SOLVING OF MULTIGROUP NON-STATIONARY TRANSPORT EQUATION IN AXIAL R-Z GEOMETRY ON GRIDS FORMED BY THE ARBITRARY CONVEX QUADRANGLES

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ABSTRACT

The numerical method for solving the multigroup non-stationary transport equation is developed in two-dimensional axial R-Z geometry on grids formed by arbitrary convex quadrangles. The semi-implicit algorithm for solving the system of non-stationary equations is developed. The conservative finite-difference scheme is obtained by the integro-interpolated method. The balance equation is augmented by linear approximations. The proposed additional relationships provide the second order of approximation at any side-visible cases under the correspondent choice of the weights of scheme. Number of additional relationships over spatial variables and their form are determined depending on one, two or three side-visible cases. The additional relationships over the time and angle variables are diamond-difference-like approximations relating the edge values to the cell-centered values. Testing of the algorithm for solving the stationary transport equation is based on the comparison of the numerical results obtained by the developed technique with the results obtained by the 1D codes KIN1D (KIAM, Russia) and ANISN (USA) by using the spherical symmetrical one-dimensional problems. The special analytical Benchmarks are developed for testing the non-stationary technique. The testing has shown the good coincidence of the results.

1. INTRODUCTION

When solving numerically the problems in complicated geometry domains there may appear the necessity to use the method of high order of accuracy. Such problems appear, for example, in calculation of accidents connected with melting of core in nuclear reactors and possible secondary criticality on prompt neutrons. In the paper [1] the conservative finite-difference scheme for two-dimensional transport equation in axial R-Z geometry on non-regular spatial grids consisting of convex quadrangles was constructed. On some special grids this scheme

converges with the second order of accuracy. At the same time on the arbitrary quadrangle grid the scheme has the first order of accuracy. In the paper [2] the additional relationships over spatial variables are improved. As a result the scheme provides the second order of approximation at any side-visible cases using the correspondent choice of weights.

In this paper the conservative semi-implicit finite-difference scheme for solving two-dimensional transport equation in R-Z geometry on grids formed by arbitrary convex quadrangles whose connectivity is logically rectangular is proposed. This scheme possesses by the simpler numerical algorithm in comparison with the numerical algorithm in the paper [2]. Additional relationships are formulated of another kind. Nevertheless, as well as in the method [2], there provide the second order of approximation at any side-visible cases under the correspondent choice of the weights of scheme.

The analytical Benchmark is considered [3] for testing of the non-stationary algorithm. The numerical results of solving the stationary and non-stationary transport equation demonstrating efficiency of proposed method are given.

2. PROBLEM DEFINISION

The neutron transport equation is written in two-dimensional axial R-Z geometry as follow:

$$\frac{1}{V^g}\frac{\partial \Psi^g}{\partial t}+\frac{\mu}{r}\frac{\partial (r\Psi^g)}{\partial r}-\frac{1}{r}\frac{\partial (\eta\Psi^g)}{\partial \varphi}+\xi\frac{\partial \Psi^g}{\partial z}+\Sigma_t^g(r,z)\cdot\Psi^g(r,z,\xi,\varphi,t)=S^g(r,z,\xi,\varphi,t) \qquad (1)$$

The right side of equation (1) is the following:

$$S^g(r,z,\xi,\varphi,t)=\frac{1}{2\pi}\cdot\sum_{h=1}^{NG}\int_{-1}^{1}\int_{0}^{\pi}\mathrm{P}^{h\to g}(r,z,\xi,\varphi,\xi',\varphi')\Psi^h(r,z,\xi',\varphi',t)\partial\xi'\partial\varphi'$$

$$+\chi_p^g(1-\sum_{d=1}^{ID}\beta_d)\sum_{h=1}^{NG}\nu\Sigma_f^h\Phi^h(r,z,t)+\frac{1}{2\pi}\sum_{d=1}^{ID}\chi^{dg}\lambda_d\ C_d(r,z,t)+\frac{1}{2\pi}Q^g(r,z,t), \qquad (2)$$

The equations for the delayed neutron precursor concentrations are:

$$\frac{\partial C_d}{\partial t}=-\lambda_d C_d(r,z,t)+\sum_{h=1}^{NG}\beta_d\nu\Sigma_f^h(r,z)\Phi^h(r,z,t)$$

where

$\Psi^g(r,z,\xi,\varphi,t)$ - angular flux in spatial point (r,z) at direction $\vec{\Omega}=(\xi,\varphi)$, at time t and in energy group g, $g=1,2,\ldots,NG$

$\Phi^g(r,z,t)$ - scalar flux in spatial point (r,z) at time t and in energy group g:

$$\Phi^g(r,z,t) = \frac{1}{2\pi}\int_{-1}^{1}\int_{0}^{\pi}\Psi^g(r,z,\xi,\varphi,t)d\varphi d\xi$$

$C_d(r,z,t)$ - precursor concentration for delayed neutron precursor group d in point (r,z) at time t, $d = 1,2,...,ID$

$Q^g(r,z,t)$ - given source in spatial point (r,z) at time t and energy group g.

The other designations are generally accepted.

The boundary condition for equation (1) is given (in particular zero) on the external surface for directions $(\vec{\Omega},\vec{n}) < 0$:

$$\Psi^g(r,z,\xi,\varphi,t)\Big|_{(r,z)\in\Gamma,\vec{\Omega}\cdot\vec{n}<0} = \Psi^g_\Gamma(r_\Gamma,z_\Gamma,\xi,\varphi,t), \tag{3}$$

where Γ - the element of mass of rotation, $\vec{n}$ - external normal to element Γ. The additional boundary conditions implying from the equation (1) are used in point $r = 0$:

$$\frac{\partial\Psi^g}{\partial\varphi}(0,z,\xi,\varphi,t) = 0 \tag{4}$$

and in direction $\varphi = \pi$:

$$\frac{1}{V^g}\frac{\partial\Psi^g}{\partial t} + \frac{\mu}{r}\left[\frac{\partial(r\Psi^g)}{\partial r} - \Psi^g\right] + \xi\frac{\partial\Psi^g}{\partial z} + \Sigma_t^g(r,z)\cdot\Psi^g(r,z,\xi,\pi,t) = S^g(r,z,\xi,\pi,t) \tag{5}$$

Initial conditions are:

$$\Psi^g(r,z,\xi,\varphi,t_0) = \Psi_0^g(r,z,\xi,\varphi) \tag{6}$$

3. SEMI-IMPLICIT METHOD FOR SOLVING OF MULTIGROUP NON-STATIONARY TRANSPORT EQUATION IN AXIAL R-Z GEOMETRY ON ARBITRARY CONVEX QUADRANGLES

The specific character of a considered non-stationary problem is the system of these non-stationary equations belongs to a class of "stiff" systems. It means, that both fast and slow variable components are present at its solution. The slow component determines the character of solution behavior through large periods. It is known that the explicit methods for solving such systems require very small time-step sizes. The implicit methods permit to make numerical integration with larger time step sizes.

For solving the system of non-stationary equations the semi-implicit method is developed. In general the transport equation (1) must be solved iteratively at each time step $\Delta t_k = t_{k+1/2} - t_{k-1/2}$. We have both inner iterations (to converge the scattering) and outer iterations (to converge the emission term). In semi-implicit method the transport equation (1) is solved iteratively at the intermediate time t_k, taking the fission term and initial fluxes at the low time $t_{k-1/2}$. The noniterative diamond-difference-like approximation is used to obtain the solution (angular flux) at the upper time $t_{k+1/2}$. It is sufficient to consider one iteration step at one time interval given that the right side (2) is known.

Let's consider a spatial domain covered by a quadrilateral grid $\{r_{i,j}, z_{i,j}\}$ assuming constant media properties in every quadrilateral cell. Let $\vec{\Omega}_{m,l}$ be a fixed direction of angular ES_n quadrature and t_k be a fixed intermediate time. For simplification of description we omit an index of group g everywhere below. Referring to figure 1, the grid values of angular flux Ψ in vertexes of a quadrangle we designate as $\overset{k}{\Psi}_{Pi,m,l} \equiv \overset{k}{\Psi}_{m,l}(r_i, z_i)$, $i = 1 \div 4$, on edges as $\overset{k}{\Psi}_{P(i,i+1),m,l}$ and at center of a quadrangle as $\overset{k}{\Psi}_{P0,m,l}$.

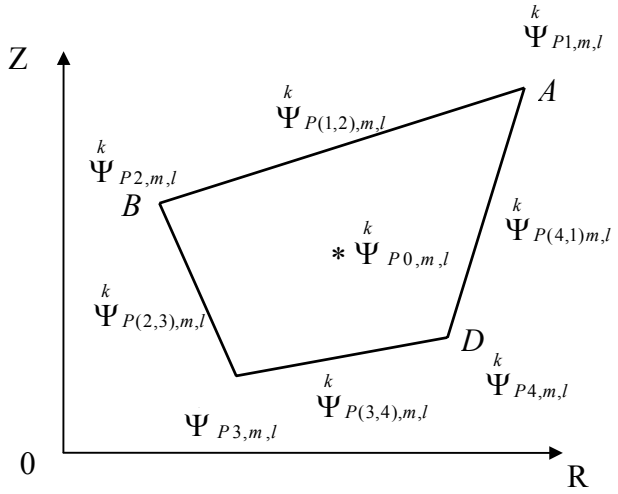

Figure 1. The values of function $\overset{k}{\Psi}_{m,l}$ in quadrangle *ABCD*.

To construct the conservative finite-difference scheme the integro-interpolated method and the formula of Gauss-Octrogradskii are used. Integrating (1) with the weight r over the quadrangle with coordinates of vertexes (r_i, z_i), $i = 1 \div 4$ and within the limits of a difference cell $[t_{k-1/2}, t_{k+1/2}] \times [\xi^-, \xi^+] \times [\varphi^-, \varphi^+]$, canceling by $\Delta t_k \Delta \xi_m \Delta \varphi_{m,l}$ we obtain the following balance equation:

$$\frac{\Delta v}{V}\left[\frac{\overset{k+1/2}{\Psi}_{P0,m,l}-\overset{k-1/2}{\Psi}_{P0,m,l}}{\Delta t_k}\right]+\sum_{i=1}^{4} R_{P(i,i+1),m,l}\overset{k}{\Psi}_{P(i,i+1),m,l}+\frac{\Delta s}{\omega_{m,l}}(\alpha^{+}\overset{k}{\Psi}_{\varphi}^{+}-\alpha^{-}\overset{k}{\Psi}_{\varphi}^{-})+\Sigma_t\overset{k}{\Psi}_{P0,m,l}\Delta v=\overset{k}{S}_{P0,m,l}\Delta v \quad (7)$$

where

$$R_{P(i,i+1),m,l}=\frac{1}{2}(r_i+r_{i+1})\left[\mu_{m,l}(z_{i+1}-z_i)-\xi_m(r_{i+1}-r_i)\right]$$

Here we use the following designations:

$$\overset{k}{\Psi}_{P(4,5),m,l}=\overset{k}{\Psi}_{P(4,1),m,l},\ R_{P(4,5),m,l}=R_{P(4,1),m,l},\ \omega_{m,l}=\Delta\xi_m\Delta\varphi_{m,l},$$

$$\Delta t_k=t_{k+1/2}-t_{k-1/2},\ \Delta\xi_m=\xi^{+}-\xi^{-},\ \xi^{\pm}=\xi_{m\pm1/2},\ \Delta\varphi_{m,l}=\varphi^{+}-\varphi^{-},\ \varphi^{\pm}=\varphi_{m,l\mp1/2},$$

$$\overset{k}{\Psi}_{\varphi}^{\pm}=\overset{k}{\Psi}_{P0,m,l\mp1/2},\ \alpha^{\pm}=\alpha_{m,l\mp1/2},\ \mu_{m,l}=\sqrt{1-\xi_m^2}\cos\varphi_{m,l}.$$

The coefficients of the angular derivative term $\alpha_{m,l+1/2}$ for given m and for all l are calculated from the relation:

$$\alpha_{m,l+1/2}=\alpha_{m,l-1/2}-\mu_{m,l}\cdot\omega_{m,l}\quad,\quad \alpha_{m,1/2}=\alpha_{m,L_m+1/2}=0$$

The value Δs is the square of the quadrilateral spatial cell in a plane R-Z and the value Δv is the volume of a domain formed by rotation of a cell round axes Z.

The additional relationships are introduced to close the obtained system of the finite difference equation (7). The relationships over variables φ and t are the diamond-difference-like approximations relating the edge values the cell-centered values:

$$\overset{k}{\Psi}_{\varphi}^{+}=(1+P_{\varphi})\overset{k}{\Psi}_{P0,m,l}-P_{\varphi}\overset{k}{\Psi}_{\varphi}^{-} \quad (8)$$

$$\overset{k+1/2}{\Psi}_{P0,m,l}=(1+P_t)\overset{k}{\Psi}_{P0,m,l}-P_t\overset{k-1/2}{\Psi}_{P0,m,l} \quad (9)$$

where $0\le P_t\le 1,\ 0\le P_{\varphi}\le 1$. The additional linear approximations over spatial variables should be constructed so that for any direction $(\xi_m,\varphi_{m,l})$ they resulted in a nonsingular system of the difference equations and for smooth solutions there should be the possibility to approximate the algebraic identities with the second order of accuracy. Number of additional relationships and their form are depended on the number of sides visible. There are three cases depending on selected direction $\Omega_{m,l}=(\xi_m,\varphi_{m,l})$. The number of visible sides of spatial cell is determined by fulfillment of the condition $(\vec{\Omega},\vec{n})<0$ on their sides (or $R_{P(i,i+1),m,l}<0$). The additional linear approximations over spatial variables are related to the edge, vertex and

cell-centered values simultaneously depending on the number of sides visible. The common weight parameter P ($0 \le P \le 1$) is used in spatial approximations. Number of additional relationships over spatial variables and their form are determined depending on one, two or three side-visible cases.

1. One side is visible (for example, *AB*) with the number $P(1,2)$. In this case the following values are known: $\overset{k}{\Psi}_{P1,m,l}$, $\overset{k}{\Psi}_{P2,m,l}$, $\overset{k}{\Psi}_{P(1,2),m,l}$, and the following values are unknown: $\overset{k}{\Psi}_{P3,m,l}$, $\overset{k}{\Psi}_{P4,m,l}$, $\overset{k}{\Psi}_{P(3,4),m,l}$, $\overset{k}{\Psi}_{P(2,3),m,l}$, $\overset{k}{\Psi}_{P(4,1),m,l}$, $\overset{k}{\Psi}_{P0,m,l}$. Additional linear relationships are obtained by using the linear interpolation of angular flux within the cell and on its sides:

$$\begin{aligned}
&\overset{k}{\Psi}_{P3,m,l} = (1+P)\overset{k}{\Psi}_{P(2,3),m,l} - P\overset{k}{\Psi}_{P2,m,l} \\
&\overset{k}{\Psi}_{P4,m,l} = (1+P)\overset{k}{\Psi}_{P(4,1),m,l} - P\overset{k}{\Psi}_{P1,m,l} \\
&\overset{k}{\Psi}_{P4,m,l} = (1+P)\overset{k}{\Psi}_{P0,m,l} - P\overset{k}{\Psi}_{P',m,l} \qquad (10) \\
&\overset{k}{\Psi}_{P3,m,l} = (1+P)\overset{k}{\Psi}_{P0,m,l} - P\overset{k}{\Psi}_{P'',m,l} \\
&\overset{k}{\Psi}_{P(3,4),m,l} = 0.5\cdot(\overset{k}{\Psi}_{P3,m,l} + \overset{k}{\Psi}_{P4,m,l})
\end{aligned}$$

Here we use in addition three points: $P0$ - the central point of quadrangle with coordinates:

$$r_0 = \frac{1}{4}\left[r_1 + r_2 + r_3 + r_4\right], \quad z_0 = \frac{1}{4}\left[z_1 + z_2 + z_3 + z_4\right];$$

P' - the point of intersection of straight line, which goes through the points $P4$ and $P0$, with the nearest side of quadrangle; P'' - the point of intersection of straight line, which goes through the points $P3$ and $P0$, with the nearest side of quadrangle. The values of function Ψ at the points P' and P'' are determined by the linear interpolation of the values of Ψ in vertexes lying on the side with these points. To solve the system (7) – (10) it is necessary to add two linear relationships with respect to unknowns $\overset{k}{\Psi}_{P',m,l}$ and $\overset{k}{\Psi}_{P'',m,l}$ depending on the case of placement of the points P' and P''.

2. Two sides are visible (for example, *AB* and *AD*) with the numbers $P(1,2)$ and $P(1,4)$ respectively. In this case the following values are known: $\overset{k}{\Psi}_{P1,m,l}$, $\overset{k}{\Psi}_{P2,m,l}$, $\overset{k}{\Psi}_{P4,m,l}$, $\overset{k}{\Psi}_{P(1,2),m,l}$, $\overset{k}{\Psi}_{P(4,1),m,l}$, and the following values are unknown: $\overset{k}{\Psi}_{P3,m,l}$, $\overset{k}{\Psi}_{P(3,4),m,l}$, $\overset{k}{\Psi}_{P(2,3),m,l}$, $\overset{k}{\Psi}_{P0,m,l}$. Additional linear relationships are obtained by using the linear interpolation of flux within the cell and on its sides:

$$\overset{k}{\Psi}_{P3,m,l} = (1+P)\overset{k}{\Psi}_{P(2,3),m,l} - P\overset{k}{\Psi}_{P2,m,l}$$

$$\overset{k}{\Psi}_{P3,m,l} = (1+P)\overset{k}{\Psi}_{P0,m,l} - P\overset{k}{\Psi}_{P'',m,l} \qquad (11)$$

$$\overset{k}{\Psi}_{P3,m,l} = (1+P)\overset{k}{\Psi}_{P(3,4),m,l} - P\overset{k}{\Psi}_{P4,m,l}$$

Here we use the point P'' of intersection of straight line, which goes through the points $P3$ and $P0$, with the nearest side of quadrangle. The value of function Ψ at the point P'' is determined by the linear interpolation of the values of Ψ in vertexes lying on the side with this point. To solve the system (7) – (9) and (11) it is necessary to add linear relationship with respect to unknown $\overset{k}{\Psi}_{P'',m,l}$ depending on the case of placement of the point P''.

3. Three sides are visible (for example, *AB*, *BC* and *AD*) with the numbers $P(1,2)$, $P(2,3)$ and $P(1,4)$ respectively. In this case the following values are known: $\overset{k}{\Psi}_{P(1,2),m,l}$, $\overset{k}{\Psi}_{P(2,3),m,l}$, $\overset{k}{\Psi}_{P(4,1),m,l}$, $\overset{k}{\Psi}_{P1,m,l}$, $\overset{k}{\Psi}_{P2,m,l}$, $\overset{k}{\Psi}_{P3,m,l}$, $\overset{k}{\Psi}_{P4,m,l}$, and the following values are unknown: $\overset{k}{\Psi}_{P(3,4),m,l}$, $\overset{k}{\Psi}_{P0,m,l}$. The additional linear relationship is used:

$$\overset{k}{\Psi}_{P(3,4),m,l} = (1+P)\overset{k}{\Psi}_{P0,m,l} - P\overset{k}{\Psi}_{P(1,2),m,l} \qquad (12)$$

The boundary conditions are given for those edges of quadrilateral cell, for which $R_{P(i,i+1),m,l} < 0$ that is equivalent to a condition $(\vec{\Omega}, \vec{n}) < 0$. To approximate the boundary conditions (4) on adjoining to axes Z edges and vertexes of quadrangles we use the following relationships at the point $r = 0$:

$$\overset{k}{\Psi}_{P(i,i+1)}(\xi_m, \varphi_{m,l}) = \overset{k}{\Psi}_{P(i,i+1)}(\xi_m, \pi - \varphi_{m,l}), \qquad \overset{k}{\Psi}_{Pi}(\xi_m, \varphi_{m,l}) = \overset{k}{\Psi}_{Pi}(\xi_m, \pi - \varphi_{m,l})$$

To approximate the boundary condition (5) in the case $\varphi_{m,1/2} = \pi$ the integro-interpolated method is used by the similar way:

$$\frac{\Delta v}{V}\left[\frac{\overset{k+1/2}{\Psi}_{P0,m,1/2} - \overset{k-1/2}{\Psi}_{P0,m,1/2}}{\Delta t_k}\right] + \sum_{i=1}^{4} R_{P(i,i+1),m,1/2}\overset{k}{\Psi}_{P(i,i+1),m,1/2} + \Delta s\left|\mu_{m,1/2}\right|\overset{k}{\Psi}_{P0,m,1/2} + \Sigma_t \overset{k}{\Psi}_{P0,m,1/2}\Delta v = \overset{k}{S}_{P0,m,1/2}\Delta v$$

This finite-difference equation together with the additional relationships over spatial variables and over time is used for solving the equation (5) for direction $\varphi_{m,1/2} = \pi$.

So, the finite-difference equation (7) together with the additional relationships (8) – (12) and initial and boundary conditions form the closed system of equation for solving the transport equation (1).

4. NUMERICAL RESULTS

4.1 STATIONARY PROBLEM

Now we present the numerical results of solving the stationary transport equation by the proposed scheme in axial R-Z geometry performed by KIN2D (KIAM, Russia) code. We consider one-group S_6 problem for 1D homogeneous sphere with radius $\tilde{R} = 1.5$. The parameters were adopted from [1]: $\Sigma_s(r,z)=1.5$, $\nu\Sigma_f(r,z) = 0$, $Q(r,z)=1$. The nodes over R are given:

$$R_k = 0.15k,\ k = 0,\ldots,8,\ R_9 = 1.332572,\ R_{10} = 1.41069,\ R_{11} = 1.456722,\ R_{12} = 1.483846,\ R_{13} = 1.5.$$

The boundary conditions are vacuum. We use the ESn quadrature set. The quadrangular spatial grid is constructed as follows: over the angle ϑ the quarter of sphere is divided on 9 sectors by the radiuses which are going out from the centre of sphere. In order that the volume of sphere and the region generated by a two-dimensional grid are coincided, the mentioned radiuses are multiplied on appropriate factor.

The flux distribution over some sectors in two-dimensional calculation by the proposed scheme and the flux distribution in one-dimensional calculation with coinciding grid over radius and the GAUSS quadrature over angle performed by KIN1D (KIAM, Russia) and ANISN (USA) codes are given in the Table I.

Table I. Results of proposed scheme.

	Two-dimensional calculation KIN2D					One-dimensional calculation	
Number point over R	$0^0 - 10^0$	$20^0 - 30^0$	$40^0 - 50^0$	$60^0 - 70^0$	$80^0 - 90^0$	KIN1D	ANISN
1	19.06	18.98	18.99	19.05	19.04	19.121	19.120
4	17.14	17.13	17.15	17.18	17.16	17.443	17.442
7	12.63	12.62	12.62	12.62	12.60	12.893	12.892
10	7.05	7.07	7.04	7.05	7.03	7.220	7.220
13	4.89	4.80	4.90	4.83	4.90	4.993	4.993

4.2 NON-STATIONARY PROBLEM

The special analytical Benchmarks are developed [3] for testing time-dependent solutions of neutron transport equation in 2D geometry. In these Benchmarks the analytical solution of system of neutron kinetic equations in diffusion multigroup approximation is used for simplest one-dimensional geometry derived by matrix exponent method. For the homogeneous sphere it is possible to represent the solution of the evolutionary system of equation in general form for three boundary conditions as product of two functions. One of them depends only on time (amplitude function), and other function depends only on spatial variable.

A homogeneous spherical zone with radius $R = 107.355cm$ and a set of macroscopic cross sections represented in Table II was considered, which describes the thermal four-group system. The initial flux distribution was calculated by solving the eigenvalue problem so that the value $K_{ef} = 0.973174$ with the same macro cross sections. The following initial condition for non-stationary problem was used as average neutron fluxes at the centre of spherical domain: $\vec{\Psi}(0,0) = \{91.7324, 14.9640, 0.5588, 1.6013\}$.

Table II. The neutron macroscopic cross sections for diffusion approximation.

Number of group	$\chi^{(g)}$	$V^{(g)}$	Σ_t^g	Σ_s^g	$\nu\Sigma_f^g$	$l \to g$	$\Sigma_s^{l\to g}$
1	1.	10^9	0.1850566	0.1601765	0.005963076	$1\to2$	0.018832
2	0.	10^8	0.4661111	0.3512645	0.061155862	$1\to3$	0.000016
3	0.	$1.2525\cdot10^6$	0.5032365	0.0330615	0.146952910	$2\to3$	0.017480
4	0.	$6.7431\cdot10^5$	1.0734197	0.6415576	0.501519400	$1\to4$	0.000031
						$2\to4$	0.032791
						$3\to4$	0.355210

The test problem was to research the qualitative behavior of the time-dependent solution of transport equation. The following property was used: the proximity of the solution of multigroup diffusion equation in one-dimensional geometry and solution of multigroup transport equation in two-dimensional axial geometry in time near the center of the homogeneous spherical domain (far from the boundary), provided that they are coincide at the initial time. Besides it is possible to compute a damping factor, which one coincides with a leading eigenvalue on asymptotics.

This algorithm was implemented in KIN2DT code. The numerical results on time are shown in Table III. The values of neutron flux in first group in the center of spatial domain are given here for the analytical diffusion solution in 1D spherical geometry. Besides, the corresponding neutron flux obtained by the code KIN2DT on the quadrangular grid and by code KIN1DT (KIAM) for 1D spherical geometry. In the Table III the values of damping factor computed by the code KIN2DT are given also.

Table III. Comparison of numerical and analytical solutions (calculated with $\Delta t = 10^{-8}$ s).

Time (s)	Analytical Solution	1D sphere KIN1DT	2D sphere KIN2DT	Damping factor λ
0.	91,7324	91,7324	91,7324	
10^{-8}	91,142	91,218	91,218	-0,48080*10^{6}
10^{-7}	88,568	88,786	88,786	-0,19153*10^{6}
10^{-6}	84,737	84,827	84,827	-0,19247*10^{5}
3.0*10^{-6}	81,934	82,021	82,021	-0,16825*10^{5}
5.0*10^{-6}	79,186	79,287	79,286	-0,17054*10^{5}
7.0*10^{-6}	76,511	76,623	76,623	-0,17119*10^{5}
9.0*10^{-6}	73,920	74,043	74,042	-0,17138*10^{5}
10^{-5}	72,657	72,785	72,784	-0,17141*10^{5}
3.0*10^{-5}	51,468	51,659	51,657	-0,17145*10^{5}
5.0*10^{-5}	36,458	36,663	36,662	-0,17144*10^{5}
7.0*10^{-5}	25,825	26,021	26,019	-0,17144*10^{5}
9.0*10^{-5}	18,294	18,468	18,466	-0,17144*10^{5}
10^{-4}	15,397	15,558	15,557	-0,17144*10^{5}
3.0*10^{-4}	0,4898	0,5045	0,5043	-0,17144*10^{5}
5.0*10^{-4}	0,01558	0,01636	0,01635	-0,17144*10^{5}

From the Table III it is seen, that the numerical solution on time of two-dimensional transport problem is close to the analytical solution in center of the sphere. Damping factor on the asymptotics $\lambda = -0.17144 * 10^5$ is close to the leading eigenvalue $\omega = -0.17240 * 10^5$ of analytic solution. Analogous results were obtained for cylindrical 1D model. The results of testing of proposed technique demonstrate high performance of this method.

ACKNOWLEDGEMENTES

This work was supported by the Grant № 01-01-00214 of Russian Foundation of Fundamental Researches.

REFERENCES

1. V.E. Troshiev, V.A. Shumilin, "The finite-difference scheme for solving the two-dimensional transport equation on irregular quadrangular grids," *Journal of Computational Mathematics and Mathematical Physics*, N2, **Vol. 26**, pp.230-241 (1986) (in Russian).
2. N.P. Pleteneva, R.M. Shagaliev, "Two-dimensional transport equation approximation on quadrangular and polygonal spatial grids with extended pattern difference scheme," *Problems of nuclear science and engineering, Sec. Techniques and programs of numerical solution of problems of mathematical physics*, Issue 3, pp.34-41 (1989) (in Russian).
3. A.V. Voronkov, E.P. Sychugova, "Non-stationary technique for calculation of the multi-group two-dimensional transport equations on grids, formed by arbitrary quadrangles," KIAM RAS, *Preprint #80*, pp.1-28 (2000) (in Russian).