PERFORMANCE OF THE ANALYTIC COARSE MESH FINITE DIFFERENCE METHOD WITH HETEROGENEOUS NODES

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ABSTRACT

This paper is focused on the performance analysis of the Analytic Coarse Mesh Finite Difference Method (ACMFD), rigorously obtained for homogeneous nodes, and generalized for heterogeneous nodes by including interface flux discontinuity factors. This new form of the ACMFD method can be implemented in design codes for generic applications, and it does not need to be used iteratively with a higher level nodal method as an acceleration tool.

On one hand, this method includes both the spectrum interaction and the coarse mesh correction effects in the own formulation, without the need of any higher level nodal method calculation. On the other hand, the method accounts for heterogeneity effects through the dependence of the nodal homogenized two-group cross sections and interface flux discontinuity factors on the flux-spectrum and burnup intranodal distributions, as well as on neighbor effects. These nonlinear feedbacks of nodal parameters have shown as a very effective way of accounting for heterogeneity effects in two-group multidimensional coarse mesh diffusion calculations.

Implementation of this method into our design code system is discussed, and numerical results are presented. Comparisons with the previous nodal method in our code system are made in terms of accuracy and computational efficiency, and they show the very favorable qualification result of this new option.

1. INTRODUCTION

Most multidimensional two-group nodal methods use the nonlinear finite difference method (FDM) iterative scheme [1] [2] to minimize the memory requirement and computing time associated with higher-order nodal methods. This nonlinear scheme consists of solving two-node problems using the higher-

order nodal method, only after a specified number of outer iterations of the FDM routine. Without the updated nonlinear correction factors, the solution of the finite difference scheme would not be equivalent to that of the higher-order nodal method itself.

Recently, Chao [3] [4] has presented an analytic coarse-mesh finite difference (ACMFD) method in two-energy groups for the case of homogeneous nodes, where the expressions to relate both nodal surface fluxes and currents to node average fluxes are analytically derived. The formulation includes by itself both the spectrum interaction and the coarse mesh correction effects, making use of matrix relationships involving nodal eigenvalues and eigenvectors. The performance of this method in multidimensional homogeneous node problems is quite good. However it is not able of handling intranodal heterogeneities, so it can not be used standalone in design code applications.

This formulation has been extended now for the general case of heterogeneous nodes, where depletion burnup intranodal gradients or directional spectrum interaction between adjacent nodes have to be accounted for. Then, the generalized ACMFD method can be used standalone without the need of any higher nodal method calculation. This provides originality in contrast to most advanced nodal methods, and high accuracy in a more computationally efficient manner.

2. UNIQUE FEATURES OF THIS GENERALIZED ACMFD METHOD

The solution of the 1-D transversely integrated nodal diffusion equation depends on the general solution of the equivalent homogeneous equation as well as on the transverse leakage profile $\boldsymbol{L}(x)$. Once the terms of the homogeneous solution are expressed in terms of the node-average flux and node-interface fluxes, and by applying the continuity condition of the neutron currents across each node interface, we obtain the nodal coupling matrix equation for a given node (a quarter of fuel assembly) with side length of h:

$$\boldsymbol{R}\boldsymbol{\phi}^{hom}_{(\pm\frac{h}{2})} = \boldsymbol{C}^{f}_{(\pm\frac{h}{2})}\,\boldsymbol{R}\,\bar{\boldsymbol{\phi}} \mp \frac{h}{2}\boldsymbol{C}^{j}\boldsymbol{R}\,\boldsymbol{D}^{-1}\boldsymbol{J}_{(\pm\frac{h}{2})} \tag{1}$$

where ($-h/2$) and ($+h/2$) refers to the left and to the right interfaces of the node respectively. $\boldsymbol{R}^{-1}$ is the 2x2 matrix of the eigenvectors, $\boldsymbol{C}^{j}$ is a node-dependent matrix, being sensitive to changes in the nodal eigenvalues, and $\boldsymbol{C}^{f}$ is an interface-dependent matrix, changing not only with nodal eigenvalues, but also with transverse leakage.

According to the equivalence homogenization theory, a 2x2 heterogeneity matrix $\boldsymbol{f}$ of interface flux discontinuity factors (IFDF), taking care of errors due to homogenization procedure, is introduced per nodal interface:

$$\boldsymbol{\phi}^{hom}_{(\pm\frac{h}{2})} = \boldsymbol{f}\boldsymbol{\phi}^{het}_{(\pm\frac{h}{2})} = \begin{bmatrix} f_f & 0 \\ 0 & f_{th} \end{bmatrix}\boldsymbol{\phi}^{het}_{(\pm\frac{h}{2})} \tag{2}$$

This provides the simplest way to establish the nodal coupling relation for the proper ACMFD formulation for multidimensional heterogeneous nodes [5]:

$$\boldsymbol{R}\,\boldsymbol{f}\,\boldsymbol{\phi}^{het}_{(\pm\frac{\boldsymbol{h}}{\boldsymbol{2}})} = \boldsymbol{C}^{f}_{(\pm\frac{h}{2})}\,\boldsymbol{R}\,\bar{\boldsymbol{\phi}} \mp \frac{h}{2}\boldsymbol{C}^{j}\boldsymbol{R}\,\boldsymbol{D}^{-1}\boldsymbol{J}_{(\pm\frac{h}{2})} \tag{3}$$

Then, the current across the interface between two adjacent nodes (*n*) (*n+1*) is related to the average fluxes in such nodes by:

$$\boldsymbol{J}_{n\to n+1} = -\frac{2}{h}\left[\boldsymbol{f}^{-1}_{n\,(+\frac{h}{2})} \boldsymbol{A}^{j}_{n} + \boldsymbol{f}^{-1}_{n+1\,(-\frac{h}{2})} \boldsymbol{A}^{j}_{n+1} \right]^{-1} \left[\boldsymbol{f}^{-1}_{n+1\,(-\frac{h}{2})} \boldsymbol{A}^{f}_{n+1\,(-\frac{h}{2})} \bar{\boldsymbol{\phi}}_{n+1} - \boldsymbol{f}^{-1}_{n\,(+\frac{h}{2})} \boldsymbol{A}^{f}_{n\,(+\frac{h}{2})} \bar{\boldsymbol{\phi}}_{n} \right] \quad (4)$$

where: $\boldsymbol{A}^{j} = \boldsymbol{R}^{-1} \boldsymbol{C}^{j} \boldsymbol{R}$ is a matrix depending only on the node, not on the interface

$\boldsymbol{A}^{f}_{(\pm\frac{h}{2})} = \boldsymbol{R}^{-1} \boldsymbol{C}^{f}_{(\pm\frac{h}{2})} \boldsymbol{R}$ is a matrix depending on the nodal interface because of the transverse leakage

To calculate the contribution of the transverse leakage, the transverse leakage profile $\boldsymbol{L}(x)$ has to be known. After analyzing lower and higher order profiles we have confirmed that a quadratic polynomial fit of transverse currents is quite good.

The above-defined interface flux discontinuity factors (IFDF) depend on the node, the nodal interface and the energy group. Since IFDF are characteristic of each interface, accuracy is maintained even if the intranodal heterogeneity is non-symmetric (which is always the case if the node is a quarter of fuel assembly).

Moreover, the IFDF depend on the neutronic and thermal-hydraulic properties of the node in the core as well as its boundary conditions. The detailed analysis of these dependencies on the main heterogeneity variables was carried out [5] [6]. And these heterogeneity effects were represented by means of local variables (neutronic and thermal-hydraulic), as well as additional generalized variables (accounting for the flux distribution and intranodal burnup gradients). The latter are non-linear since are a function on the own flux solution. The suitable variable choice yields to linear functional dependencies and therefore only a reduced number of conditions are needed to synthesize the IFDF dependencies on the heterogeneity effects.

The so-synthesized dependencies for each type of node are included in a cross-section library per node type and are used in the multidimensional coarse mesh diffusion calculation to interpolate, node-by-node at the actual local conditions and locations, the 2-G parameters of the actual core configuration. In other words, intranodal heterogeneity is included in the own cross sections and IFDF, through local and non-linear feedbacks, instead of being included in the nodal calculation algorithm.

3. NUMERICAL RESULTS AND DISCUSSION

The new nonlinear iterative scheme has been implemented in our 2-G SIMULA2D code for two-dimensional PWR steady-state calculations, before being implemented in our 3-D transient SIMTRAN [7] code.

SIMULA2D solves the 2-G diffusion equation through a CMFD scheme, based on the Eq. (4). The problem is solved simultaneously for both neutron energy groups. The problem involves a linear system with a block penta-diagonal matrix, where each block is a full 2x2 matrix.

In SIMULA2D, the PWR reflector has been replaced by consistent relationships between the nodal leakage and the node average fluxes. These are formulated as a function of response matrix and heterogeneity factors in the core-reflector interface. Both of these parameters take into account their variability with the heterogeneity effects.

The applicability and accuracy of this method were first tested on the first cycle of Ascó II NPP, which is a Westinghouse-type PWR rated at 2775 MW (thermal). The loading pattern contains seven different types of 17x17 fuel assemblies, identified by the enrichment and the number of burnable poison pins. In all calculations, the solution of the new nonlinear iterative scheme of SIMULA2D are compared first, with that of the 2-D mesh-fine transport-corrected diffusion COBAYA code (reference code), and second with that of the former conventional nodal method in our code system [6], called hereafter PRE-SIMULA2D. The nodal libraries to be used by SIMULA2D were generated for each type of fuel assembly, and include all the local and generalized derivatives.

3.1. CONVERGENCE RATE

Table I illustrates the iteration procedure, in double precision, of SIMULA2D (including the generalized ACMFD method) at beginning of life/hot-zero power. The convergence looks quite satisfactory. The big deviations in the first iteration are due to the initialization with single-assembly 2-G constants, without non-linear corrections. The convergence along the burnup cycle is illustrated in Figure 1.

Table I. SIMULA2D iteration procedure results at 0.GWd/t and HZP

Iteration	Core Keff	Maximum Relative Differences (%) in nodal power
1	.9845774	14.87
5	.9947825	1.04
10	.9948250	0.02
15	.9948259	0.01
20	.9948258	0.00

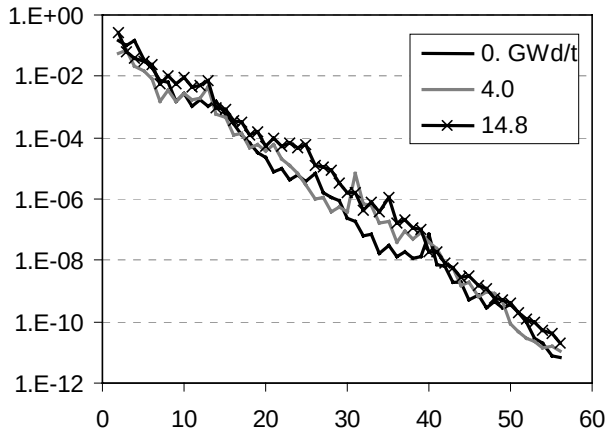

Figure 1. Convergence ratio: maximum nodal deviation versus iteration number (double precision), at the beginning, middle and end of the cycle.

3.2. COMPARISONS WITH REFERENCE CODE

COBAYA reference code [8] performs two-group detailed pin-by-pin 2-D calculations of full core planes, including the radial reflector with shroud. This code has been widely validated by comparison with collision probability methods in detailed 2-D geometry, international benchmarks and measured data in about 60 cycles of 6 PWR units operating in Spain.

Illustrative results of the comparison between SIMULA2D and reference code are given in Table II. Computing times along the cycle burnup are shown. Errors in boron concentration (ppm) as well as errors in powers are presented too. The results indicate that the mean error in power is < 0.5% and the relative percent discrepancy in maximum average power per assembly never overpasses 0.5 %.

Table II. Comparison of SIMULA2D Nodal Solutions with Reference Calculations at HFP

Cycle burnup (GWd/t)	Δppm (1)	CPU time (s) (2)		RMS (%) error in power	% error in max. average power per assembly
		Reference	Nodal		
0.	-3	129.2	1.69	0.27	-0.08
1.	-3	128.5	1.81	0.30	-0.32
4.	-4	117.8	1.83	0.26	0.16
10.	-4	124.7	1.85	0.25	0.25
14.8	-1	130.9	1.86	0.32	0.01

(1) $\Delta ppm = ppm^{reference} - ppm$
(2) Solution time on HP-715 to achieve a relative error of 10^{-6} for the multiplication factor and a relative maximum error of 10^{-5} for the source. The total time includes libraries access and all calculations in the iteration process, including non-linear feedbacks.

A detailed radial power distribution comparison over the whole core at the beginning of the burnup cycle is shown in following figures. Figure 2 shows deviations using nodal parameters (both nodal homogeneized two-group cross sections and interface flux discontinuity heterogeneity factors) for each node equal to those obtained in the corresponding single-assembly calculations. At the beginning of the cycle and full power, relative percent discrepancy is almost 2%, and increases with the cycle burnup.

	-0.24	-1.87	-1.78	-0.18	-0.11	-1.55	-1.42	0.25	0.32	-0.47	-0.48	0.89	1.11	0.75	0.41
		-0.22	-0.22	-1.51	-1.45	0.02	0.13	-1.11	-0.90	0.58	0.83	-0.87	0.07	1.24	0.46
			-0.21	-1.43	-1.35	0.02	0.11	-1.00	-0.81	0.62	0.92	-0.19	0.93	1.27	0.23
				-0.10	-0.03	-1.20	-1.07	0.39	0.52	-0.62	0.18	0.96	1.05		
					0.02	-1.11	-0.97	0.44	0.59	0.06	0.93	0.98	0.36		
						0.31	0.38	-0.77	-0.02	0.06	0.71	1.31	0.54		
							0.45	-0.56	0.34	0.46	1.14	1.00	0.21		
								0.99	1.06	0.89	0.89				
									0.93	0.51	-0.03				

Figure. 2. Errors (%) in power distributions to 0. GWd/t, HFP, ARO, with both cross sections and interface flux discontinuity heterogeneity factors from assembly calculations.

When intranodal gradient and flux-spectrum distribution effects are considered only into the cross sections, the error decreases until about 1% (Figure 3). When these non-linear effects are included also in the IFDF, Figure 4 shows that maximum errors in nodal power are less than 0.8%, and it has been noticed that the assemblies facing the core baffle with two interfaces do not have the higher errors.

0.14	0.76	0.78	-0.03	-0.07	0.65	0.60	-0.23	-0.32	0.35	0.32	-0.63	-1.06	0.33	-0.60
	0.08	-0.01	0.67	0.62	-0.12	-0.18	0.48	0.45	-0.37	-0.59	0.59	0.39	0.24	-0.52
		-0.08	0.68	0.64	-0.21	-0.28	0.50	0.43	-0.46	-0.72	0.40	0.28	-0.04	-0.58
			-0.17	-0.21	0.57	0.52	-0.32	-0.42	0.23	-0.03	0.38	-0.38		
				-0.27	0.53	0.47	-0.40	-0.61	0.06	-0.12	0.03	-0.82		
					-0.29	-0.38	0.34	0.15	0.36	0.16	0.09	-0.54		
						-0.50	0.28	0.14	0.19	0.09	-0.46	-0.68		
							-0.46	-0.79	0.43	-0.48				
								-0.98	-0.23	-0.89				

Figure. 3. Errors (%) in power distributions to 0. GWd/t, HFP, ARO, with non-linear effects in cross sections but interface flux discontinuity heterogeneity factors from assembly calculations.

0.77	-0.07	-0.05	0.43	0.39	-0.14	-0.17	0.33	0.27	-0.28	-0.20	0.34	0.45	-0.64	-0.38
	0.43	0.30	0.15	0.12	0.21	0.18	0.07	0.10	0.21	0.36	-0.46	-0.19	-0.52	-0.45
		0.21	0.12	0.11	0.10	0.08	0.07	0.06	0.10	0.32	-0.50	0.12	-0.21	-0.26
			0.13	0.09	0.07	0.04	0.05	0.03	-0.08	0.04	-0.22	-0.31		
				0.07	0.02	0.02	0.04	-0.03	-0.15	0.01	-0.19	-0.35		
					0.06	0.03	-0.03	-0.05	0.34	0.22	-0.26	-0.34		
						0.02	-0.03	0.08	0.17	0.32	-0.32	-0.22		
							0.17	0.34	-0.37	-0.38				
								0.75	-0.53	-0.46				

Figure. 4. Errors (%) in power distributions to 0. GWd/t, HFP, ARO, with non-linear effects in both cross sections and interface flux discontinuity heterogeneity factors.

3.3. COMPARISONS WITH FORMER NODAL METHOD

Table III let us compare the performance of SIMULA2D using ACMFD versus PRE-SIMULA2D using our previous conventional nodal method. It is interesting to see how the computing times in SIMULA2D were reduced with respect to the previous nodal solution.

Table III. Comparison of PRE-SIMULA2D Nodal Solutions with Reference Calculations at HFP

Cycle burnup (GWd/t)	Δppm (1)	CPU time (s) (2)		RMS (%) error in power	% error in max. average power per assembly
		Reference	Nodal		
0.	-2	129.2	3.69	0.51	-0.66
1.	-1	128.5	4.69	0.40	-0.40
4.	-5	117.8	2.78	0.40	-0.56
10.	-4	124.7	5.58	0.51	-0.33
14.8	-2	130.9	6.53	0.68	-0.43

4. CONCLUSIONS

In summary, the test results demonstrate that the new nodal ACMFD method overcomes the limitations of the ACMFD method strictly valid for nodewise-constant nodes. This method allows us to treat the heterogeneity effects on the nodal parameters explicitly, as a function of the heterogeneity variables inside the node. Therefore, nonlinear two-node problems do not need to be solved during the CMFD calculation. In this way, the computational efficiency is very high, since the nodal parameter feedback method has been shown as a very effective way of accounting for the nonlinear effects.

5. REFERENCES

1. K.S. Smith, "Nodal Method Storage Reduction by Nonlinear Iteration", *Trans. Am. Nuc. Soc*., **44**, pp. 265-266 (1983).
2. P.J. Turinsky et al., "NESTLE: A Few-Group Neutron Diffusion Equation Solver", EGG-NRE-11406, North Carolina State University (June 1994).
3. Y.A. Chao, "A Theoretical Analysis of the Coarse Mesh Finite Difference Representation in Advanced Nodal Methods", *Proc. Mathematics and Computation, Reactor Physics and Environmental Analysis in Nuclear Applications*, Madrid, Spain, September 27-30, **Vol. 1**, pp. 117-126 (1999).
4. Y.A. Chao, "Coarse Mesh Finite Difference Methods and Applications", *Proc. ANS Int. Top. Meeting on Advances in Reactor Physics and Mathematics and Computation into the Next Millennium*, 9404, PHYSOR 2000, Pittsburgh PA, USA, May 7-12 (2000).
5. N. García-Herranz, O. Cabellos, J.M. Aragonés, C. Ahnert, "Analytic Coarse Mesh Finite Difference Method for Heterogeneous Multidimensional Two-Group Diffusion Calculations", submitted to *Nucl. Sci. Eng*.
6. N. García-Herranz, J.M. Aragonés, O. Cabellos, C. Ahnert, "Dependence of the Nodal Homogenized Two-Group Cross Sections on Intranodal Flux-Spectrum, Burnup and History", *Proc. Mathematics and Computation, Reactor Physics and Environmental Analysis in Nuclear Applications*, Madrid, Spain, September 27-30, **Vol. 1**, pp. 127-138 (1999).
7. J.M. Aragonés, C. Ahnert, O. Cabellos, "Methods and Performance at the 3-D PWR Core Dynamics SIMTRAN Online Code", *Nuc. Sci. Eng*., **124**, 111-124 (1996).
8. O. Cabellos et al., "Generalized Effects in Two Group Cross Sections and Discontinuity Factors in the DELFOS Code for PWR Cores", *Proc. Mathematics and Computation, Reactor Physics and Environmental Analysis in Nuclear Applications*, Madrid, Spain, September 27-30, **Vol. 1**, pp. 700-709 (1999).