ANALYSES OF EXPERIMENTS FOR RELATIONSHIP BETWEEN FLUX TILT IN TWO-ENERGY-GROUP AND EIGENVALUE SEPARATION

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ABSTRACT

For a relationship between the flux tilt in two-energy-group and the first-mode eigenvalue separation, it was found that the flux tilt in two-energy-group is proportional to the perturbed reactivity and inversely proportional to the first-mode eigenvalue separation on the basis of the Modified Explicit Higher Order Perturbation method (the MHP method) with two-energy-group. To show the validity of the relationship, experiments were carried out in a coupled-core at the Kyoto University Critical Assembly (KUCA), and numerical calculations were performed for inferring the eigenvalue separation from the perturbed reactivity and the degree of flux tilt induced by a perturbation. In the coupled-core at the KUCA C-core, comparing with the results of the experiments and the numerical calculations, it was shown that the validity of the relationship is confirmed and that the flux tilt is independent on the neutron energy.

1. INTRODUCTION

The perturbation theory leads to a reactivity expression which indicates significant and interesting physical interpretations that an inserted perturbation excites not only the

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fundamental-mode component but also the higher-mode ones. The relationship between the flux tilt and the eigenvalue separation was first presented by R. A. Rydin *et al.* [1], under an assumption that all higher-mode components excluding the first-mode one were neglected on the basis of the first-order perturbation theory. From this relationship, the eigenvalue separation can be easily inferred from the degree of flux tilt when the perturbed reactivity is known, and it has been analyzed by numerical calculations and observed through experiments in several reactor cores [2-5].

For a relationship [6] between the flux tilt in two-energy-group and the first-mode eigenvalue separation, it was found that the flux tilt in two-energy-group is proportional to the perturbed reactivity and inversely proportional to the first-mode eigenvalue separation on the basis of the Modified Explicit Higher Order Perturbation Method (the MHP method [7]) with two-energy-group. For inferring the eigenvalue separation from the degree of flux tilt, through numerical simulations, one could strictly evaluate the neutron energy dependence of the flux tilt by using the MHP method with two-energy-group, and could confirm the validity of an ordinary methodology with one-energy-group [8] for inferring the eigenvalue separation.

The purpose of this paper is to show the validity of the proposed relationship through experiments and analyses of experiments in a coupled-core at the Kyoto University Critical Assembly (KUCA), and finally to examine the neutron energy dependence of the flux tilt obtained from the experiments.

In Sec. 2, a formulation of flux tilt in two-energy-group is briefly derived by using the MHP method, and the relationship between the flux tilt in two-energy-group and the eigenvalue separation is presented through the derivation procedure. The results by the experiments and the analyses of experiments, on inferring the eigenvalue separation, are presented in Sec. 3. Finally, the conclusion of this paper is summarized in Sec. 4.

2. FLUX TILT IN TWO-ENERGY-GROUP AND FIRST-MODE EIGENVALUE SEPARATION

In the MHP method, the perturbed flux is assumed to be expressed by using the λ-mode eigenvalues and eigenfunctions in unperturbed system. In the neutron diffusion theory in two-energy-group without the up-scattering from the thermal group to the fast group, the λ-mode eigenvalue problem in the unperturbed system can be expressed as follow:

$$\begin{pmatrix} A_{1,1} & 0 \\ A_{2,1} & A_{2,2} \end{pmatrix} \begin{pmatrix} \varphi_{l,1} \\ \varphi_{l,2} \end{pmatrix} = \mu_l \begin{pmatrix} M_{1,1} & M_{1,2} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \varphi_{l,1} \\ \varphi_{l,2} \end{pmatrix} , \tag{1}$$

where $\varphi_{l,g}$ indicates the l-th expansion mode eigenfunction of the g-th energy group in the unperturbed system, and μ_l indicates the inverse of eigenvalue of the l-th expansion mode in the unperturbed system.

In two-energy-group, the perturbed flux ϕ_g' in the g-th energy-group can be expressed as follow:

$$\underline{\phi}' = \begin{pmatrix} \phi_1' \\ \phi_2' \end{pmatrix} = \begin{pmatrix} \phi_{0,1} \\ \phi_{0,2} \end{pmatrix} + \sum_{n=1}^{N} \sum_{l=0}^{L} \begin{pmatrix} a_{l,1}^{(n)} \varphi_{l,1} \\ a_{l,2}^{(n)} \varphi_{l,2} \end{pmatrix} = \begin{pmatrix} \left(1 + \sum_{n=1}^{N} a_{0,1}^{(n)}\right) \varphi_{0,1} \\ \left(1 + \sum_{n=1}^{N} a_{0,2}^{(n)}\right) \varphi_{0,2} \end{pmatrix} + \sum_{n=1}^{N} \sum_{l=1}^{L} \begin{pmatrix} a_{l,1}^{(n)} \varphi_{l,1} \\ a_{l,2}^{(n)} \varphi_{l,2} \end{pmatrix} , \quad (2)$$

where $\phi_{0,g}$ indicates the fundamental-mode flux in the unperturbed system, and $a_{l,g}^{(n)}$ indicates the expansion coefficient in the l-th expansion mode and the n-th perturbation order.

Here, by using the perturbed flux ϕ_g' in the g-th energy-group, one can derive the following equation of the flux tilt τ_{1+2} in two-energy-group, which is the extension to the two-energy-group version from the ordinary formulation of the flux tilt in one-energy-group defined by R. A. Rydin *et al.* [1], and it includes the effects of the neutron energy dependence:

$$\tau_{1+2} = \frac{\left\langle (\alpha + \delta\alpha)\,\phi_1' + (\beta + \delta\beta)\,\phi_2' \right\rangle_L - \left\langle (\alpha + \delta\alpha)\,\phi_1' + (\beta + \delta\beta)\,\phi_2' \right\rangle_R}{\left\langle (\alpha + \delta\alpha)\,\phi_1' + (\beta + \delta\beta)\,\phi_2' \right\rangle_{L+R}} , \quad (3)$$

where α and β indicate arbitrary constants of the fast and thermal groups in the unperturbed system, respectively. Moreover, $\langle\ \rangle_L$, $\langle\ \rangle_R$ and $\langle\ \rangle_{L+R}$ are space integrals over the left, the right and both core regions, respectively, in the coupled-core as shown in Fig. 1.

Assuming that the total values $\alpha + \delta\alpha$ and $\beta + \delta\beta$ in perturbed system are equal to the reference values α and β in the unperturbed system, respectively, Eq. (3) can be approximately expressed as follow:

$$\tau_{1+2} = \frac{\left\langle \alpha\phi_1' + \beta\phi_2' \right\rangle_L - \left\langle \alpha\phi_1' + \beta\phi_2' \right\rangle_R}{\left\langle \alpha\phi_1' + \beta\phi_2' \right\rangle_{L+R}} . \quad (4)$$

Substituting Eq. (2) into Eq. (4) and using the higher-mode and higher-order reactivity expression defined by the MHP method with two-energy-group, the higher-mode and higher-order formulation of the flux tilt τ_{1+2} in two-energy-group can be expressed as follow:

$$\tau_{1+2} = \frac{\langle \alpha\phi_1' + \beta\phi_2' \rangle_L - \langle \alpha\phi_1' + \beta\phi_2' \rangle_R}{\langle \alpha\phi_1' + \beta\phi_2' \rangle_{L+R}}$$

$$= \sum_{l=1}^{L} \Big[\sum_{n=1}^{N} \frac{1}{(E.S.)_l} \Big\{ -\frac{\mu_l \langle \varphi_{l,1}^{+}, M_{1,2}\,\varphi_{l,2} \rangle}{\langle \varphi_{l,2}^{+}, A_{2,2}\,\varphi_{l,2} \rangle} \langle \varphi_{l,2}^{+}, (\delta A_{2,1}\,\phi_1^{(n-1)} + \delta A_{2,2}\,\phi_2^{(n-1)}) \rangle - \langle \varphi_{l,1}^{+}, \delta A_{1,1}\,\phi_1^{(n-1)} \rangle$$

$$+ \Big\langle \varphi_{l,1}^{+}, \sum_{i=1}^{n} \mu^{(i-1)} (\delta M_{1,1}\,\phi_1^{(n-1)} + \delta M_{1,2}\,\phi_2^{(n-1)}) \Big\rangle + \Big\langle \varphi_{l,1}^{+}, \sum_{i=1}^{n-1} \mu^{(i)} (M_{1,1}\,\phi_1^{(n-i)} + M_{1,2}\,\phi_2^{(n-i)}) \Big\rangle \Big\}$$

$$\times \Big(\frac{\mu_l}{\mu_0}\Big) \times \Big(\frac{\langle \alpha\varphi_{l,1} + \beta\varphi_{l,2} \rangle_L - \langle \alpha\varphi_{l,1} + \beta\varphi_{l,2} \rangle_R}{\langle \alpha\varphi_{0,1} + \beta\varphi_{0,2} \rangle_{L+R}} \Big) \Big] \quad , \tag{5}$$

where $\mu^{(i)}$ indicates the inverse of eigenvalue of the *i*-th perturbation order in the perturbed system, and $\varphi_{l,g}^{+}$ indicates the adjoint eigenfunction of the *l*-th expansion mode and the *g*-th energy group in the unperturbed system. Moreover, $(E.S.)_l$ indicates the eigenvalue separation in the *l*-th expansion mode and is defined as the following equation in accordance with W. M. Stacey Jr.[9]:

$$(E.S.)_l = \mu_l - \mu_0 \quad . \tag{6}$$

Applying the first-order approximation ($N=1$) to Eq. (5), and taking into account only the first-mode expansion approximation ($L=1$) in Eq. (5), which is considered to be excited more intensely than any other higher-mode in the core, the flux tilt τ_{1+2} in two-energy-group can be expressed as follow:

$$\tau_{1+2} = \frac{1}{(E.S.)_1} \Big\{ -\frac{\mu_1 \langle \varphi_{1,1}^{+}, M_{1,2}\,\varphi_{1,2} \rangle}{\langle \varphi_{1,2}^{+}, A_{2,2}\,\varphi_{1,2} \rangle} \langle \varphi_{1,2}^{+}, (\delta A_{2,1}\,\varphi_{0,1} + \delta A_{2,2}\,\varphi_{0,2}) \rangle - \langle \varphi_{1,1}^{+}, \delta A_{1,1}\,\varphi_{0,1} \rangle$$

$$+ \langle \varphi_{1,1}^{+}, \mu^{(0)} (\delta M_{1,1}\,\varphi_{0,1} + \delta M_{1,2}\,\varphi_{0,2}) \rangle \Big\} \times \Big(\frac{\mu_1}{\mu_0}\Big) \times \Big(\frac{\langle \alpha\varphi_{1,1} + \beta\varphi_{1,2} \rangle_L - \langle \alpha\varphi_{1,1} + \beta\varphi_{1,2} \rangle_R}{\langle \alpha\varphi_{0,1} + \beta\varphi_{0,2} \rangle_{L+R}} \Big) \quad . \tag{7}$$

Considering that, in the unperturbed system, the fundamental-mode eigenfunction $\varphi_{0,g}$ is initially symmetric while the first-mode eigenfunction $\varphi_{1,g}$ is asymmetric (refer to Fig. 2), the numerator of the last term in Eq. (7) can be expressed as follow:

$$\begin{aligned} \langle \alpha\varphi_{1,1} + \beta\varphi_{1,2} \rangle_L - \langle \alpha\varphi_{1,1} + \beta\varphi_{1,2} \rangle_R &= \langle | \alpha\varphi_{1,1} + \beta\varphi_{1,2} | \rangle_{L+R} \\ &= \langle \alpha | \varphi_{1,1} | + \beta | \varphi_{1,2} | \rangle_{L+R} \quad . \end{aligned} \tag{8}$$

Assuming that the core is a decoupled system where neutron coupling is very loose and the first-mode is very close to the fundamental-mode in the core region (refer to Fig. 2), the relationship can be obtained as follow:

$$\left\langle \left| \varphi_{1,g} \right| \right\rangle_{L+R} \approx \left\langle \varphi_{0,g} \right\rangle_{L+R} \quad . \tag{9}$$

Applying Eq. (9) to Eq. (8), it can be expressed as follow:

$$\left\langle \alpha\varphi_{1,1} + \beta\varphi_{1,2} \right\rangle_L - \left\langle \alpha\varphi_{1,1} + \beta\varphi_{1,2} \right\rangle_R \approx \left\langle \alpha\varphi_{0,1} + \beta\varphi_{0,2} \right\rangle_{L+R} \quad . \tag{10}$$

In the MHP method, l-th expansion mode reactivity ρ_l excited by the perturbation can be expressed as follow in accordance with D. C. Wade *et al.* [10] :

$$\rho_l = \frac{\mu_l \left\langle \varphi_{l,1}^{+}, M_{1,2}\,\varphi_{l,2} \right\rangle}{\left\langle \varphi_{l,2}^{+}, A_{2,2}\,\varphi_{l,2} \right\rangle} \left\langle \varphi_{l,2}^{+}, (\delta A_{2,1}\,\phi_1^{(n-1)} + \delta A_{2,2}\,\phi_2^{(n-1)}) \right\rangle + \left\langle \varphi_{l,1}^{+}, \delta A_{1,1}\,\phi_1^{(n-1)} \right\rangle$$
$$- \left\langle \varphi_{l,1}^{+}, \sum_{i=1}^{n} \mu^{(i-1)} (\delta M_{1,1}\,\phi_1^{(n-1)} + \delta M_{1,2}\,\phi_2^{(n-1)}) \right\rangle - \left\langle \varphi_{l,1}^{+}, \sum_{i=1}^{n-1} \mu^{(i)} (M_{1,1}\,\phi_1^{(n-i)} + M_{1,2}\,\phi_2^{(n-i)}) \right\rangle \ , \tag{11}$$

applying Eq. (11) to Eq. (7), and using ρ_0 and ρ_1 for $l=0$ and $l=1$ in Eq. (11), respectively, the flux tilt τ_{1+2} in two-energy-group can be expressed as follow:

$$\tau_{1+2} \approx \frac{1}{(E.S.)_1} \times |\rho_0| \times \frac{|\rho_1|}{|\rho_0|} \times \frac{\mu_1}{\mu_0} \ . \tag{12}$$

Apparently, the flux tilt τ_{1+2} in two-energy-group can be expressed as seen in Eq. (12) without arbitrary constants α and β. Assuming that the first-mode adjoint eigenfunction is close to the fundamental-mode one in the core region as well as Eq. (9), the absolute value of the first-mode reactivity $|\rho_1|$ can be approximately expressed by that of the fundamental-mode reactivity $|\rho_0|$. Moreover, the value of μ_1/μ_0 in Eq. (12) can be approximately considered to be 1, since the first-mode and the fundamental-mode eigenvalues are close with each other in the decoupling system. Finally, applying these above approximation conditions to Eq. (12), one can obtain the following relationship between the flux tilt in τ_{1+2} in two-energy-group and the first-mode eigenvalue separation $(E.S.)_1$:

$$\tau_{1+2} \approx \frac{|\rho_0|}{(E.S.)_1} \ . \tag{13}$$

Through the derivation of the flux tilt τ_{1+2} in two-energy-group based on the MHP method, it was found that the flux tilt τ_{1+2} in two-energy-group is proportional to the fundamental-mode reactivity ρ_0 excited by the perturbation and inversely proportional to the first-mode eigenvalue separation $(E.S.)_1$, which is the same as the formulation of the flux tilt based on the one-energy-group perturbation method [8] .

3. RESULTS BY EXPERIMENTS AND NUMERICAL CALCULATIONS

3.1 INVESTIGATED COUPLED-CORE

For evaluating the flux tilt in two-energy-group and the eigenvalue separation, experiments were carried out in the coupled-core at the KUCA C-core, which is a water moderated and reflected core with a rectangular geometry shown in Fig. 1.

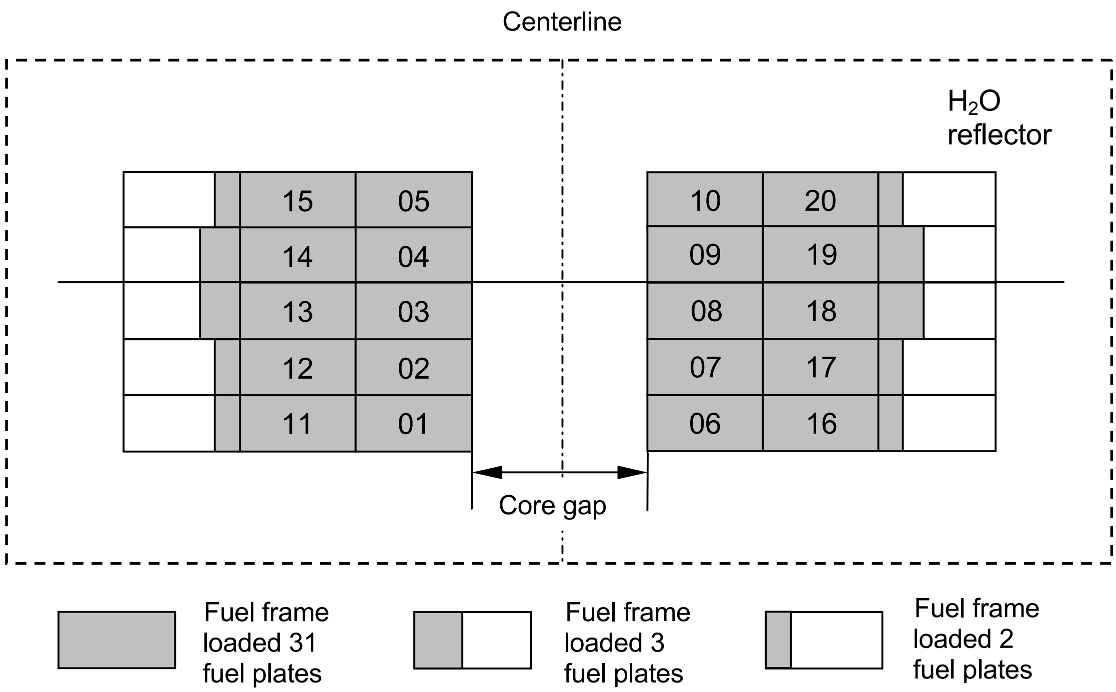

Fig. 1 Top view of investigated coupled-core configuration at the KUCA C-core.

For the experiments, the attention is paid to the case where an asymmetric perturbation is locally inserted in the coupled-core that is considered to be initially symmetric to the core center shown in Fig. 1 and the neutron flux distribution is distorted more intensely in one half of the core than the other, since the coupled-core can be regarded as a typical system where a large flux tilt is caused by the asymmetric perturbation inserted locally. In these experiments, to induce the flux tilt, the perturbation was asymmetrically introduced by the withdrawal of fuel plates, according to patterns as shown in Table I.

Table I
Patterns of fuel plates withdrawal in the coupled-core at the KUCA C-core.

Case name	Number of fuel plates withdrawal	Contents
Case 1	5	Withdrawal of each 4-th fuel plate from the fuel frames [06] through [10] of the opposite side to the core gap, respectively.
Case 2	2	Withdrawal of the 3-rd and 4-th fuel plates from the fuel frame [08] of the opposite side to the core gap.
Case 3	4	Withdrawal of the 3-rd and 4-th fuel plates from the fuel frames [08] and [09] of the opposite side to the core gap, respectively.
Case 4	4	Withdrawal of the 1-st and 2-nd fuel plates from the fuel frame [09] of the opposite side to the core gap and of the 1-st and 2-nd fuel plates from fuel frame [19] of the closer side to the core gap.

[ ] : Name of fuel frame shown in Fig.1

The value of experiment $(E.S.)_1^{Exp}$ was inferred by both the fundamental-mode reactivity ρ_0^{Exp} and the degree of flux tilt τ^{Exp}. The reactivity ρ_0^{Exp} was measured by the excess reactivity of the reactor cores before and after the withdrawal of fuel plates, and the degree of flux tilt τ^{Exp} was obtained from the measured reaction rate distribution. Moreover, as shown in Figs. 3 through 5, the reaction rate distribution along a horizontal solid line shown in Fig. 1 was measured by using an optical fiber mounted with a mixture of LiF and ZnS(Ag) scintillator [11].

Subsequently, numerical calculations were performed by inferring the first-mode eigenvalue separation $(E.S.)_1^{Cal}$ from both the fundamental-mode reactivity ρ_0^{Cal} and the degree of flux tilt τ^{Cal} ($= \tau_2$). The perturbed reactivity ρ_0^{Cal} was obtained from the results of eigenvalue calculations executed for the perturbed and unperturbed systems, and the degree of flux tilt τ^{Cal} was obtained from the calculated reaction rate distribution along the horizontal direction shown in Fig. 1.

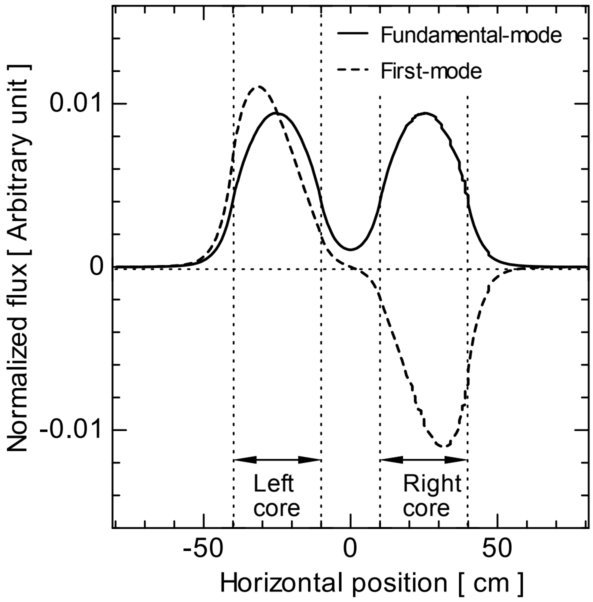

Fig. 2
Calculated neutron flux distribution in the fundamental-mode and first-mode, respectively, of the fast group for the coupled-core at the KUCA C-core along the horizontal solid line shown in Fig. 1 before the perturbation. (The neutron flux distribution is normalized as $<\varphi_l,\varphi_l>=1$.)

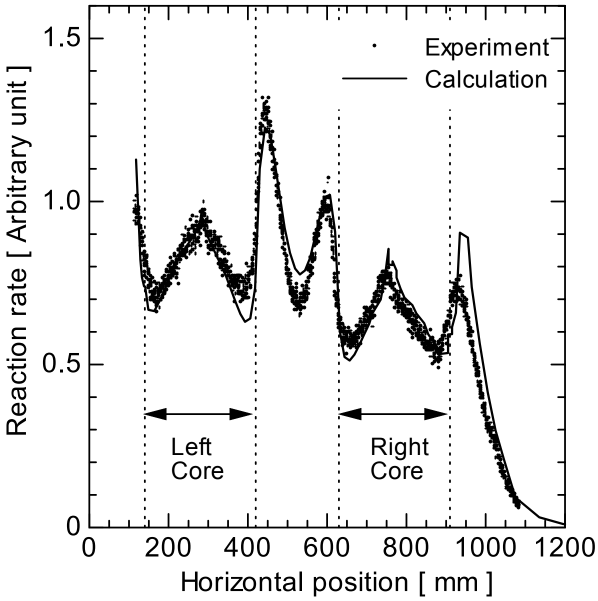

Fig. 3
Calculated reaction rate in the coupled-core at the KUCA C-core after the perturbation of Case 2 shown in Table I along the horizontal direction shown in Fig.1.

3.2 RESULTS BY EXPERIMENTS AND NUMERICAL CALCULATIONS

The results of the reactivity ρ_0^{Exp} and ρ_0^{Cal} are shown in Table II. As shown in Table II, comparing the result of the numerical calculation with two-energy-group in 2-D by SRAC 95 [12] with that of the experiment, it was observed that the relative difference in the reactivity worth between the experiment and the numerical calculation is less than 7%. Moreover, as shown in Figs. 3 through 5, it was observed that the calculated reaction rate distribution approximately agrees with that of the experiment, excluding reflector regions in the right- and left-halves and the core gap, and reproduces exactly the change measured in the experiment, which occurred in the right half of the core by the withdrawal of fuel plates.

From these results, it was considered that the numerical calculations by both the reaction rate distribution and the eigenvalue of cores before and after the withdrawal of fuel plates approximately reproduced the results of experiments.

Table II
Comparison of perturbed reactivity by the experiments and the numerical calculations for perturbations.

Perturbation	ρ_0^{Exp} (%Δk/k)	ρ_0^{Cal} (%Δk/k)	Difference (%)
Case 1	-4.327×10^{-2}	-4.624×10^{-2}	4.8
Case 2	-6.049×10^{-2}	-6.351×10^{-2}	6.9
Case 3	-9.320×10^{-2}	-9.806×10^{-2}	5.2
Case 4	-12.340×10^{-2}	-13.133×10^{-2}	6.4

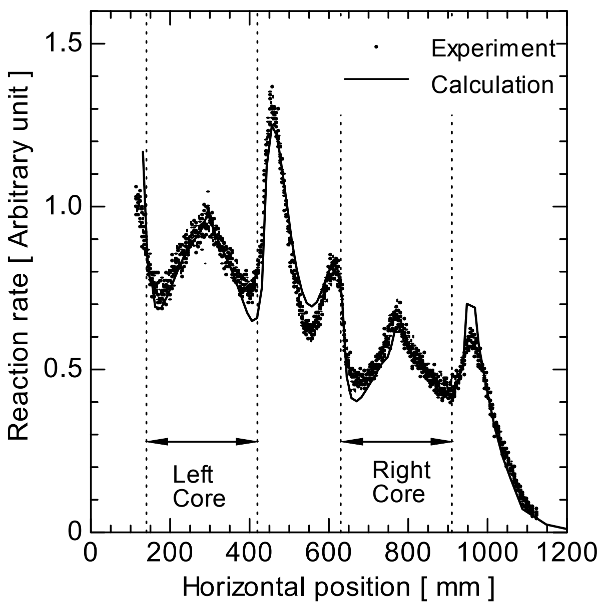

Fig. 4
Calculated reaction rate in the coupled-core at the KUCA C-core after the perturbation of Case 3 shown in Table I along the horizontal direction shown in Fig. 1.

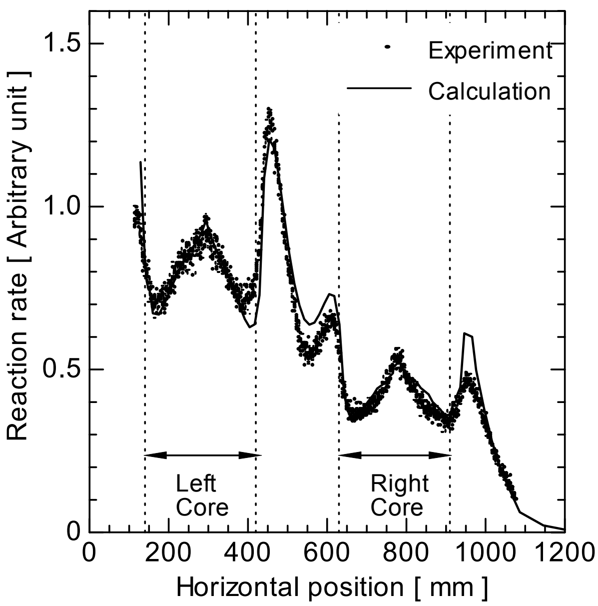

Fig. 5
Calculated reaction rate in the coupled-core at the KUCA C-core after the perturbation of Case 4 shown in Table I along the horizontal direction shown in Fig. 1.

The results of the eigenvalue separation obtained as $(E.S.)_1^{Exp}$ and $(E.S.)_1^{Cal}$ are shown in Tables III. As shown in Table III, the relative difference between the results of the experiments $(E.S.)_1^{Exp}$ and the numerical calculations $(E.S.)_1^{Cal}$ becomes less than 5% at each case. Moreover, in Table III, one can examine the neutron energy dependence of the flux tilt by comparing $(E.S.)_1^{\tau_{1+2}}$ with $(E.S.)_1^{\tau_1}$ and $(E.S.)_1^{\tau_2}$ (= $(E.S.)_1^{Cal}$). As shown in Table III, the relative difference between $(E.S.)_1^{\tau_{1+2}}$ and either $(E.S.)_1^{\tau_1}$ or $(E.S.)_1^{\tau_2}$ is found to be within 1% for all the cases, regardless of the location of the perturbation inserted.

From these results, it was concluded that the validity of the relationship shown in Eq. (13) is confirmed through the experiments and the analyses of the experiments, and that the flux tilt is independent on the neutron energy in these decoupling cores.

Table III
Comparison of the eigenvalue separation between the experiments and the numerical calculations inferred from the flux tilt obtained by the reaction rate distribution for perturbations.

Perturbation	$(E.S.)_1^{Exp}$ (%Δk/k)	$(E.S.)_1^{Cal}$ (%Δk/k)	$(E.S.)_1^{\tau_{1+2}}$ (%Δk/k)	$(E.S.)_1^{\tau_1}$ (%Δk/k)
Case 1	0.426	0.443 (4.0%)	0.443 (3.9%)	0.442 (3.8%)
Case 2	0.454	0.465 (2.4%)	0.464 (2.2%)	0.461 (1.5%)
Case 3	0.415	0.428 (3.1%)	0.428 (3.0%)	0.425 (2.4%)
Case 4	0.387	0.404 (4.3%)	0.404 (4.3%)	0.404 (4.2%)

( ) : Relative difference between the experiments and numerical calculations.

4. CONCLUSION

A formulation of flux tilt in two-energy-group is briefly presented by using the MHP method with two-energy-group. To show the validity of the formulation, the experiments were carried out in the coupled-core at the KUCA C-core. The following conclusions were obtained through the present study:

1. Through the derivation of the flux tilt in two-energy-group, it was found that the flux tilt in two-energy-group is proportional to the perturbed reactivity and inversely proportional to the first-mode eigenvalue separation.
2. The validity of the formulation was demonstrated by the experiments and the analyses of experiments in the coupled-core at the KUCA C-core for the withdrawal of fuel plates.
3. The neutron energy dependence of the flux tilt was not found clearly in the experiments and the analyses of experiments.

Therefore, through the experiments and the analyses of experiments, it was concluded that the conventional methodology on the basis of the one-energy-group perturbation method is sufficiently valid for inferring the first-mode eigenvalue separation from the degree of the flux tilt and the perturbed reactivity.

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