end{bmatrix}, \quad \left[v^{-1}\right] = \begin{bmatrix} \frac{1}{v_1} & 0 \\ 0 & \frac{1}{v_2} \end{bmatrix},$$

and

$$M = \begin{bmatrix} \nu\Sigma_{f1} & \nu\Sigma_{f2} \\ 0 & 0 \end{bmatrix}, \quad \phi = \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}, \quad \chi = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

To discretize the spatial part of these equations, the reactor core is divided in N neutronic nodes and a nodal collocation method [5], [6] is applied. This method makes use of an expansion of the neutron flux in each node in terms of Legendre polynomials and it allows to approximate equations (1) and (2) by the following set of ordinary differential equations

$$\left[v^{-1}\right]\frac{\partial\psi}{\partial t} + L\psi = (1-\beta)M\psi + \sum_{k=1}^{K}\lambda_k X C_k \ , \tag{3}$$

$$X\frac{\partial C_k}{\partial t} = \beta_k M\psi - \lambda_k X C_k \, , \tag{4}$$

where now ψ and C_k are vectors whose components are the corresponding coefficients of the Legendre polynomials expansions of ϕ and C_k in each node, at each time step. L and M have the following block structure

$$L = \begin{bmatrix} L_{11} & 0 \\ -L_{21} & L_{22} \end{bmatrix}, \quad M = \begin{bmatrix} M_{11} & M_{12} \\ 0 & 0 \end{bmatrix},$$

and we have introduced the matrix X,

$$X = \begin{bmatrix} I \\ 0 \end{bmatrix}.$$

The modal method consists of assuming to integrate equations (3) and (4) the modal expansion

$$\psi \approx \sum_{l=1}^{M_d} n_l(t)\psi_l \ , \tag{5}$$

where ψ_l, $l=1,\ldots,M_d$ are the eigenvectors associated with the M_d dominant eigenvalues of a static configuration of the reactor. That is, they are the solutions of a static problem of the form [6]

$$L_0\psi_1 = \frac{1}{k_1} M_0\psi_1 \ , \tag{6}$$

where the matrices L_0 and M_0 are the matrices L and M of equations (3) and (4), for a given configuration of the reactor core. Problem (6) is known as a Lambda modes problem for the reactor core. Since L_0 is not a hermitic matrix, we consider the adjoint problem [7]

$$L^{+}{}_0\psi^{+}{}_1 = \frac{1}{k_1} M^{+}{}_0\psi^{+}{}_1 , \tag{7}$$

To obtain two sets of biorthogonal modes ψ_n and $\psi^{+}{}_m$. Thus, these modes satisfy the relation

$$\left\langle \psi^{+}{}_m , M_0\psi_n \right\rangle = \left\langle \psi^{+}{}_m , M_0\psi_m \right\rangle \delta_{n,m} = N_m \ \delta_{n,m} \ , \tag{8}$$

where $\delta_{n,m}$ is the Kronecker delta function.

Multiplying equations (3) and (4) by $\psi^{+}{}_m$, writing $L=L_o+\delta L$, $M=M_o+\delta M$, and making use of expansion (5) and equation (6), we obtain [1]

$$\sum_{l=1}^{M_d} \Lambda_{ml} \frac{dn_l}{dt} = (\rho_m - \beta) N_m n_m + (1-\beta)\sum_{l=1}^{M_d} A^{M}_{ml} n_l - \sum_{l=1}^{M_d} A^{L}_{ml} n_l + \sum_{k=1}^{K} \lambda_k C_{mk} \ , \tag{9}$$

$$\frac{dC_{mk}}{dt} = \beta_k N_m n_m + \beta_k \sum_{l=0}^{M_d} A^{M}_{ml} n_l - \lambda_k C_{mk} \ , \quad k = 1,\ldots,K, \tag{10}$$

where we have used the biorthogonality relation (8), we have introduced the notation

$$\Lambda_{ml} = \left\langle \psi^{+}_{m}, \left[v^{-1}\right]\psi_l \right\rangle, \quad A^{L}_{ml} = \left\langle \psi^{+}_{m}, \delta L\psi_l \right\rangle, \quad A^{M}_{ml} = \left\langle \psi^{+}_{m}, \delta M\psi_l \right\rangle, \quad C_{mk} = \left\langle \psi^{+}_{m}, XC_k \right\rangle,$$

and the reactivity of the m-th mode, defined as

$$\rho_m = \frac{k_m - 1}{k_m} .$$

If the number of modes used in expansion (5) is not very large, the dimension of the system of differential equation (9) and (10) associated with the modal method is quite smaller than the initial system (1) and (2). To solve numerically the system of equations (9) and (10), we have used a variable step implicit method [8] because of the stiffness of the equations.

For realistic transients, the nuclear cross-sections are time dependent functions, and to use the modal method exposed above, generally it is necessary to calculate a large amount of modes, but this process is prohibitive from a computational point of view. Thus, instead of calculating a large amount of modes, we calculate only a small number of them and we update these modes each certain time step Δt_i. In this way, to integrate the neutron diffusion equation in a time interval $[t_i, t_{i+1}]$, we use the modes solution of the problem

$$L^i \psi_l^i = \frac{1}{k_l^i} M^i \psi_l^i \ ,$$

where L^i and M^i are the matrices associated with the reactor configuration at time t_i. And the differential equations to be integrated are the following ones

$$\sum_{l=1}^{M_d} \left\langle \psi_m^{i+}, \left[v^{-1}\right] \psi_l^i \right\rangle \frac{dn_l^i}{dt} + \sum_{l=1}^{M_d} \frac{1}{k_l^i} \left\langle \psi_m^{i+}, M^i \psi_l^i \right\rangle n_l^i + \sum_{l=1}^{M_d} \left\langle \psi_m^{i+}, \delta L^i \psi_l^i \right\rangle n_l^i =$$
$$(1-\beta) \sum_{l=1}^{M_d} \left\langle \psi_m^{i+}, M^i \psi_l^i \right\rangle n_l^i + (1-\beta) \sum_{l=1}^{M_d} \left\langle \psi_m^{i+}, \delta M^i \psi_l^i \right\rangle n_l^i + \sum_{k=1}^{K} \lambda_k \left\langle \psi_m^{i+}, XC_k \right\rangle ,$$

$$\frac{d}{dt} \left\langle \psi_m^{i+}, XC_k \right\rangle = \beta_k \sum_{l=1}^{M_d} \left\langle \psi_m^{i+}, M^i \psi_l^i \right\rangle n_l^i + \beta_k \sum_{l=1}^{M_d} \left\langle \psi_m^{i+}, \delta M^i \psi_l^i \right\rangle n_l^i - \lambda_k \left\langle \psi_m^{i+}, XC_k \right\rangle . \tag{11}$$

To calculate the initial conditions $n^i_m(t_i)$, we reconstruct the vector

$$\psi(t_i) = \sum_{l=1}^{M_d} n_l^{i-1}(t_i) \psi_l^{i-1} \ ,$$

from the variables $n^{i-1}_m(t_i)$ and ψ^{i-1}_l obtained in the last time step, and we calculate

$$n_m^i(t_i) = \frac{1}{\left\langle \psi_m^{i+}, M^i \psi_m^i \right\rangle} \left\langle \psi_m^{i+}, M^i \psi(t_i) \right\rangle . \tag{12}$$

To obtain the variables related to the concentration of the delayed neutron precursors, we assume that

$$\psi_m^{i+} \approx \sum_{l=1}^{Md} a_{ml} \psi_l^{i-1+} \ ,$$

where

$$a_{ml} = \frac{\left\langle \psi_m^{i+}, M^{i-1} \psi_l^{i-1} \right\rangle}{\left\langle \psi_m^{i-1+}, M^{i-1} \psi_l^{i-1} \right\rangle} ,$$

and this yields

$$\left\langle \psi_m^{i+}, XC_k \right\rangle (t_i) \approx \sum_{l=1}^{M_d} a_{ml} \left\langle \psi_l^{i-1+}, XC_k \right\rangle (t_i).$$

which completes the initial conditions to integrate equations (11) in $[t_i, t_{i+1}]$.

The nodal modal method with the modes updating strategy generalizes in a certain way the quasi-static method [8]. The quasistatic method factorizes the neutronic flux as the product of an amplitude function, which only depends on time, by a shape function, that carries the spatial dependence of the flux and varies slowly in time. The nodal modal method uses the lambda modes as shape functions and the amplitudes are calculated with the modal equations (11).

The nodal modal method exposed above has been implemented in a module called MODKIN that uses as inputs the reactor core geometry, the nuclear cross-sections at each time step and the Lambda modes which are calculated using an implicit restarted Arnoldi method [9]. The output of this module is the nodal power distribution at each time step.

2.2 TRAC/BF1-MODKIN coupling

As thermal-hydraulic module we have used TRAC/BF1 code [2]. This code has models for the usual components of a BWR reactor as the Vessel, the Channels, pumps, Separator-dryer, etc. The fuel channels in the core are modelled with multiple channels components. To model the heat transfer in the fuel, an axial-radial heat transfer equation is used. The thermal-hydraulics processes are modelled solving six balance equations of mass, momentum and energy for liquid and vapor phases. For the numerical integration of the fluid flow equations a semi-implicit two step method is used for the time discretization, and a first order finite difference method with staggered mesh is applied to discretize the spatial part of the equations.

The nuclear cross sections associated to each neutronic node are obtained interpolating the values of multiple entries tables in terms of the thermal-hydraulic variables and the control rods insertion pattern. These nuclear cross sections are used to obtain the power distribution with MODKIN module. This power distribution is used as an input for TRAC/BF1 in the POST stage [2] to obtain the thermal-hydraulic variables, which are used to obtain a second set of cross sections. After this, TRAC/BF1 uses the nodal power distribution provided by MODKIN in the PREP and OUTER stages. Both sets of cross sections are used for the implicit integration of the nodal equations (11). The cross sections at intermediate time steps are obtained by linear interpolation from both sets of cross sections. In this way, we obtain an explicit coupling between MODKIN and TRAC/BF1 codes, performed in a sequential way and allowing the use of different time steps for the internal neutronic calculations and the thermal-hydraulic calculations.

3. NUMERICAL RESULTS

To test the performance of the TRAC/BF1-MODKIN code we have simulated a turbine trip transient of Peach Bottom reactor, which constitutes the third exercise of the NEA-NRC BWR TT benchmark exercise that is defined with detail in [3]. This transient begins with a sudden turbine stop valve closure. The pressure oscillation generated in the main steam piping propagates with little attenuation in the reactor core. The induced core pressure oscillation results in dramatic changes of the core void distribution and fluid flow, resulting in a sudden peak in the neutronic power, which is slowed down by the feedback from the increased direct and conducted flux to the coolant. A scram is triggered at a defined power level.

Particularly, we have simulated the first second of the transient, that is when the power peak takes place. For the calculations we have used an accumulated thermal-hydraulic time counter selecting the updating time for the nuclear cross sections, when the counter is larger than 10^{-3} s. After the actualisation the counter is set to 0. In the MODKIN module we have made use of 1 lambda mode and we have performed the calculation without updating the mode, updating it each 0.2 seconds, and updating it each 0.1 seconds. In figure 1, we compare the obtained results for the fission power evolution in the different calculations with the results obtained using an implicit method, called NOKIN [11], where the integration time step has been set to 10^{-3}s.

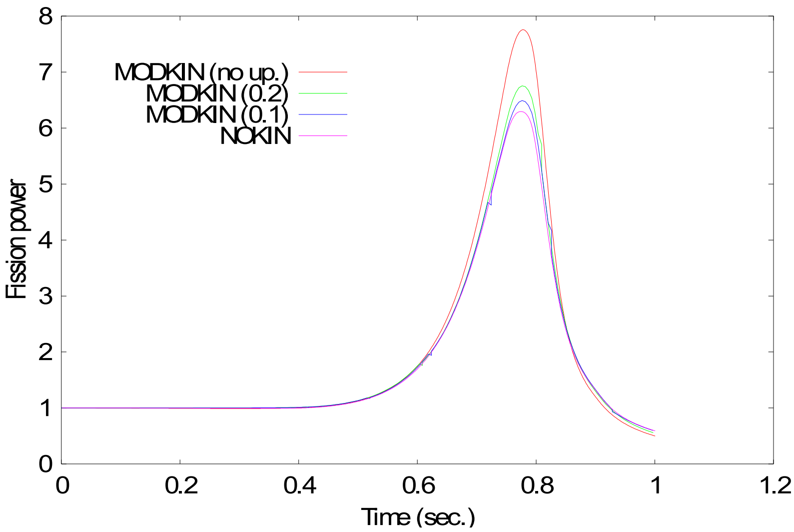

Figure 1. Fission power evolution for the Peach Bottom turbine trip transient.

We observe that MODKIN module reproduces the power evolution quite accurately with only one mode using the updating strategy each 0.2 or 0.1 seconds. This clearly indicates that although the transient analysed is quite fast the 3D spatial shape of the power distribution inside the core does not suffer large variations, and the 3D capabilities of the coupled code are not fully required in the analysis.

To compare the time efficiency of the nodal modal method, in table I we present the CPU times needed to simulate the transient using NOKIN module and MODKIN module with the different options, using as platform a Pentium III 1 Mhz personal computer.

Table I. Different CPU times needed to simulate the transient

Module	CPU time	Relative cost
Neutronics NOKIN	10965 s	29.8
Thermal-hydraulics	1685 s	4.6
Neutronics MODKIN (no up.)	368 s	1
Neutronics MODKIN (0.1 s)	1453 s	3.95
Neutronics MODKIN (0.2 s)	850 s	2.3

We observe that the MODKIN method without the modes updating is the fastest method, but it is the worst method with respect to the accuracy of the obtained solution. To update the modes each 0.2 or 01 seconds is a good compromise between computational cost and accuracy.

CONCLUSIONS

In this paper, we have presented the coupled code for BWR reactor analysis TRAC/BF1-MODKIN code. This code uses the best estimate code to simulate the heat transfer and thermal-hydraulic processes taking place in the reactor. MODKIN module is used to integrate the neutron diffusion in 3D geometry. This module implements a nodal modal method based on the expansion of the neutronic flux in terms of the dominant lambda modes associated with a static configuration of the reactor core.
To test the performance of the coupled code, the third exercise proposed in the Peach Bottom turbine trip benchmark exercise has been simulated and the results have been compared with the ones obtained using a 3D implicit method called NOKIN. From this simulations we observe that it is possible to obtain quite accurate results with a reasonable computational cost using the modal method using only one mode, specially if we make use of the modes updating strategy. This indicates that in the conditions of this transient, the shape of the spatial power distribution inside the reactor core does not experiment large changes and a full 3D kinetics code is not necessary. Nevertheless, TRAC/BF1-MODKIN codes succeeds in solving the transient in a moderate amount of time.

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