DEVELOPMENT AND APPLICATION OF THE REGIONAL ANGULAR REFINEMENT TECHNIQUE AND ITS APPLICATION TO NON-CONVENTIONAL PROBLEMS

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ABSTRACT

We have investigated new quadrature sets for the solution of the Boltzmann transport equation for non-conventional problems. For problems where the angular flux and/or source are highly peaked, it is necessary to utilize a high order quadrature set. Moreover, if the physical system contains large regions of low density or highly absorbent materials, the ray-effects will appear in the flux distribution. In these circumstances the Level Symmetric quadrature set is not suitable, because of its limitation to order S_{20}. We discussed in a previous paper these limitations and we developed new quadrature techniques including the EW, P_N-EW and P_N-T_N with a new biasing approach called Ordinate Splitting (OS).

In this paper we derive a new biasing technique, called Regional Angular Refinement (RAR). The discrete directions generated with the RAR technique satisfy the moments of direction cosines of transport equation. We have simulated a CT-Scan device using these new quadrature techniques and benchmarked the results.

1. INTRODUCTION

In this paper we present a new biasing technique called Regional Angular Refinement (RAR). This methodology has been utilized for simulating a CT-Scan device used for medical/industrial applications. A CT-Scan device utilizes a collimated X-ray source (fan-beam) to scan an object or a patient[1]. We utilized the new quadrature sets generated with the RAR technique to simulate a CT-Scan model.

The S_N method is extensively used to solve the neutron transport equation. The angular variable in the transport equation is discretized into a finite number of directions, with the associated weights.[3] The set of directions generated, must meet a number of requirements in order to preserve the physics of the problem. These conditions are the preservation of symmetry and the moments of direction cosines of the transport equation.

The Level Symmetric[3] (LQ_N) and the Equal Weight[3] (EQ_N) quadrature sets are commonly available in S_N codes. The former set preserves the two aforementioned conditions but is limited to a maximum order of S_{20}. The latter set has no limitation on the order, but it only preserves zeroth moment of the direction cosines. Because of this drawback, a high order EQ_N quadrature set must be used to avoid loss of accuracy.

In a previous paper[4], in order to simulate a CT-Scan device, we developed new techniques including P_N-EW, P_N-T_N and a biasing approach referred to as Ordinate Splitting (OS). These techniques provided the possibility of using a high order quadrature set with the possibility of direction refinement (i.e. biasing). Comparing the flux distribution with Monte Carlo predictions[5] indicated that the most effective approach is P_N-T_N with OS. The only shortcoming of this technique, is that for

biasing (i.e. OS), we did not preserve the higher moments of direction cosines, henceforth it may require higher order refinement to avoid loss of accuracy.
In this paper, we propose a new technique, referred to as RAR, which does preserve higher moments of direction cosines.
The remainder of this paper is organized as follows. Section 2 derives the RAR method. Section 3 presents the results and Section 4 gives the summary and conclusions.

2. DERIVATION OF THE REGIONAL ANGULAR REFINEMENT (RAR) TECHNIQUE

The RAR method is developed for solving problems with highly peaked angular flux and/or source. The approach consists of two steps. In the first step, we derive a P_N-T_N quadrature set of arbitrary order on one octant of the unit sphere. In the second step, for a range of polar and azimuthal angles, we fit an additional P_N-T_N quadrature set.
The P_N-T_N quadrature set[4] is derived by setting the ξ levels, on the z-axis of the unit sphere, equal to the roots of Legendre polynomials (P_N). The azimuthal angles on each level are set equal to the roots of the Chebyshev polynomials (T_N) of first kind. The Chebyshev polynomials of first kind have the following formulation:

$$T_l\left[\cos(\omega)\right] \equiv \cos(l\omega) \tag{1}$$

The Chebyshev polynomials are orthogonal and satisfy the following condition:

$$\int_{-1}^{1} dy T_l(y) T_k(y)(1-y^2)^{-1/2} = \begin{cases} 0, l \neq k \\ \pi, l = k = 0 \\ \pi/2, l = k \neq 0 \end{cases} \tag{2}$$
$$y = \cos(\omega)$$

By using the ordering of the LQ_N quadrature set, we set the azimuthal angles on each level using the following formulation:

$$\omega_{l,i} = \left(\frac{2l-2i+1}{2l}\right)\frac{\pi}{2} \qquad \omega_{l,i} \in \left(0, \frac{\pi}{2}\right) \tag{3}$$
$$\mathrm{i} = 1 \ldots \mathrm{l}$$

In Eq. 3, l is the level number. The P_N-T_N set does not present negative weights for S_N orders higher than 20. In Fig. 1 we show a S_{30} P_N-T_N without RAR.

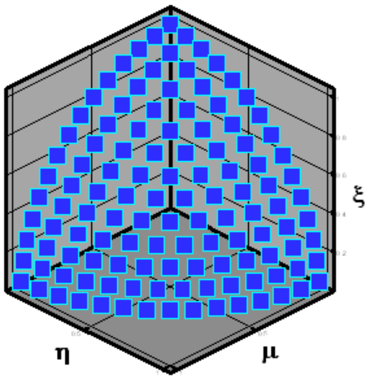

Fig. 1. Discrete directions selected with S_{30} P_N-T_N quadrature set without RAR.

In Fig. 2, we show a S_{12} P_N-T_N quadrature set biased with RAR of order S_{26} P_N-T_N quadrature.

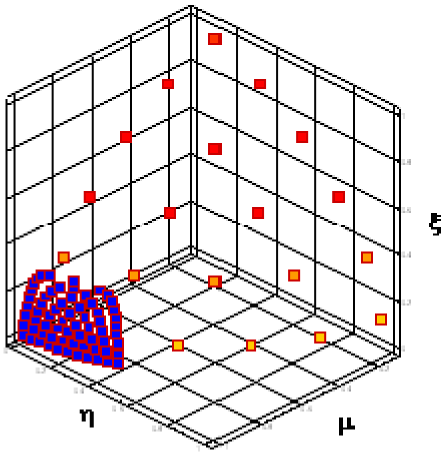

Fig. 2. S_{12} P_N-T_N quadrature set biased with RAR technique.

The P_N-T_N quadrature set fitted in the sub-domain is defined as follows:

$$\begin{aligned} &\xi_i \in \{P_N(\xi) = 0\}_{i=0\ldots N} \\ &0 \le \xi_i \le \xi_{\max} \\ &\varphi_i \in \{T_N(\varphi) = 0\}_{i=0\ldots N} \\ &0 \le \varphi_i \le \varphi_{\max} \end{aligned} \tag{4}$$

In Eq. 4, $\xi_{\max}$ and $\varphi_{\max}$ are the limits of the polar and the azimuthal angles in the biased region.

3. RESULTS

A CT-Scan device is shown in Fig. 3; the main components are a X-ray source mounted on a rotating gantry, doughnut shaped and an array of sensors. The patient is placed on a sliding bed that can be positioned inside the CT-Scan. The full PENTRAN[2] model for the CT-Scan device is shown in Fig. 4; this model includes the rotating gantry along with the array of sensors and the X-ray source.

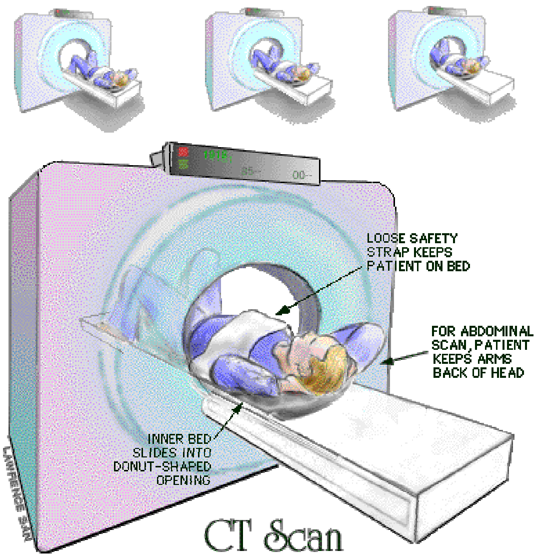

Fig. 3. CT-Scan Device

We tested the RAR technique on a simplified model and the black lines in Fig. 4, show the limits of this model.

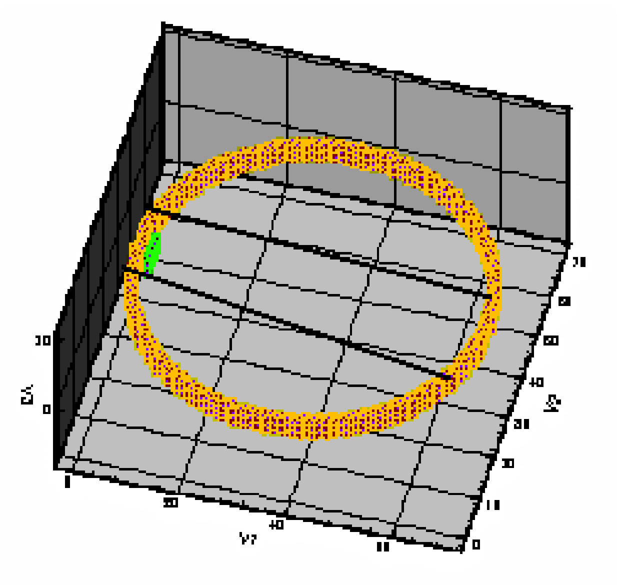

Fig. 4. Full model of the CT-Scan Device

We simulated the simplified model of the CT-Scan using the PENTRAN transport code[2] with the new quadrature sets. Fig. 5 shows the simplified PENTRAN model which represents the X-ray directional source (“fan” beam), a large region of air and a detector. The size of this model is 74 cm along the y-axis and 20 cm along the x-axis. The array of detectors is located at 70 cm from the source along the x-axis.

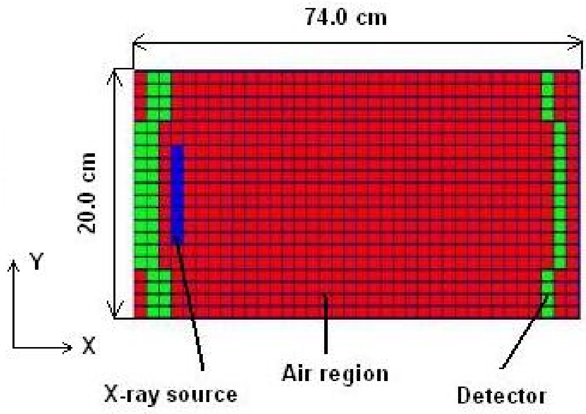

Fig. 5. Simplified CT-Scan model.

Because of the presence of large void regions and a directional source, the solution of the transport equation is affected by ray-effects. One remedy is to use higher order quadrature sets[6] with biasing such as RAR technique. We compared the results to a reference solution obtained with a S_{80} P_N-T_N quadrature set. The RAR technique has been applied to a S_{12} P_N-T_N quadrature set; the biased region on the octant extends from z = 0.0 to z = 0.3 and the azimuthal angle spans from 0 to 25 degrees. In the biased region, we used a S_{26} P_N-T_N quadrature set. The S_{12} P_N-T_N biased with S_{26} P_N-T_N resulted in 110 directions per octant. The S_{80} P_N-T_N quadrature set yielded 820 directions per octant. The S_{20} LQ_N quadrature set yielded 55 directions per octant. We have chosen these parameters based on the knowledge of the X-rays fan-beam. In Figs. 6, 7 and 8 we show the scalar flux at different positions along the x axis. In Fig. 6 we show the scalar flux at x = 30.0 cm from the source, along the y-axis.

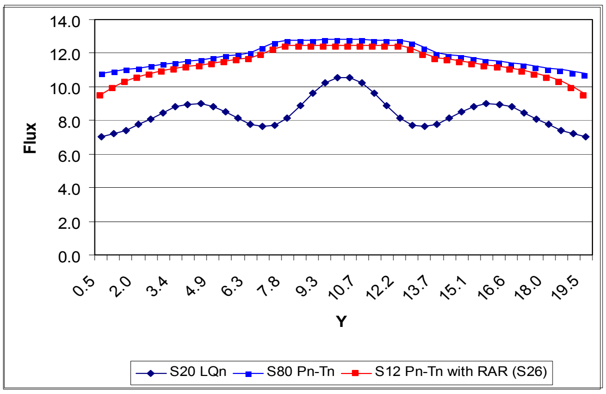

Fig. 6. Scalar flux at x = 30.0 cm from the source.

It is clear that RAR technique is in agreement with the S_{80} P_N-T_N quadrature set, while the solution obtained with the S_{20} LQ_N quadrature set is inaccurate and affected by ray-effects. In Fig. 7 we show the scalar flux at x = 40.0 cm along the y-axis. At these positions, the RAR technique still yields accurate results compared to the S_{80} P_N-T_N solution.

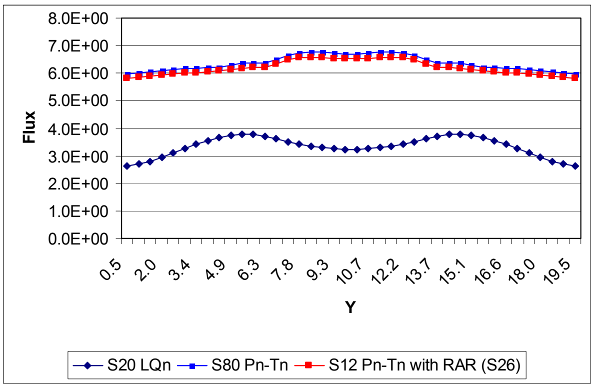

Fig. 7. Scalar flux at x = 40.0 cm from the source.

In Fig. 8, we show the scalar flux at the detector position (x=70.0 cm) along the y-axis. As shown in Fig.8, the S_{12} P_N-T_N biased with S_{26} shows excellent agreement with S_{80} P_N-T_N. Fig. 8 also demonstrates that the S_{20} LQ_N quadrature set underpredicts the flux distribution at the detector position by more than one order of magnitude. It is worth noting that the range of oscillation occurring in the predicted flux distributions from P_N-T_N S_{80} and P_N-T_N S_{12} with RAR is less than 10%.

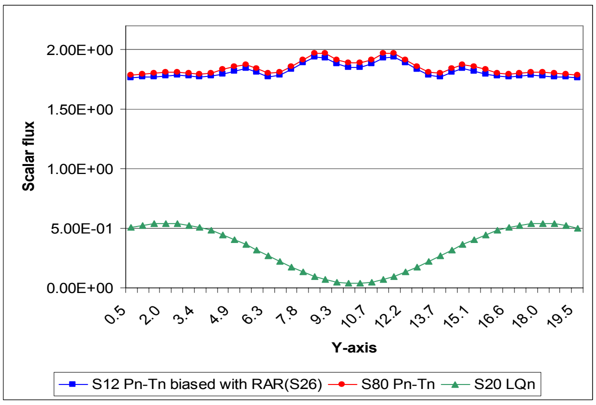

Fig. 8. Scalar flux at detector position (x=70.0 cm).

Table 1 compares the computational times of the three quadrature sets.

Table 1. Computational time and memory required for the simulation[a].

Quadrature Set	Directions[b]	CPU Time(sec)
S_{80} P_N-T_N	6560	682.9
S_{12} P_N-T_N RAR (S_{26})	880	56.3
S_{20} LQ_N	440	25.0[c]

[a] These results have been achieved on a PC-Workstation with 1 GHz Pentium III processor and 256 MBytes RAM.
[b] Total number of directions on the unit sphere.
[c] S_{20} LQ_N underpredicts the results by one order of magnitude.

We can observe that the RAR technique greatly reduces the computational time, by more than one order of magnitude compared to S_{80}, while resulting in accurate solution. Fig. 9 shows a 3-D plot of the scalar flux obtained with the S_{20} LQ_N quadrature set; the ray-effects increase as we move far away from the source.

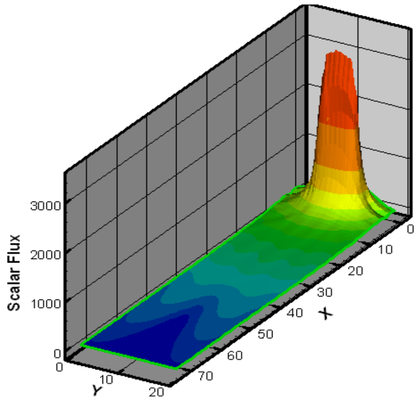

Fig. 9. Plot of the scalar flux calculated with S_{20} LQ_N quadrature set.

Fig. 10 shows the scalar flux calculated with the S_{12} P_N-T_N with S_{26} RAR. In this case the solution exhibits small ray-effects, even far away from the source.

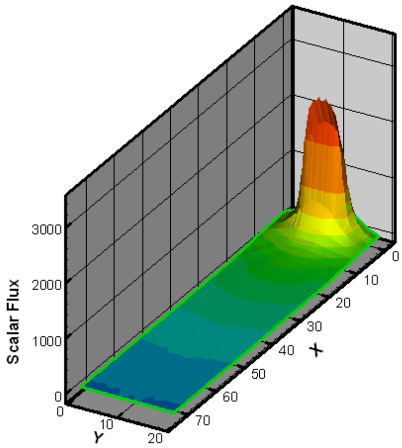

Fig. 10. Plot of the scalar flux calculated with S_{12} P_N-T_N quadrature set biased with S_{26} RAR.

4. CONCLUSIONS

We have developed a new technique referred to as Regional Angular Refinement (RAR) for use in the discrete ordinates codes. We have demonstrated that this methodology is effective in improving the solution of the Boltzmann transport equations for non-conventional problems with large regions of low density material. For a CT-Scan problem, which contains large void regions, we have achieved accurate solutions with practically no ray-effects in a relatively short time. We have concluded that for regional refinements, RAR is more effective than our previous technique referred to as the Ordinate Splitting (OS).

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